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16. The Role of Symmetry in Event Perception

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Symmetry in the Laws of Nature

Ernst Cassirer (1944), in his article discussing the role of group theory in perception, asserts that the primitive form of understanding is that of the intuitive concept of a *group*. The usefulness of the group concept in contemporary mathematics and theoretical physics offers strong support to the validity of this insight. One of the chief functions of group theory in mathematics and physics has been to describe what properties of objects, events, or even natural laws remain invariant, or symmetrical, across different domains, or under modification by transformations.

In geometry, Felix Klein, in pursuing his Erlangen program, succeeded in showing that different geometries can be represented by specific groups of transformations which leave only certain geometric properties of objects invariant when applied. Consequently, Herman Weyl provided a very simple definition of the term "invariant," indicating that it is merely a synonym of the term "symmetrical," namely, that *a thing is symmetrical if there is something we can do to it so that after we have done it, it appears the same as it did before.*[2] The precise statement of what the thing is, and what can be done to it without changing its essential form, has been the main task of symmetry group theory. In spite of the apparent simplicity of the concept of group symmetry, its explanatory power in science has been truly surprising.

Physicists have succeeded in discovering a broad class of symmetry operations under which physical phenomena (for example, objects, events, and even laws) remain invariant. For instance, a physical phenomenon can be *translated* in space or over time without destroying the identity of that phenomenon. It is for this reason we can say that the laws of physics hold throughout the universe, given, of course, that we satisfy certain initial conditions which control for the effects of nonessential variables due to the change in locale. Other well-known symmetry operations in physics are rotation through a fixed angle, uniform velocity in a straight line, reversal of time, reflection in space, interchange of identical atoms or particles, quantum-mechanical phase, and charge conjugation (that is, matter-antimatter).

Wigner (1967), the Nobel laureate in physics, expresses the theoretical value of the symmetry approach to understanding nature in this way: "There is a structure in the laws of nature which we call the laws of invariance. This structure is so far-reaching in some cases that laws of nature were guessed on the basis of the postulate that they fit into the invariance [symmetry] of structuring." One immediately thinks of the examples of Mendeleev predicting the existence of elements on the basis of a profound belief in the symmetry of the periodic table of chemical elements; or Kepler stubbornly agonizing for years over Brahe's astronomical data in order to discover the geometric curve (the ellipse) with the highest form of symmetry that might, as he put it, restore "the harmony to the spheres." As Weyl (1952) points out, Einstein's theory of relativity can also be interpreted as an attempt to restore symmetry to the laws of nature, a symmetry not preserved under Newton's laws of mechanics.

[2] The key idea of this approach is that usually in a mapping of a set A on a set B some geometric properties of the objects in A are also properties of their images in B, whereas other properties in A are not possessed in B. Hence we say that each mapping *preserves* some properties, or leaves them *invariant*, and that it fails to preserve or *destroys*, others.

For instance, if a circle is reflected over a diameter where P', Q' are the images of any two points P, Q then distance $|P'Q'| - |PQ|$, and the mapping is said to be a *symmetry* operation which *preserves distance or leaves distance invariant*. On the other hand, if the order of points (P, Q, R) on the circle read clockwise, then the order of their images (R', Q', P') will be *counterclockwise*, and the operation of reflection is not a symmetry operation with respect to order since order does not remain invariant. A rotation of the circles, however, is a symmetry operation since both order and distances remain invariant after the application of the operation.

Wigner even goes so far as to argue that laws of nature could not exist without principles of invariance (the symmetry postulate).

Might we not also expect then that the concept of symmetry will figure intimately in our theories of psychology? What form might the symmetry postulate assume in general cognitive theory to which perceptual theory belongs as a particular mode of epistemic adaptation? This paper is an attempt to explore some particular aspects of the general problem of how organisms attain relatively persistent perceptual adaptation to their environment. Our first attempt to provide a symmetry postulate for cognitive theory can be tentatively stated as follows: *An organism possesses the highest degree of adaptation to its environment when the greatest degree of symmetry exists between its states (both biological and psychological) and the states of its environment.* (See Shaw and McIntyre, *in press.*)

In what follows, we shall attempt to discover both the theoretical as well as empirical implications of this assumption. In particular, we attempt to make the symmetry postulate the basic postulate of Gibson's theory of perception as the direct pick-up of information.

Adaptation as the Dynamic Expression of Symmetry

Mach (1902) once observed that "in every symmetrical system every deformation that tends to destroy the symmetry is complemented by an equal and opposite deformation that tends to restore it." The law of entropy in thermodynamics, homeostasis in many biological mechanisms, adaptation in the evolution of species, and epistemic adaptation in psychology (that is, an organism's attainment of knowledge and skills in dealing with events sufficient to meet the needs of health and survival) are all manifestations of Mach's principle.

Mach goes on to explain how such equilibria exist as dynamic expressions of the symmetry structure of nature: "One condition, therefore, though not an absolutely sufficient one, that a maximum or minimum of work corresponds to the form of equilibrium, is thus supplied by symmetry. Regularity is successive symmetry. There is no reason, therefore, to be astonished that the forms of equilibrium are often symmetrical and regular."

The key to understanding the application of Mach's principle for psychology is to see that where the existence of equilibria among the states of an energy system implies a symmetry of forces within the system, so the existence of equilibria among the states of a psychological system similarly implies a symmetry of information within the system. The domain of application of Mach's principle apparently ranges from physical systems through biological ones to psychological ones. Consider the following illustrations of the way in which various equilibria states imply a symmetry among energetic as well as informational states. A spinning top remains in a state of balance (equilibrium) so long as there exists a symmetry between the gravitational force tending to upset the top and the rotational forces which tend to lift the top. Biological homeostases can be described in a similar fashion. For instance, the temperature of a mammal's body remains invariant in spite of changes in environmental temperatures so long as a symmetry is maintained between the climatic changes in heat around the body and the body's oxidative reactions (for example, peripheral vasoconstriction and piloerection, shivering) which reduce heat irradiation, or the contrary processes which increase heat irradiation from the body (for instance, peripheral vasodilation, perspiring). The argument to be made is that, presumably, psychological adaptation to an environment of events, that is, how an organism learns what it needs to know about its world in order to maintain its health and well-being, similarly requires that a symmetry relation somehow hold between the information resource states of the environment and the states of the cognitive structures by which such needed information can be processed.

Neither, then, as psychologists, should we be surprised to find that the laws of psychological adaptation also turn out to be merely another expression of natural symmetry—the symmetry, perhaps, between events and an organism's experiences of them.

A psychology of events and experiences. To rephrase Wittgenstein, the world is a totality of events and not of things. The environment of any organism is in dynamic process so that the smallest significant unit of ecological analysis must be an *event* rather than a simple stimulus, object, relation, geometric configuration, or any other construct whose essence can be captured in static terms. Each terrestrial event that occurs is a distal source of potential information about itself as a local phenomenon as well as about the global properties of the total environment. Similarly, each organism's experience of events is a proximal source of information about itself and its biographical place in that environment.

An ecologically based cognitive psychology, as envisioned here, should be the study of *events, experiences,* and the *adaptive relationships* between

the two. By analyzing the organism's context of physical stimulation into events with adaptive significance, we have a means of conceptually distilling from the ambient flux of stimulation those aspects most relevant to the maintenance of equilibrium in the organism's ecosystem. Similarly, by analyzing the organism's life into significant experiences, we have a means of emphasizing those aspects that contribute most to his adaptation.

The basic assumption of this approach is that ecologically significant events will be accompanied by ecologically significant experiences. By an ecologically significant event or experience we mean those events or experiences which possess sufficient efficacy to significantly modify the adaptive life style of the organism if they but occur or are omitted. The sign or value of such events and experiences can, of course, be either positive (supportive) or negative (damaging), such as the event of falling off a cliff or the experience of vertigo that prompts a hasty retreat from the cliff's edge.

