range of hundreds of milliseconds which have been reported recently (Jaeger & Bower 1994; Larson-Prior et al. 1990) would require respective values of Δx of 52 and 520 mm (assuming a length of the EPSP of 100 msec).

Conclusion. In order to formulate a general theory, the distance that the *smallest* cerebelli in regions of their *smallest* laterolateral extent can accommodate for the build-up of tidal waves has to be considered. A small cerebellum (e.g., that of the bat or the mouse which are demonstrated in Figs. 1 and 2 of the target article) displays laterolateral extensions as small as 1 mm (e.g., in posterior regions which readily receive cerebral inputs via pontocerebellar fibers, see for review Brodal & Bjaalie 1992). Thus, as stated above, these extensions support just two sequential cortical inputs based on the most optimistic assumptions about their temporal precision. Longer sequences consisting of many successive inputs, which seem to be desirable, are not supported by these regions. Furthermore, the requisite length for achieving a satisfactory level of sequential activity in the output of a given tidal wave is not reached at all, even if one assumes the most opportune values of Δx as given above. Therefore, based on the present considerations, the prerequisites for the generation of tidal waves are not offered in every region of small cerebelli. To make the theory acceptable as a general one (which has to be provided in view of the highly preserved cerebellar morphology across species and across cerebellar regions), the precision in the cerebellar inputs as well as the precision of Purkinje cells in the detection of synchronous inputs must be shown to be substantially higher than has hitherto been reported.

The job description of the cerebellum and a candidate model of its "tidal wave" function

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Abstract: A path space integral approach to modelling the job description of the cerebellum is proposed. This new approach incorporates the "tidal wave" equation into a kind of generalized Huygens's wave equation. The resulting exponential functional integral provides a mathematical expression of the inhibitory function by which the cerebellum "sculpts" the intended control signal from the background of neuronal excitation.

The job of the cerebellum. In a nutshell, the job of the cerebellum involves a distribution of well-timed inhibitory volleys that has been described as a process that, without benefit of positive feedback (target article, sect. 4), "sculpts" the motor sequences out of background excitation (sect. 25). Furthermore, it does so in keeping with the cerebrally stipulated intention and the requirements of the perceptually informed physics of the action system. The "sculpting" function is envisaged as a unidirectional sum of local "tidal wave" effects acting transversely over neighboring folia that each contain "beams" of neural impulse in their parallel fibers. These effects reach maximal amplitude when the speed of a neural impulse approximates the conduction rate of the parallel fibers, diminishing otherwise, and virtually disappearing if the signal moves counter-directionally. The formal description offered here purports to capture the chief characteristics of the function that sculpts out of the spatio-temporal distribution of all possible motor control signals the intended motor control signal.

First, for a most natural description, the sculpting function requires a functional integral formulation. As it now stands, with continuous variations in all parameters, the tidal wave equation (sect. 14) implies a system of differential equations that is potentially infinite in number (Kramer 1970).

Second, the fact that the distribution function sculpts away from the excitation the control signal needed, suggests a need for a generalized form of Huygens's famous principle of constructive and destructive wave interference (where his time variable is replaced by Lagrange's action variable) (Lanczos 1970).

Third, because the cerebellum must accommodate a potentially astronomical number of such sculpting functions, all possible paths through the lattice-like structure of the cerebellum are needed. Selection of the proper control signal by the cerebellum (as constrained by perception, with or without learning) entails solving a two-point boundary problem (e.g., the time interval for serial volleys crossing the folia, sect. 9); this involves a kind of "least action" principle, similar to variational solutions (but see caveat in commentary by Kadar et al., this issue). Fortunately, a functional integral approach for solving such problems exists. It is called *a Feynman path space integral* (Feynman & Hibbs 1965), and is our candidate for modeling the proposed job of the cerebellum; moreover, its use in perceptually informed motor control tasks has already been introduced (Shaw et al. 1994).

Formulating that job with path space integrals. The equation for the inhibitory functional distribution that will sculpt away all excitation except the intended path (normalized here to be the stationary path solution) takes the following generic form:

$$K(b, a) = \lim_{\varepsilon \to 0} \frac{1}{k} \int \int \mathsf{L} \int e^{(i/\alpha)S[b,a]} \frac{dx_1}{k} \frac{dx_2}{k} \mathsf{L} \frac{dx_{N-1}}{k}$$

$$= k \int_a^b e^{iS/\alpha} Dx(t) \tag{1}$$

This is a path space integral that defines all the possible paths in the space-time distribution, $Dx(t)$ (for a vector function of 1, 2, or 3 spatial dimensions), from an initial time site $a = x_0(t_0)$ to a final time site $b = x_f(t_f)$. Here the "tidal wave" term (sect. 14) is inserted into the exponent to redefine the (Lagrangian) action term, $S = Axt$, and k is the normalizing factor to keep the series from diverging (Feynman & Hibbs 1965). The constant α refers to the smallest unit of neural action possible (possibly related to the refraction period of conduction in the parallel fibers) – descending on Planck's constant if the volleys are quantum effects and on zero if beams approximate continuous functions. Perhaps, better still for neural purposes is to define α as an elementary Gabor function, $\alpha = a$ *logon unit* (Pribram 1991).

Two caveats. A. If complex number solutions are not needed, Equation (1) can be transformed into a real number (Brownian) path space functional by a Wick rotation, according to the Feynman-Kac Theorem, by replacing t by it in the exponential (Roestorff 1994). However, if the quantum nature of the brain is established (e.g., vis-à-vis micro tubules), probability amplitudes might be needed to handle quantum interfering paths, demanding both the real and complex conjugate solutions to Equation (1) (Jibu & Yasue 1995). B. To model the cerebellum as a lattice of orthogonal inhibitory and excitatory dimensions (sect. 3), Equation (1) can be rewritten for physics of "zig-zag" paths on a lattice (Roestorff 1994).