Symmetry and Symmetry-Breaking in Thermodynamic and Epistemic Engines: A Coupling of First and Second Laws

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Abstract. Science traditionally models interactions at the same scale treating systems as isolated first and then as interacting second. This approach has well-recognized limitations. It imputes an original symmetric state space to a system followed by symmetry-breaking operations until systems of a desired order of complexity are reached. Once a level of order is reached they converge back onto their original symmetric state space. This sequential alternation of conservative and nonconservative strategies is tantamount to the assumption that first principles (the First and Second Laws) are so weakly coupled that system dynamics are reversible; hence it assumes the laws operate essentially at the same scale. For systems with complex interiors, where much symmetry-breaking is needed, it is impossible to prevent anomalous modes of organization from arising that inflate the dimensionality of the system—an acknowledged but unavoidable curse of open systems analysis when all parameters are distributed at the same scale (i.e., over the same state space). To prevent this unwelcome outcome, ad hoc algebraic filters or post hoc (super) selection rules must be added to *same-scale* theories—a violation of both the parsimony and integrity of the putative first principles. *Same-scale* theorists so far have been unable to show how such ad hoc or post hoc provisos may either be avoided, used to formulate new first principles, or to reformulate the old ones. An alternative strategy is to assume that the first principles are coupled nonlinearly rather than linked linearly. The nonlinear strategy assumes that the smallest unit of analysis for explaining complex system dynamics is the nonlinear *law-couple* itself, which applies irreversibly across macro-and micro-scales and reversibly across modes at the same scale. Thus symmetry-preserving and symmetry-breaking strategies are treated as a single strategy with no need to postulate theoretically unredeemable provisos. Evidence for the nonlinear coupling of the two laws is found across micro- and macro scales with respect to ascending orders of organization—spaces coupled by geometries, geometries coupled by fields, fields by engines, and engines by systems. These issues are discussed in terms of open complex systems examples (e.g., nest-building by termites, craniofacial growth, perceptual illusions) to illustrate the need for applying the same nonlinearly coupled first principles across the scales of physical, biological, and psychological modes.

1. Introduction: Same-scale versus Cross-scale Interactions

To even the most naive observer, Nature presents a sequence of levels of energy interactions that extends from small-scale ones between nuclei, atoms and molecules, through biological-scale interactions involving cells, organs, and organisms, up to large-scale interactions involving planets, stars and galaxies. Yet, despite the ubiquity of these *cross-scale* energy interac-

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tions, physical science has a long history of limiting its investigations to phenomena whose interactions only involve entities ('atomisms') at the same dynamic scale.

Implicit in this assumption are the isolated restrictions associated with conservative systems, out of which were born the scientific enterprises of celestial mechanics, classical mechanics, relativistic mechanics, and quantum mechanics. None of these approaches are designed to account for the behavior nor the experiences of systems with complex interiors. One might argue, rather, that such accounts must await more adequate biological and psychological theories. However, we propose a different tack.

Perhaps if we had a physcial theory that applied across scales to systems with complex interiors, then it would not only reveal the workings of ordinary physical phenomena, but biological and psychological ones as well. This is not a reductionistic claim. Rather, it is a claim that theoretical extrapolations across scales may follow the same physical principles. Such principles cannot be the principles of *same-scale* physical theory. To claim otherwise would, indeed, be crass reductionism.

From physics we have learned that predictive models based strictly on interactions between *same-scale* atomisms may provide successful approximations for the behavior of very large (celestial) and very small (quantum) scale systems. In these models the predictive power is derived from the identification of symmetries that are invariant over different interactions at the same scale. Unfortunately, the application of this same modeling strategy to the behavior of biological and psychological systems has not been as successful. We suspect that this is because such phenomena are only defined across different scale interactions.

Thus it seems that science must pursue its course under the auspices of both *same-scale* and *cross-scale* reasoning. In the next section we compare and contrast these two styles of scientific reasoning and make certain explicit recommendations as to how they may be amalgamated into a single scientific strategy.

1.1 Side-side Versus Up-down Reasoning

Let us begin by contrasting the two styles of thinking about science alluded to above. One style attempts to produce theories whose major concepts are coordinated at the same scale of analysis, and thus treats interactions between scales as if they were reducible to interactions at a single scale. We call this concentration on *same-scale* concepts and *same-scale* interactions "side-side" reasoning. Although this approach has had notable successes, it has been shown to lead to unresolved and, apparently, unresolvable theoretical perplexities.

A more fruitful approach to scientific explanation, we believe, is to look beyond the *same-scale* interactions and to recognize the need for theories with concepts that are fundamentally tailored to address *cross-scale* interactions. We call this style of scientific thinking "up-down" reasoning. With the growing successes of nonlinear, nonconservative dynamical (open) systems theory to draw upon, and being mindful of the limitations of *side-side* reasoning inherent in linear, conservative dynamical (isolated) systems theory, we believe, like others, that an approach is needed that embraces both styles of thinking [1; 2; 3; 4; 5; 6; 7].