Experiences are logically coextensive with events. The relation between the two is direct rather than mediated. An event is not the cause of an experience nor the experience a response to an event. (One experience may be, however, a consequence of a prior experience, just as one event may be a consequence of a prior event.) Experiences and events accompany one another as different but equally important aspects of the same dynamic phenomenon in the ecosystem—that is, as the terms of the relationship between organism and environment. To paraphrase the distinction made by Bertrand Russell: An *event* is the experience described from the point of view of the environment, while an *experience* is the event described from the point of view of the organism.

The concept of ecological information. Physics provides the necessary principles for describing the energy distributions determined by any event. It does not, however, attempt to provide principles sufficient to account for those aspects of energy distributions which are adaptively significant to a given organism. Biology, and its kindred sciences, attempts to discover the spectra of energy forms to which the sensory systems of organisms respond. But even here there is no attempt systematically to determine what the various patterns of stimulation "mean" to the organism, that is, what ecologically significant properties of events composing its world may be specified by them. As necessary as these two levels of analysis are, they still leave unanswered much that must be known if psychological adaptation is to be understood.

In addition to knowing the energy forms to which the perceptual systems of organisms may be genetically 'pre-attuned," or to which they may become attuned by experience, we also need to know what information about the environment is *specified* by the stimulation made available to organisms by the events. If the information specifies nontrivial properties which facilitate an organism's achievement or maintenance of an adaptive relationship with its environment, then we call it *ecological* information.

The Affordance Structure of Events

It would, however, be convenient to have a single concept by which to denote those types of ecological information which play a prominent role in adaptation. Gibson has provided such a concept in his notion of *affordances* of events. The affordances of events are those invariant properties which imply directly the meaningful dimensions of interaction an organism might have with its world. An important implication of Gibson's theory of *direct* realism for perceptual theory is the hypothesis that whatever organisms perceive about events (or objects as limiting cases of events) is directly perceived *vis à vis* the invariants of ecological information determined by the events. Hence what organisms perceive are the affordances of events rather than their intrinsic physical structures. In this sense affordances can be considered as the potential experiences for an appropriately attuned organism to whom an event occurs.

A further implication (and one that is nothing more than an expression of the postulate of symmetry alluded to earlier) is that a scientific understanding of the affordance structure of events provides at the same time an objective analysis of the experience structure of any organism perceiving the event. This means that a study of events, at least in principle, can also be a study of mind. If so, then we might expect that the invariant properties of events to which nature and/or experience have adaptively attuned organisms will exist among the significant symmetries of events. For instance, the invariant physical properties of substances that render them relatively *rigid* might specify that they are *walk-upon-able*, or those that render them *less rigid* might specify that they are *penetrable*; or invariant properties that render substances *brittle* might also specify their *breakability*.

As we assay the affordance structure of events in general, we might seek experimentally to determine what invariant physical properties carry

over symmetrically into potential experiences. To do this we must ulti- mately determine which properties of events remain invariant under symmetry transformations wrought by nature, the contraints on observa- tion, as well as the perceptual attunement of the organism in question. What invariant optical information, for example, specifies the potential experience of grasping, burning, catching, eating, or even loving an object?

In addition to the affordances of simple events (for example, objects and surfaces), there exist higher forms of ecological information which specify still more complex affordance structures. Later in the paper, in order to illustrate the ecological approach to event perception, we will present several studies in which researchers have tried experimentally to determine the nature of ecological information which invariantly specifies the separation in depth of moving surfaces, the shape and orientation of objects under displacements, as well as the age level of human faces. All of these problems involve the transformation of events over time or space in such a way that the essential identity of the event is preserved (recall Weyl's definition of symmetry).

In all the above cases something essential about the event remains invariant under the transformations, that is, the respective phenomena of depth, shape, and age level must be definable by appropriate symmetry operations. Since we are discussing events which can be perceived as being transforms of previously perceived events, the invariance must be ex- pressed in ecological information. This leads us to assert another working hypothesis that we believe may be the theoretical key to understanding the age-old problems of the perceptual similarity of different events, and the persistent phenomenal identity of transformed events. The hypothesis, although not at all simple, can nevertheless be stated quite simply: *Events are similar to the extent that they share the same affordance structure.*

Attensity: A measure of ecological significance. The view that events can be distinguished in terms of their affordance structure suggests that some dimensions of ecological information made available by events are more important than others to properly attuned organisms. Indeed, if organisms are attuned to pick up ecological information which most reliably specifies the adaptive value afforded by an event, then it follows that they might be more likely to pick up the information for event properties following an order of priority dictated by the degree of signifi- cance of the affordance specified. The properties of events determining ecological information will be said to vary in *attensity* level in direct proportion to their cognitive salience or probability of being attended to.

The term "attensity," therefore, should not be confused with the term "intensity." Intensity is an observer-independent parameter for the measure of physical stimulation (such as how bright a light is in foot candles, or how loud a noise is in decibels), while attensity, on the other hand, is a parameter for measuring the significance of ecological informa- tion, and hence is dependent on the organism's selective attunement to modulate the affordance properties of events in the order most greatly favoring adaptation. In later sections, we will see that the concept of attensity plays an important role in attempts to account for the perceptual organization of events.

Events That Afford the Perception of Layout

A great deal of research has been directed toward so-called "space" perception. As Gibson so rightly points out, we do not perceive spaces, rather we perceive surfaces. Similarly, we do not perceive "depth of space" but the extent of separation between surfaces which are located at dif- ferent places on an opaque, textured, terrestrial ground plane. Gibson (1950) and others have done extensive research on the problem of deter- mining the invariants of optical information which specify the layout of the visual world. His discovery of the role of texture gradients contributed significantly to our understanding of how relatively frozen optic arrays can reliably specify the separation of objects on the terrestrial ground plane.

It is well known, of course, that a single static projection (that is, a sample of a frozen optic array) of a textured or shaded surface is quite insufficient for specifying its slant or the relative distribution of textural inhomogeneities on it, just as a single perspective of an object is not sufficient information for specifying its true shape. In all such cases, more than one perspective is required to specify the orientation or shape of the surface or object to the observer. This means that the observer must be provided with several samples (the exact number or selection depends upon the degree of structural complexity of the object or surface), a condition that is most readily satisfied by nature having made organisms active, investigative perceivers instead of passive receivers of static infor- mation. Since not all animals have conjugate binocular visual systems which are focusable, kinetic samples are the rule rather than the exception.

Regarding the perception of objects, logically it does not matter whether

the observer or the object moves, for in either case the relative motion alters the pattern of adjacent light-contrast values in the optic array sample projected to the observer's point of observation. In such a kinetic optic array sample, the edges of the objects are specified, as is the relative location of the object with respect to other objects included in the environmental sample.

Moreover, if there is relative motion, then it does not matter whether the observer is monocular or binocular since it is also well known that the information picked up through binocular parallax can just as easily be picked up through head-motion parallax. Taking all the above facts into consideration, we can greatly simplify our question regarding the perception of layout by asking what monocular kinetic information is minimally sufficient to specify the separation of surfaces in the environment?

If we are able to discover the minimally sufficient optical information that does so, then we might also attempt to determine if it is necessary information, that is, if it might not prove to be the invariant optical information shared by all situations in which there is a so-called "depth" effect. The empirical answer to these questions would pave the way to the theoretical formulation of a unitary law of layout perception.

Conditions on the Kinetic Separability of Surfaces

In what follows, we will attempt to show that none of the well-known monocular kinetic variables of optical information (for example, accretion and deletion of texture, brightness changes, motion parallax, motion gradients, harmonic motion) known to be sufficient for specifying separation of surfaces are really necessary. We will do this by presenting evidence that far simpler transformations of optical patterns suffice to determine events that afford ecological information which reliably specifies surface separation. In order to understand the success of these simpler cases, three conditions on optical information known to be sufficient to specify separations of surfaces must be discussed in some detail, namely, the *kinetic* condition, the *light-contrast* condition, and the *figural* condition.

The kinetic conditions. One of the best known monocular kinetic "depth" phenomena is the so-called "kinetic depth effect" as investigated by Wallach and O'Connell (1953). Wallach showed that the shadows of rotating geometric objects (for example, various solids, wire figures, and straight pegs), when observed on a back-projection screen, provide

sufficient optical information for the correct identification of the rotating three-dimensional objects. However, when the objects were static their shadows were not sufficient to correctly specify their three-dimensional shape. The important finding here is that observers did not report the elastic transformations of the two-dimensional shadowgraphs, but merely the perception of rigid three-dimensional objects, although the kinetic shadowgraphs afforded optical information for both events. Here seems to be clear evidence that the invariant ecological information specifying the affordance of rigidity which is so important to the specification of still other significant affordances, such as graspability and penetrability, has higher attensity value than the variant optical information specifying two-dimensional elastic forms upon which no other significant affordances depend.

The empirical problem is to design optical displays sufficiently complex to determine the information specifying a given phenomenon but sufficiently simple to be precisely analyzed. This is the proper role of *demonstrations* in perceptual psychology. For instance, in the above case, the Wallach display captures sufficient optical information to specify the *rigidity* of kinetic physical structures as well as their relative depth. The ultimate theoretical problem, however, can only be solved by experimental manipulation of the relevant variables of optical information determined in such demonstrations so as to reveal the invariant properties across such displays sufficient to determine the desired phenomena. This is the proper role of *experimentation* in perceptual psychology. Put more briefly, then, demonstrations provide the dimensions of relevant optical information which determine the phenomenon to be studied, while experiments provide the analysis of that information into the invariant optical structures which specify the phenomenon.

What kinetic condition is necessary for the perception of the rigid three-dimensional shape of the objects in these experiments? Wallach concluded that the essential condition for this effect is a change in at least two spatial dimensions. He states, "Shadows whose only deformation consists in an expansion and contraction in one dimension will look flat," while, "shadows which display contour lines that change their direction and their length will appear as turning solid forms."

A number of years later, Johansson (1964) reported similar findings from his research. While investigating phase relations between changes of length and width of rectangles, he found they were rarely perceived

as undergoing elastic transformations. Changes in length and width of the rectangles were always taken to be due to rigid rotations and translations of the rectangle in depth. Like Wallach, Johansson also concluded that the effective information for rigid three-dimensional motions was simultaneous change in at least two spatial dimensions.

Metzger, in 1934, demonstrated that two-dimensional shadowgraphs projected from a series of vertical pegs revolving on a turntable provided an optical event affording the perception of a three-dimensional configuration. The back projection screen was masked in such a way that the top and bottom of the pegs were hidden from view. Furthermore, the light source was far enough away to approximate an isometric, or so-called "parallel" projection. With such projection the kinetic shadowgraph of the pegs was not seen to undergo perspective change in either length or width as they revolved from near to far relative to the screen.

Wallach recognized that his principle asserting that three dimensionality is due to perspective changes in at least two spatial dimensions could not explain Metzger's kinetic depth phenomenon; but he (Wallach) dismissed it on the grounds that it was a weak and unreliable effect. White and Mueser (1960), on the other hand replicated Metzger's findings and, therefore, concluded that it constituted strong evidence against Wallach's and Johansson's explanation.

There is, however, still a source of projective information that must be considered as a possible explanation for Johansson's, Wallach's and Metzger's kinetic depth phenomena, namely, harmonic motion. Motion projected from a rotating object onto a plane surface (for example, shadowgraph of rotating pegs) is called *harmonic*; if, given any projected point on the plane of projection, it periodically translates back and forth on a linear path and accelerates when moving inward from the end points of its path and decelerates when moving outward from the midpoint of its path. Oscilloscope watchers have known for some years that certain combinations of wave patterns appear to specify rigid surfaces rotating in depth (see, for example, Fisichelli, 1946). These are called Lissajous patterns. All of the kinetic depth phenomena discussed so far involve projected harmonic motion. That such motion is not a necessary condition for the specification of the separation of surfaces in depth becomes clear when we consider still other situations in which kinetic depth effects are obtained, although all projective information has been systematically excluded.

The light-contrast condition. It can be shown that an appropriate configuration of just *four* light-contrast values is sufficient to specify a semitransparent surface through which an opaque surface is seen (Metelli, 1966). Shaw and Knöbil made a computer generated motion picture in which they showed that a systematic linear change in only *two* light-contrast values across a randomly textured display was sufficient to specify a semitransparent surface moving over a static opaque one. If, however, as MacLeod (1940) predicted, the kinetic margin of light-contrast values follows a less steep gradient of change, a penumbra appears which specifies a shadow crossing a surface rather than a partially occluding, semitransparent surface.

Still another depth effect is obtained, on the other hand, if the kinetic change in light-contrasts values follows a steep gradient, as in the first case, and is such that the brightness level of the random texture units becomes low enough to submerge totally into the background, then a dark opaque surface is seen as a moving occluding surface. What is seen is just another version of Michotte's so-called "rabbit hole" phenomenon, in which a bright disk is seen to disappear into a dark slit due to the systematic deletion of the bright area at the fixed linear margin (Michotte, Thines, and Crabbé, 1964). Michotte's rabbit-hole phenomenon constitutes a limiting case specifying the boundary conditions on both the kinetic transparency depth effect and Kaplan's (1969) kinetic occluding edge effect in which the edge of an opaque, randomly textured surface is specified by an appropriate accretion and deletion of texture at a moving linear margin.

In all three cases, the margin of light-contrast changes is moving at a fixed linear rate across the face of the screen. We must conclude, therefore, that harmonic motion plays no necessary role in determining events that afford the perception of the relative separation of surfaces in three dimensions. Similarly, the adjacent light-contrast values sufficient to produce the kinetic transparency depth effect are not necessary since among the phenomena investigated by Johansson, Wallach, and O'Connell, and Metzger, only the Wallach situation actually satisfies these light-contrast conditions.

From the above cases, we can also conclude that parallactic motion is not a necessary source of optical information for specifying the relative depth of surfaces, since none of the above cases satisfies the conditions for parallactic motion. In fact, in the next study to be reviewed, Gibson,

Gibson, Smith, and Flock (1959) were able to demonstrate that parallactic motion is not even sufficient to specify a determinate order of surfaces in depth. They conclude instead that a rather simple figural condition may be the necessary source of information for separation in either two or three dimensions.

The figural condition. In the Gibson *et al.* (1959) experiment, shadows from two parallel transparent surfaces, randomly textured with sprinkled talcum powder, were presented on a back-projection screen. The two surfaces were rigidly yoked on a common carriage so that when the carriage was moved on a line perpendicular to the observer's line of regard, the two fields of talcum-powered shadows were relatively translated in such a way as to determine parallactic motion. When the motion disparities of the texture shadowgraphs were sufficiently large to be perceived, subjects reported that the shadowgraphs appeared separated in depth. However, they reported that the ordering of the surfaces was indeterminate, since either one could be seen as being in front. If the depth information had been specified by motion parallax, then the faster moving shadows should have been seen as specifying the nearer surface. However, the data indicate only a slight bias for seeing the faster moving shadows as nearer. What then might be the necessary source of depth information?

Gibson suggests that the necessary optical information specifying separation might be due to what he calls *topological breakage.* This concept refers to the kinetic margin separating two subsets of points which are distinguishable via membership in a particular subset composed of texture elements sharing velocity vectors which are related in some principled way. The principle explaining the perceived coherence of texture elements into a single subset (that is, an optical whole) was called "the law of common fate" by Gestalt psychologists. Simple examples would be flocks of flying birds, or platoons of marching soldiers. When texture elements sharing common fate are packed with sufficient density and aligned in the proper way, the optical wholes are seen as optical surfaces. Thus, a single optical surface is defined by texture elements having a common kinetic fate (that is, proportional velocity vectors), while topological breakage, as a higher-order concept, is defined by subsets of texture elements having different kinetic fates.