Although the two styles of reasoning complement each other, such that no theoretical effort can be considered complete if either is omitted, they perform different duties. *Side-side, same-scale* reasoning emphasizes the First Law, and *up-down, cross-scale* reasoning the Second Law. To address the issue of how dynamical constraints on natural systems originate and where change comes from, the *cross-scale* approach logically assimilates the *same-scale* approach. But there is no issue for which the reverse is true.

1.2 Side-side Reasoning in Physics and Psychology

As mentioned earlier, *side-side*, or *same-scale*, reasoning has a long and illustrious history in physical science—most notably in classical mechanics and relativity theory. The program of classical mechanics was an attempt to reduce all physical effects to mechanical interactions at the same scale; hence all phenomena had to be treated as if they were linearly predictable and causally reversible even though, in the final analysis, they are not. In relativity theory Einstein inaugurated a program aimed at reducing all physical phenomena—whether matter or energy based—to the *same-scale* field processes.

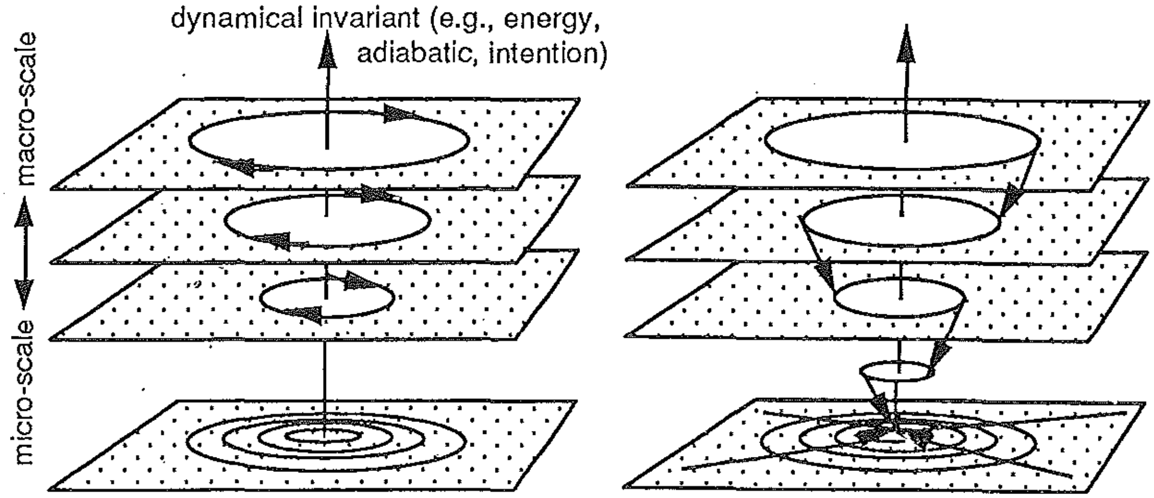
In psychology, reductionistic forms of behaviorism and mentalism are, perhaps, the most notable examples of *side-side* reasoning. The behaviorism program represented an attempt to reduce all psychological phenomena to *objective same-scale interactions*, while cognitive psychology, in most guises, remains a thinly veiled attempt to reduce all psychological phenomena to *subjective same-scale interactions*. In contrast to these, reductionistic neuro-cognitive approaches assume that all effects of interest are to be located functionally side-side in the "brain" field.

In the next section, we introduce concepts that will be used throughout to characterize the symmetries that systems may share across scales and the asymmetries that account for their relative instabilities. As we shall see, *cross-scale* symmetries reciprocally relate macro-and micro-scales. The fundamental challenge for physical sciences is to find *cross-scale* symmetries (critical sets) provided by the First Law that order the space (gradient sets) under the Second Law.