This is an extremely simple but powerful principle since the information for topological coherence applies to all known cases in which kinetic

depth phenomena are perceived. In this concept we seem to have an implicit principle of sufficient generality and logical necessity to account for separation of surfaces not only in three dimensions but in two dimensions as well. For instance, it has been demonstrated that distinct coplanar textured surfaces which move in relatively contrary directions (that is, that have different velocity vectors) appear to be separated by a margin of topological breakage (for example, a crack of zero width).

Another virtue of the concept of topological breakage is that like "common fate," it can be defined in terms of symmetry factors. Margins of topological breakage (for instance, a three-dimensional edge or a two-dimensional crack) can be characterized as cases where *reversal fronts* exist between two sets of linearly independent vectors. A *reversal front* is a point or line of points at which an opposing pair or opposing sets of vectors nullify or balance one another (that is, a point or line of points where sign or direction reverses). When an object rotates, for instance, the point where the centrifugal vector balances the force of the centripetal vector is an invariant point known as the reversal point. When a sheet of rubber is stretched, the reversal front is that line of reversal points around which the opposing vectors of elasticity symmetrically balance one another. In neither of these cases, however, would the reversal fronts be margins for sets of independent vectors since the opposing forces or motions are symmetrical. On the other hand, the reversal front becomes a margin of topological breakage precisely when the distribution of opposing forces on either side becomes imbalanced or asymmetrical. At such a time the spinning object flies apart because centripetal vectors no longer symmetrically balance the centrifugal ones, or the stretched sheet of material tears as the elastic limit is reached and the vectors of elastic transformation can no longer symmetrically compensate for the kinetic vectors. Analogously, when a planar object, such as a disc, rotates in a plane, the crack that is seen is optically determined by the kinetic vectors belonging to the disc becoming disparate with the vectors of the surrounding material. Or a three-dimensional edge is seen when the kinetic vectors of a translating object relative to those of a background of texture determine opposing optical transformations specifying an asymmetry of motion on either side of a reversal front. Hence a margin of topological breakage (that is, a crack or edge) is specified by a lack of symmetry in optically determined vectoral transformations, that is, whenever what might be thought of as an "optical tearing" is determined at a reversal

front that is optically specified. (Later on we will present a sketch of a mathematical proof of this notion). In all of the above examples, we would ultimately want to show that specific symmetry operations exist for precisely characterizing the optical invariants specifying reversal fronts as well as those optical variants or asymmetries which specify the independence of opposing vectors by which we see the optical tearing which Gibson has called "topological breakage."

There is a problem yet to be resolved regarding the precise interpretation of this concept, namely, what is the nature of the ecological information which specifies separation of surfaces in three dimensions as opposed to separation in two dimensions? The concept of topological breakage is surely necessary for explaining separation in any spatial dimension; but without the stipulation of other conditions on event perception, it is not alone sufficient to explain separation of surfaces in a given dimension (for example, in three dimensions).

Mace and Shaw (*in press*) have succeeded in developing a set of kinetic displays in which separation of surfaces in depth is specified by relative motion of regularly textured surfaces. The displays were designed so that none of the usual variables known to be sufficient to specify three-dimensional separation of surfaces was present.

To illustrate the symmetry conditions we think necessary and sufficient to define topological breakage, consider a typical display from Mace's study. A moderately dense rectangular lattice of dots of light was displayed on the computer controlled CRT of a PDP-12. The lattice pattern consisted of a 16 × 16 dot display with discrete linear parallel columns (or rows). In order to introduce motion into the display, an 8 × 16 sublattice of dots composed of alternate columns was displaced in one of three possible directions—vertical, horizontal or diagonal—while the complementary sublattice remained static. The sublattices were slightly out of spatial alignment, so that the dots in the displaced sublattice never moved over the static dots but moved past them.

When the kinetic sublattice was displaced in a vertical or horizontal direction, subjects reported only linear coherence of columns (or rows, as the case may be) with little coherence of the dots into a surface and very weak relative depth. (It should be noted that throughout experiments of this type subjects consistently report a very weak relative depth of distinguished optical figures which is most likely due to figure ground rather than any hidden variables of kinetic depth.) A typical report of

subjects was that the subsets of dots appeared to intermingle as one platoon of soldiers might march through another. However, a dramatic effect was achieved when the kinetic sublattice was displaced in an approximately 45° diagonal direction; namely, the two sublattices of points not only cohered into two quasi-surfaces, but also appeared strongly separated in depth.

How might we explain the fact that direction of relative motion seems to be both a necessary and sufficient condition for the appearance of relative depth in these types of patterns? Furthermore, if we can explain it in this case, can we show how our explanation might generalize to other kinetic depth phenomena? Our attempt to do so requires an excursion into group symmetry theory. We will, however, eschew the details of a rigorous proof at this time and attempt only to sketch the general idea behind the proof.

Sketch of the proof. We can begin by showing that the dot light arrays used in the displays constitute geometric structures called two-dimensional *lattices*, that is, an array of points whose positions are all generated by a group of two *independent* translations. Independent translations are rectilinear displacements whose directions are neither parallel nor opposite. Next we define the two sublattices in the manner discussed earlier (that is, let one sublattice be all the even numbered columns of points and the other sublattice be the odd numbered ones).

We now come to the first important assumption of the proof. Any displacement of a sublattice that leaves its shape invariant is, by our definition, a symmetry operation. In particular the three translations chosen for the sublattice must be symmetry operations because they displace the sublattice without altering its shape. Translation, as a symmetry operation, is a "congruent transformation" in that it takes all points in the lattice into new positions across the same distance and in the same direction. To be well defined for congruencies, this distance must be traversed in discrete movements—in other words, no near approximations or convergences on the next positions as *limit points* are permitted. (Recall the race between the tortoise and the hare; the paradox arises because once ahead, the tortoise is defined as the limit point for the translation of the hare.) This assumption is also a very natural one, given the evidence that perception is a synchronous process by which discrete optical samples of the world are taken.

Our next problem is to decide what properties characterize the *space*

in which the lattices are perceived to move. We need a space of three dimensions in which congruent transformations, such as translation, can be well defined. This means that a natural space for symmetry operations in which lattices can be constructed and displaced would be one in which no limit points can exist, but only points at discrete distances from one another.

To construct the minimally sufficient space we need, we begin with an infinite three-dimensional lattice of points. By definition the intercepts on the coordinates by which the position of each point in a lattice is determined must be defined by integral (whole number) values. This follows from the assumption that the smallest distance between points is a discrete unit value. Clearly, if we take the discrete unit distance to be very small, say a unit of distance smaller than can be perceived, then we not only can construct the lattices we need, but any other figure as well. Notice carefully that we have not assumed the unit distance to be *finite* but only discrete. We do not need to assume finiteness at all, nor do we want to, since we also want to be able to speak of a displacement arbitrarily small.

Therefore, we relax the assumption that the intercepts on the coordinate axes must be integral values, and allow them to be discrete rational values (that is, nonrepeating fractions of distance where the numerator and denominator are both whole numbers). Now we have the space we need in which to account for the perceptual phenomena. It is called *discrete rational* space for obvious reasons; it is infinite, and it allows arbitrarily small discrete distances.

We can now prove, given this space, that all displacements such as translation and rotation can be defined as congruent transformations (symmetry operations), since they must be discrete *rational functions* (that is, functions which are defined over the field of rational numbers). We are also at the end of our quest for an explanation of why diagonal displacement of one of the two sublattices specifies a separation in three dimensions, while vertical or horizontal displacements do not.

Discrete rational three-dimensional space consists of an infinite number of parallel discrete rational planes (that is, two-dimensional spaces whose point positions have discrete rational intercepts). When both sublattices are static, or move vertically or horizontally, it is obvious that their points are always in positions that can be defined as lying within a single discrete rational plane. This is so because vectors that are parallel are linearly

dependent, that is, can be defined over the discrete distance as a basic unit of measurement. The two coordinate axes of a rational plane are themselves independent vectors (recall the earlier definition of independence). A vector of displacement defining the vertical or horizontal translation of a sublattice is dependent on, and can be expressed in terms of, one of these two coordinate vectors defining the position of the static sublattice, since it must be parallel to it. Therefore, the static and kinetic sublattices can be co-planar.

On the other hand, a diagonal displacement of a sublattice, or of almost any other oblique displacement, is necessarily independent of the two coordinate vectors defining the location of the static sublattice in a particular plane in discrete rational space. This is so because the diagonal direction is the hypotenuse on a right triangle for which the plane's coordinate vectors are the other two sides. By the Pythagorean theorem, the square of the hypotenuse is equal to the sum of the squares of the other two sides. Let us take the value of the diagonal translatory displacement to be the minimum distance which can be perceived; since it must be discrete, we can set it at a unit value. But this implies a contradiction, for the intercepts defining the new position of points in the diagonally displaced sublattice would have to be different from the intercepts defining the old position of any point in the sublattice by a value less than the unit value selected to be the measurement basis of the coordinate vectors. This follows directly from the Pythagorean theorem, since the hypotenuse (the diagonal displacement vector) of a right triangle is longer than either of the other two sides (the coordinate vecotrs). In other words, the unit of measurement for the diagonal translatory displacement would have to be an irrational fraction. But this is impossible since an *irrational* fractional distance (for example, $2a$ where each coordinate vector is defined over a discrete unit distance equal to a) cannot be composed as a resultant of two orthogonal discrete rational vectors. Thus the diagonal translation must be defined over a basis linearly independent of the coordinate vectors in the given rational plane, which, you will recall, is the plane in which the static sublattice is defined. Therefore, the diagonally displaced sublattice, unlike the orthogonally displaced ones, cannot be co-planar with the static sublattice but must move in some other parallel plane in rational three-dimensional space. It follows that the optical information determined by these two-dimensional physical displays is sufficient to specify separation of the lattice surfaces in depth.

In a related experiment, Mace and Shaw were also able to demonstrate that a randomly structured subarray of dots when moved in any direction whatsoever appeared separated in depth relative to its complementary static subarray. Hence we can only conclude that it is the symmetry of the structure of the lattice arrays that makes the difference. Presumably, only a single pair of independent coordinate vectors are perceptually specified in the optical information from the symmetrical displays, while apparently no particular pair of coordinate vectors are specified in the unstructured displays. This seems to us to provide support for two fundamental precepts proposed by the ecological approach to event perception alluded to earlier. First, perceptual experiences are directly related to events by symmetry relations, and secondly, the affordance structure of events can be predicted on the basis of symmetry theory.

The application of symmetry theory is no doubt tedious but logically straightforward. The generalization of the principle involved in this particular situation to other forms of kinetic transformations besides translation (for example, rotation), or to other types of structures besides lattices, is no trivial matter. Unfortunately, this must be left to a later paper (Shaw and McIntyre, in press). However, there is one immediate general conclusion about kinetic depth phenomena that can be drawn. Separation in depth seems dependent on the manner in which kinetic transformations interact with the intrinsic structure of the objects transformed. The form of this interaction specifies the affordance structure of the perceived event. These results also suggest that the affordance structure seems dependent upon both the symmetry of the structures involved in the event and the transformations applied to them. Consequently, the precise agreement between the analysis of the phenomena captured by these events and the reported experiences of them provide some independent support for the principle of cognitive symmetry as being a useful postulate for psychological theory.

Symmetry Conditions on the Perceived Invariance of Shape

Clearly, an important part of the affordance structure of an object is the invariant ecological information specifying its shape, since so many other affordances are dependent on this one (for example, graspability, stability, ability to be used as tools, vessels, or weapons). Shape, as mentioned earlier, is dependent on rigidity, since the more elastic the substance of an object, the less likely that many events will leave its shape

invariant. It is generally believed that shape is the primary means by which objects are recognized by men and other higher animals. Thus, there is good reason to expect that ecological shape-information might possess a very high attensity value, since the ability of animals and humans to reliably recognize objects is obviously essential to adaptation.

In spite of the great magnitude of research undertaken in order to understand shape perception, there have been surprisingly few fundamental insights into the problem. If it is true that shape is perceived relative to its affordance value and that it depends on what events are most salient in the environment of the organism, then there is reason to question whether it is wise to consider shape as a truly objective, intrinsic physical property of objects. Perhaps shape is best considered as an ecological aspect of objects, to be understood in terms of how organisms are most likely to interact with them.

In other words, shape is not a property of static physical objects but is rather a property of events, necessarily afforded by the physical structure of the object to be sure, but not identical with it. This is a very radical departure from the traditional approach which attempts to incarcerate the perceived shape of objects in the ideal, formal descriptions of classical geometry. Such geometries invariably begin with the empiricist assumption that shape is a primary sense datum expressible in essentially static terms.

Gibson suggests another way of viewing the nature of shape, that is, by denying that shape perception is based on "form" perception. His reason seems quite paradoxical at first, but it rests on the radical assumption that not even so-called "form" perception is based on the perception of form as interpreted in the neoempiricistic tradition. He might put it this way: *Form perception is not based on the perception of static forms, but on the perception of formless invariants detected over time.* Surprisingly, there is a straightforward interpretation of this statement within the framework of symmetry theory, as we will try to show.

When is a cube not a cube? In a demonstration in which a wire cube is rotated at a constant speed on each of its three axes of symmetry and strobed at appropriate rates, several effects are perceived which can be accounted for in terms of symmetry theory. When a cube is rotated on a face, the period of symmetry is four, since every 90° brings the cube into a position of self-congruence; when rotated on an edge, the cube has two twofold symmetries such as a rectangle does when rotated in the plane;

and finally, when rotated on a vertex, the cube has a threefold symmetry, coming into positions of self-congruence every 120°. If the strobe rate is selected to be synchronous with the period of rotational symmetry of the cube on any of the three axes (that is, is strobed at a frequency which is a multiple of its period of symmetry), then one of three effects is observed: (1) the cube either appears to rotate in the direction of the physical rotation, or (2) it appears to rotate in a counter direction to the actual rotation, or (3) it appears stroboscopically stopped. (We have all experienced these three effects when watching western movies where wagon wheels often appear to rotate (or not rotate, as the case may be) contrary to the apparent speed and direction of the wagon.

In all three of the above conditions the cube appears rigid to an unconstrained normal observer, and it seems to rotate on an axis that is fixed at an angle perpendicular to the turntable. By contrast, however, if the rotating cube is strobed at a frequency which is asynchronous with its period of symmetry, say on the axis running through opposite vertices, then something very surprising takes place: the cube appears to rock abruptly back and forth at some asynchronous strobe frequencies and even to tumble at other asynchronous frequencies. Moreover, the shape no longer even appears recognizable as a cube. These effects are not due to excessive difference in the strobe frequencies as compared to the synchronous frequencies in the earlier cases, since the rates can be selected to be very close in value.

Moreover, if speed of rotation and strobe frequency are held constant, but different axes of symmetry are selected, say the two twofold axes versus the threefold one, the earlier (synchronous) effects may occur for rotation on one of these while the latter (asynchronous) effects occur for the other. The main conclusion is that the particular intrinsic symmetry of the structure under each condition interacts with the symmetry of the dynamic transformation (that is, the speed of rotation and frequency of strobing). This seems to provide very strong independent support for the generality of the principle concluded from the studies of kinetic depth phenomena.

Again these effects can all be predicted from an application of symmetry group theory to event perception. We will, however, again omit the details of the proof and merely provide a sketch.

Sketch of the proof. The perspective group of an object, that is, the full range of *successively ordered* perspectives of all sides of an object, contains in it the rotational symmetry groups of the object as discrete subgroups.

In other words, all the invariant information for the shape of an object which is contained in the symmetry of its structure remains invariant under projection. The invariance of successive order of perspectives of the rotating cube is only guaranteed under the synchronous strobe conditions. When the strobe frequency is asynchronous relative to the period of symmetry of the cube on the given axis, this has the effect of destroying the correct successive ordering relationship among the perspectives. Hence the period of symmetry appropriate to the shape of the cube is destroyed and the resulting optical information projected to the eye is nonecological, since it specifies some other object undergoing a different displacement than is actually the case.

This result supports the notion that invariant ecological information specifying the shape and kinetic orientation of an object is surely dependent on the joint symmetry of the structure and the transformations defining the event. Destroy one of these symmetries and you destroy the other. Or put more generally, change the affordance structure of an event, and you change the nature of the event specified.

Symmetry theory then provides an explicit interpretation of Gibson's assertion that shape perception is not based on the perception of form but on the perception of formless invariants over time. (1) If by the form of an object one means the variant perspectives projected from it, then this demonstration indicates that different successive orderings of perspectives specify different shapes for the same physical object. Since it is the rotational invariants or symmetries that must be preserved in projection, if the perspective group is to correctly specify the shape of the object, then the hypothesis that shape is specified by *any* set of variant forms of the object is refuted. (2) On the other hand, if the claim is that *all* the perspective forms of the object are required to specify its shape, the hypothesis is simply false, since all the necessary information can be proven to be carried by a special ordered subset of such perspectives, namely, those perspectives symmetrical with the rotational symmetry subgroups. (3) Still one might argue that these invariant perspectives which are the basis for the veridical perception of shape can themselves be legitimately termed "forms" of the object. But do they really qualify as objective and intrinsic aspects of the physical object? They are at best only variants of the shape whose nature depends as much on the conditions of observation, such as the orientation of the observer and medium through which they are projected, as on the intrinsic physical structure of the object. A very simple example shows this to be so: The period of rotational symmetry of a

planar square object is four; it is self-congruent every 90°. The period for a trapezoid is one, returning to a self-congruent position only after a full 360° rotation. Yet if one views these forms from any angle other than a perpendicular line of regard, under some angles of regard the static perspective forms of each might easily be taken as specifying the shape of some other object. At an appropriate angle of regard the square may even project a trapezoidal perspective, while the trapezoid may project the form of a rectangle. In none of the static oblique perspective views will the correct angularity or relative length of sides be the same as the true proportions of the object, and rarely will they even be like one another.

 However, as soon as the figures are rotated through a 360° angle, their true periods emerge in spite of their variant perspective forms. The square will still project identical trapezoidal perspectives, but only every 90°, while the trapezoid will project identical perspectives only after a 360° rotation. Thus, in spite of the conditions on observation, the symmetry of the object still specifies its shape whenever the full rotational event occurs. In this sense, the symmetry on which the specification of shape depends is intrinsic to the event (the group of rotations), while the apparent form of the object is variant under most conditions. Therefore, it is quite proper for Gibson to claim that shape perception is based on the perception of "formless" invariants extracted *over time*.

If we accept this interpretation of Gibson's insight into shape perception, an exciting possibility arises that the generalizations of this principle may prove useful in other areas of event perception. Perhaps events involving transformations other than kinetic will be amenable to a similar symmetry analysis. In the next section, we attempt to show that even transformations which take a very long time to occur, and therefore define what we call "slow" events, can be explained by principles similar to those discussed above.

The Perception of Events Involving Viscal Elastic Changes

A taxonomy of events would surely include such polar categories as "fast" events, where dynamic change is immediately perceived (for example, the movement of the second hand of a watch), versus "slow" events, where the dynamic change is not immediately perceived but can be surmised from its effects (for example, the movement of the hour hand of a watch); *reversible* versus *irreversible* events; *rigid* versus *elastic* events (that is, nonrigid); and *animate* versus *inanimate* events.

In the preceding discussion, much was said about the perception of reversible, fast, rigid, inanimate events, but can anything significant be said about irreversible, slow, elastic, animate events? As one might expect, only a modicum of research has been directed toward such problems, mainly because the popular mathematical techniques generally favor the analysis of reversible and rigid, rather than irreversible and elastic processes. A brief survey of the events which fall under this category suggests other reasons for the dearth of psychological research on such problems.

Evolutionary events. The biological evolution of animal species exemplifies this class. It is obviously an irreversible, slow, animate event, but why it should also be called elastic may not be quite so obvious.

If you trace what is known about the phylogenetic development of a species, say from the twelve-inch high eohippus through mesohippus (24 inches), merychippus (40 inches), pliohippus (50 inches) to equus, the modern horse, the structural changes that have taken place are much more than a mere size change. Rather it is more accurate to say that each new stage of development in a species is characterized by a thorough "remodeling" of the old structure. Yet sufficient structural features remain invariant under the remodeling transformation so that we can still reconstruct the evolutionary lineage of modern man, animals, or plants. Note that by Weyl's definition, this evolutionary remodeling transformation must be considered to be a symmetry operation, since phenomenal identity of the species is preserved.

The logic of symmetry operations that applies globally to nonrigid, complex structures is not well understood. For this reason there have been numerous mathematical challenges to the evolutionary mechanisms proposed by the neo-Darwinian theory by contemporary theorists. (See, for instance, the Wistar Institute report [1967], especially the Schützenberger paper.) The evolutionary remodeling transformation can legitimately be considered a case of what Gibson has called a "viscal elastic change." More generally, any real event which involves a natural symmetry transformation, as opposed to an abstract one, can be analyzed into two logical components: those factors which contribute to change (the elastic component) and those factors which impede change (the viscal component). Clearly, the remodeling transformation of biological evolution (or, for that matter, any other process of evolution in which evidence for earlier stages is preserved in later ones) qualifies as a viscal elastic change. The process of biological reproduction is a symmetry transformation in

which the transmission of genotypic information from parent to offspring is the viscal component, since it is part of the evolutionary process that preserves the species and ancestral invariant properties. By contrast, spontaneous mutation as well as genetically extrinsic factors (such as nutrition, disease, and so on) introduce changes into the phenotype and, therefore, constitute the elastic component.

Aging event. Another well-known animate event also involving viscal elastic changes is the process of aging. Moreover, the aging transformation is a symmetry operation possessing great attensity value. Human observers are for the most very good judges of relative age level.

Not only are we perceptually attuned to detect the structural invariants of events but also the dynamic invariants of transformations. For instance, we can detect the invariant properties of a face sufficient to recognize the same person at different age levels; but we can also perceive the invariants of the aging process with sufficient accuracy to rank order different persons of the same age level. In most cultures, age level plays an important role in determining the social pecking order. Therefore, there is ample reason to believe that the aging transformation, like expressive transformations, might possess great attensity value.

Hence, reasonable evidence for the fruitfulness of the ecological approach to perceptual theory would be provided, if a symmetry analysis could be given which defines the ecological information specifying the invariants of the aging transformation. It would not be sufficient, however, merely to isolate the ecological information specifying different age levels, although this must be included in the analysis. In addition, a really adequate theory of what we mean by the perceptual invariants of the aging process must include also a set of criteria for distinguishing the aging transformation from other formally similar transformations which determine related but distinct classes of events. In other words, we must say both what aging is and what it is not.

Our hope is that if we can gain some insight into one class of events involving viscal elastic changes, then we might find the key to understanding the logic of the whole family of classes. The most general hypothesis we want to explore asserts that the degree of persistent phenomenal identity of slow events transformed by remodeling, such as aging or evolution, depends on the degree to which the viscal component of the process significantly outweighs the elastic component. Specifically, we wish to discover the symmetry group of aging transformations which

(a) leaves invariant the general structural information specifying the species and (b) which leaves invariant the more specific structural information by which the individuality of a face can be recognized.

Some work has been done by biologists, anatomists, anthropologists, and paleontologists on the problem of mathematically characterizing the remodeling transformation which is involved in both growth and evolution. Since much of the motivation and inspiration for the ecological approach to such nonrigid, animate events has been derived from the work of others, such as D'Arcy Wentworth Thompson (1917), the great British naturalist, Mikhail Gerasimov (1971), the late Russian paleontologist, and Donald Enlow (1968), Professor of anatomy at the University of Michigan, a brief review of this literature will be helpful.

The Logic of Coalitions and Global Symmetry Transformations

A lump of clay can be considered a plastic aggregate which can be modeled and remodeled. Since it is an inert, homogeneous material, any motive force for its deformation must originate from external sources. Similarly, any directed modeling of its shape must be governed by some agent outside of itself. Since clay is a plastic substance, however, a sculptor can work it into an artful facsimile of a human head. He does it by shaping, gouging, poking, or even pounding it until it simulates the visage. We can call such shaping processes *plastic* transformations, since there is no greater resistance offered by the clay to remain one particular shape than another.

The group of plastic transformations belongs to topology, a primitive geometry of shapeless things. The elastic component in such plastic changes completely outweighs the viscal one which, unlike the viscal component in the previous examples, is geometrically trivial. Only the nature of the material composition remains invariant under plastic transformations. Since there are no significant structural invariants, we hardly wish to call these transformations symmetry operations.

Now contrast this modeling event with the growth and development of complex, heterogeneous living systems. As symmetry operations, growth and aging transformations are surely nontrivial. In the former there are emergent properties following a group of remodeling transformations governed by an intrinsic (genetic) policy of change, while in the latter there are many complex structural properties, defying precise geometric description, which remain invariant under remodeling.

The sculptor creates the individual features of the clay head by *local* plastic transformations, while the intrinsic natural process of growth creates the visage of the human face by *global* symmetry transformations which simultaneously remodel every region. The developing human head, like the whole body, then, is best characterized as a dynamic *coalition* of interdependent heterogeneous substances rather than merely as a homogeneous reactive aggregate.

There is bone, cartilage, layers of skin, fluids, arterial and neural tissues, muscles, and hair. As a coalition of interacting growth processes, its development is easily one of the most complex events in nature. The primary mathematical difficulty of accurately defining the group of remodeling transformations characterizing its growth is no doubt due to the fact that it is a highly laminated structure, where each lamina possesses a different coefficient of elasticity, but yet somehow manages to contribute at any given moment to the overall relative rigidity of the whole structure. There is little wonder that a geometry of such dynamic, quasi-elastic, quasi-rigid structures has been difficult to come by. Nevertheless, a significant beginning has been made, we believe, by Thompson (1917).

The perception of the human face no doubt depends in part upon the ability to abstract the invariants of such remodeling processes as growth and aging. If so, then it must be the case that the invariant information determined by these global processes can be given a precise description. We set about this task in the following way. First, we attempted to derive a general formula for the resultant stage of the growth process, for example, the adult human head. Next, we attempted to discover the group of symmetry transformations that would reverse the growth process so as to retrieve the shape of the head at earlier stages of growth. Finally, we sought to determine the range of parametric values for variables in our formula which fit all human heads at all age levels. In the course of achieving this final result we also perchanced to discover that disproportionate parametric values, those going outside the range of normal human heads, yielded a dimension that suggested monsters.

The Growth of the Human Head

The following is a brief summary of the general growth processes of the craniofacial complex abstracted from Enlow (1968). The growth processes determine the invariants of the shape of the head. The neonatal human skull exhibits an exaggerated cranium and a diminutive face. The facial

complex, however, grows progressively more rapidly than the skull so that the overall dimensions of the head and the face become disproportionately altered. The most important fact of growth is that the craniofacial growth, no less than that of other parts of the body, does not take place merely by enlarging or even by deposition of interpolated tissue layers. Rather, growth follows a process that can best be thought of as a *remodeling*. New bone tissue additions in any portion of the skull result in successive relocation of all other parts. As each individual bone grows, a remodeling transformation is systematically multiplied over the whole structure.

The composite of all morphogenetic remodeling changes determines the form taken by topographic features that characterize the human face from childhood to maturity. The resultant growth determines both a *shearing* and a *strain* transformation over the entire head. A shearing transformation effects a progressive change in the overall angulation of the profile of the face, while a strain is reflected in a kind of stretching of the face in the vertical dimension, as the mandible, sinuses, nasal pocket, and other bony processes grow. (The notions of *strain* and *shear* transformations will play a very important role later in our attempt to provide a precise characterization of the invariant information for age level and degree of monstrosity.) The growth of soft tissue, unlike bone, is characterized essentially as an increase in size due to several combined processes: cellular proliferation (epithelia), enlargement of cells (muscle and cartilage), and by increase of the material between cells. All these are *interstitial* growth processes since they involve expansive changes of already existing tissue. Bone, because of its hardness, is not capable of growing in any of these ways.

The Invariants of Faces

Clearly, if the recognition of faces depends on perceiving their style of change, which in turn depends on the complex remodeling process, then we must be highly dubious of the success of theories which attempt to explain recognition in terms of isolatable, static lists of features, or as a process of matching figural templates stored perhaps as images in memory. Both of these approaches ignore the fact that the information for recognizing faces might be due to invariants resulting from the history of the head as a dynamic coalition. But exactly how does the policy of growth of heads determine the invariant optical information for their recognition?

Gerasimov (1971), the late Russian paleontologist, spent most of his

adult life developing fairly explicit procedures by which the detailed struc-
ture of human faces might be inferred from just their bare unfleshed skulls.
He was dramatically successful in reconstructing the minute details of the
faces of several historical personages such as Tamerlane, Ivan the Terrible,
and Schiller. He even prepared identifiable busts from skulls which on
several occasions led to the solution of murder mysteries. How might we
explain the fact that a skull is somehow a logically unique fossil of the face?

Our best hypothesis is that bones are not capable of growing and
remodeling themselves. Rather it seems more likely that the source of
control on bone growth resides in the soft tissues associated with them;
but, unfortunately, the details of these control processes are not yet fully
understood (Enlow, 1968). If this is the case, then in a sense one might
expect that the soft-tissue structures of the face somehow leave distinctive
residual traces implying their size and shape in the coalition of relations
defining the bone processes of the skull. Gerasimov, for instance, was able
to determine precisely the shape of the nose from the nature of the inter-
relationship of the structure of the nasal bones, contour of the nasal
opening, the configuration of the glabella, the structure of the whole
supraorbitory region, the outer corner of the eye, together with the overall
profile of skull and alveolar region of the upper jaw. His methods are quite
tedious but have proven dramatically effective.

If the skull truly carries invariant structural information specifying the
detailed shape of the face, then perhaps age level for faces will also be
sufficiently specified by the geometry of the skull.

The Method of Coordinate Transformations

Thompson (1917) early this century pioneered a mathematical tech-
nique for precisely characterizing the symmetry properties of the evolution
and growth of plants and animals, especially with respect to the way their
parts become remodeled during evolution. He called the method he used
"the method of the transformation of coordinate systems." This is a
method by which closed geometric curves of best fit are given for plant or
animal parts (for example, leaves or skulls) at different periods of growth.
Each curve is then defined over a rectangular coordinate system. Finally,
Thompson then attempted to discover an elegant set of global symmet-
rical transformations which applied to the coordinate system (that is,
to the two- or three-dimensional space) rather than locally to individual
points or parts of a given curve which transformed each curve into the
other. Hence, this technique transforms the space in which an object is

defined, simultaneously moving every point of the object rather than each
point separately. (This method of coordinate transformation is often
called the "alibi" interpretation of geometric transformations since it
assigns the object to a new space by effecting a *global* geometric transfor-
mation on the old space. The more ordinary Cartesian method is called
the "alias" interpretation of geometric transformations since it simply
applies a *local* transformation to the coordinates of the points, thus
renaming them without affecting the space as a whole.) This technique
developed by Thompson is very convenient for transforming truly com-
plex objects, such as skulls or faces, since it applies equally well to any
object in the space, complex or simple. Also it has the virtue of allowing
for the full development of the group of symmetry transformations even
before the final details of the equation of the particular objects to be
transformed are known.

The limitations of this technique reside in two problems which must be
solved to apply it. First, can the object be properly oriented with respect
to the origin of the coordinate system, and, second, can there be selected
only those simple objects or subparts of complex objects for which no
significant invariant properties of their structure are lost by global trans-
formations? These two problems are obviously independent of the pro-
blem of discovering the proper equation for the object (or object part)
as well as the appropriate group of symmetry transformations.

The mathematization of growth transformations. To locate the skull in
the coordinate space we must be able to determine the nodal point of
growth for each individual bone process or the resultant nodal point
derived from them. The overall intrinsic policy of growth of a human
skull, as viewed in two-dimensional profile, is determined by the inter-
action of the growth of the mandible, temporal, occipital, and parietal
bony processes. It is our belief that, in general, global invariants of events
possess a greater attensity value than do local ones. This assumption
motivated us to select as our first approximation for the curve of the
two-dimensional profile of the human face, the inverted heart-shaped
curve called the *cardioid* (see Figure 1). The choice of this curve, rather
than, say, a circle, ellipse, or other closed curve, to represent the head was
by no means arbitrary.

Foci of closed curves that are bounded in a space of n-dimensions are
fixed-point properties of the curves under nearly all kinds of continuous
translations (i.e., excluding translations), according to an extension of
Brouwer's famous "fixed-point property" theorem. (See, for instance,

Figure 1. A regular cardioid $r' = r(1 - \sin\theta)$ and the fit of the transformed cardioid ($r' = r(1 - k\sin\theta)$) to the points of a characteristic skull. Note how the shear transformation is represented in the selection of parametric values of k where $k = 0$ or $k = 1$. The strain on the cardioid is defined by $x' = jy + x$ where x, y are the coordinate axes and j is a constant.

Tompkins, 1964). This theorem asserts that any continuous transformation from a bounded figure onto itself leaves at least one point fixed. For instance, a rotation of any figure in a plane leaves the axis point fixed, whereas a translation, of course, leaves no single point fixed.

Since growth and other remodeling transformations are symmetrical,

they are also continuous; therefore, they must have a fixed-point property: a nodal point of growth is such an invariant point. Most leaves have a single fixed-point property, a few have a line of such points, around which growth is symmetrical over time. Thompson showed that the focus of the cardioid represents the nodal point of growth in leaves such as the *begonia daedalia* because this is the origin of two growth vectors representing the *radial* growth component that dilatates the leaf, and the *tangential* growth component that represents the circumferential growth. The former is represented as a global one-dimensional homogeneous strain transformation, while the latter is represented as a global shearing transformation in the method of coordinate transformation. In the real skull, vectoral descriptions of its resultant growth indicate this nodal point to lie near the tympanic annulus (Enlow, 1968). We use this fact to orient our skulls on the axes. These two transformations form an affine group of symmetry operations that preserve the properties of parallelism and "betweenness" among points. In Figure 1, note the extreme goodness of fit of the cardioid when transformed by these operations as compared to the untransformed points of the cardioid lying above the skull.

Experimental test. The above symmetry analysis bearing on the nature of the perceptual information for age level determined by the invariants of growth processes was tested in the following way. Computer generated drawings of the profiles of three arbitrarily selected skulls at different age levels were used in the study. By showing that each of these drawings could be transformed into the other when the face area was omitted and they were corrected for scale, it was possible to test the adequacy of our selection of the affine group of transformations in order to define the process of remodeling due to growth. It was so successful that no test seemed required. In any case, a more adequate test of the appropriateness of the group of transformations selected would be found in the results of the experiment.

For one condition of the experiment, we had the computer plotpak construct 34 (it missed one!) transforms of a single infant skull to which an arbitrary face was added (see Figure 2). The standard infant skull is in the second row of the second column. Looking at the anthropological data values incorporated into the parametric values of the two transformations, we have the following a priori pattern of predictions: (1) Age increases monotonically with strain. (2) On the dimension of shear, aging is curvilinear from the standard shear of $-5°$. In the negative direction it moves very slightly toward a geriatric skull, while in the positive direction, the
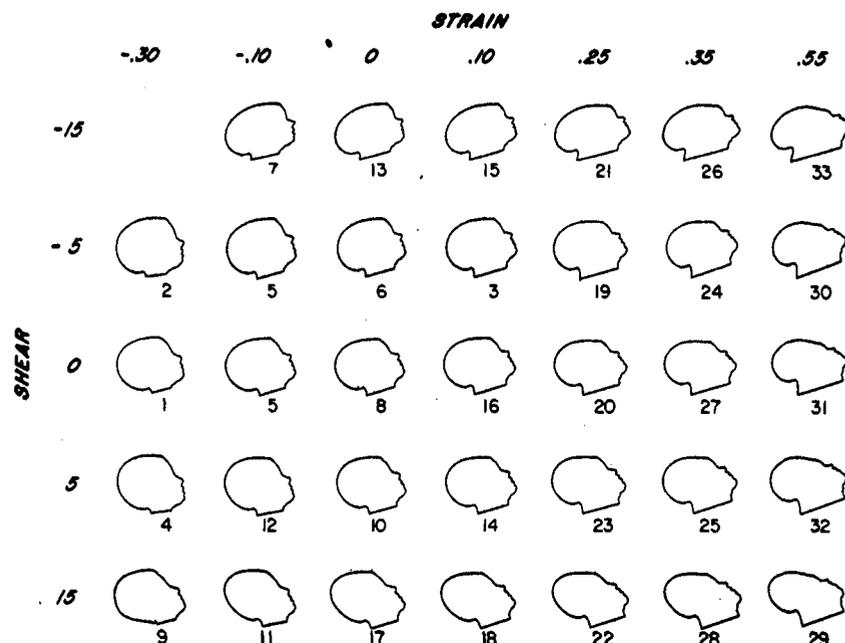
Figure 2. Set of stimuli with the mean rank order age attributed to stimuli indicated below figure outlines.

growth parallels the normal aging of an infant, through adolescence to adulthood. (3) Post hoc, but independent, agreement by two judges indicated that non-normal transforms defining "monstrosity" was a complex dimension which spread out increasingly, down from the normal face.

A group of 14 subjects recruited from introductory psychology classes was presented with the 34 stimuli of Figure 2. They were used to consider these pictures in terms of a rating scale on which the criteria would be the age level of each figure. They were further instructed that if the second figure looked twice as old as the first, the score assigned to the second figure was to be double that for the first figure. All 34 slides were presented three times, once for practice and twice for actual data collection. The randomization was different for each presentation.

The scores for each subject were transformed into rank-order scores, and the rank-order scores of all subjects were combined to determine the average rank order for each stimulus figure. The results are presented in Figure 2.

Our general hypothesis in this study was that figures should appear older or younger as a direct function of how the parameters on these two transformations are changed. The data in Figure 2 indicates that subjects were, in fact, able to consistently make fine age-level discriminations. Hence, we get the strict ordering of age-level judgments corresponding to the transformational distance of the drawings from the standard face. In other words, the evidence seems to support our claim that the selected group of transformations does provide the perceptual invariants of both aging and age levels.

Of course, these two transformations do not specify all the invariant information by which faces are recognized, but we believe, buttressed by this initial success, that the persistent identity of faces under other transformations (such as caricaturing or expressive changes) will also yield to techniques of analysis suggested by the ecological optics point of view.

Consequently, we feel encouraged to apply symmetry analysis to other events, and to retain the postulate of symmetry as the basis for our ecological approach to psychology.

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