1.3 The Dynamical Coupling of the First and Second Laws

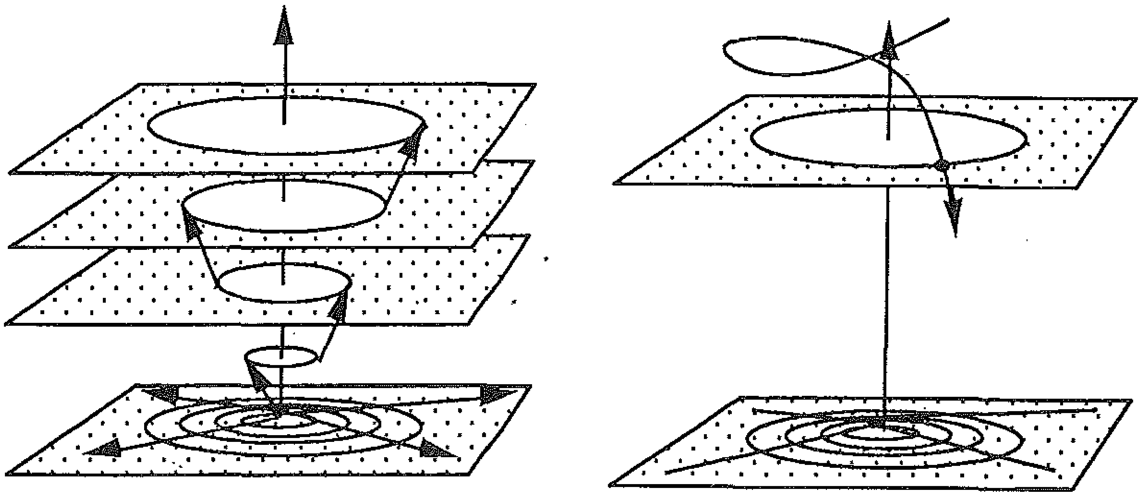
Of Newton's three laws only the third is considered truly a law [8]. The first two "laws" are definitions—defining force in terms of mass and acceleration. Physical laws, such as the laws of gravitation, electromagnetism, etc. tell us what forces are acting in specific situations. The equation F = ma is not all encompassing; for it is not applicable to statics. Since statics deals with forces acting in structures at rest, the definition of force in terms of acceleration would make statics incomprehensible as a science. Likewise, for living systems, whose dynamics are guided by information, the direction a system's trajectory takes can not be explained by the direction of forces applied since the force originates from within the system. Rather the direction taken is that in which the system applies the forces. Directional information may specify forces but it does so geometrically or kinematically rather than kinetically. Information has length, distance, and time dimensions but has no mass or energy dimensions. Perhaps, then, it would be wiser to reformulate the laws of nature at a level of abstraction not dependent on force or mass per se. In this way dynamical anlysis will not be restricted to inanimate systems or aimless animate systems but might address complex goal-directed systems as well.

Laws of nature can be construed as operations that relate all relevant dynamic variables, including informational variables as well as kinetic ones. At the most abstract level the relations that relevant variables take are defined by symmetry-preserving and symmetry-breaking operations. Indeed, in what follows, we shall argue that the active content of such laws may be expressed by the coupling of symmetry-preserving and symmetry-breaking operations. Figure 1 shows what this means with respect to three kinds of systems. Figure 1a depicts a conservative system whose dynamics is governed by the First Law alone. Such a system must be considered to be isolated from interactions with other systems. No real systems are so isolated.



(a) Conservative System: Reversible First Law dynamics. Different iso-similarity contours correspond to different initial conditions of the motion invariant. No trajectories cross the contours.

(b) Nonconservative System. Irreversible Second Law dynamics. Trajectories are *contracting* spirals that cross the contours as they move toward the point-attractor.



(c) Nonconservative System: Irreversible Second Law dynamics. Trajectories are *expanding* spirals that cross the contours as they move away from point-repellor.

(d) NOTE: The trajectories in the nonconservative dynamics intersect the iso-similarity contours in only one point.

Figure 1: Same-scale and Cross-scale Coupling of the First and Second Laws

Figure 1b and 1c depict nonconservative systems that have *cross-scale* interactions as well as *same-scale* interactions. These systems are governed by the Second Law.

While conservative transport processes never cross the iso-similarity contours, nonconservative transport processes do. In conservative transport processes there is a symmetry that holds over the boundary conditions (intitial and final conditions). We identify this symmetry as a conservation of something. But we do not have to be specific in order for the mathematical description to be explicit. This symmetry relation over boundary conditions is responsible for the reversibility of transport process operating at a single scale. If the dynamical invariant were total energy, then the conservative transport would allow potential energy to flow into the kinetic energy and vice-versa. However, if the dynamical invariant were an adiabatic invariant, and the transport an adiabatic transform, then the conserved quantity would be action (energy x time) [9]. If the dynamical invariant were intention, then the conserved quantity would be still more abstract, perhaps, a generalized quantity consisting of energy and information components [10]).

By contrast, nonconservative transport processes are not reversible because they operate across scales. There is no exact symmetry over boundary conditions defined across scales as there is for transport processes confined to the same scale (macro-macro transport or micro-micro transport). Furthermore, it is commonly assumed, although it is as yet not certain, that the irreversibility of cross-scale transport process which keeps them from being conservative derives from the fact that macro-modes are less stable than micro-modes. This allows for an asymmetry by which the arrow of flow might be set.

The main point to be developed is that the interplay of the First and Second Laws is sufficiently abstract when rendered in terms of symmetry-preserving and symmetry-breaking operations to apply to any dynamical variables. Let's look a little more closely at the important role that symmetry and asymmetry have to play in this regard.

1.4 Why Search for Symmetry?

A simple definition of a symmetry was given by Herman Weyl [11]: A thing is symmetrical if there is something we can do to it so that after we have done it, it appears the same as it did before. An operation is a way of doing something. Hence, in the most general sense, a symmetry-preserving operation conserves the property in question. Does this mean, however, that such an operation does nothing? If so, having no effects, the operation would be invisible since nothing happens to draw attention to it? If this were all there was to symmetry, it would hardly be worth the candle required to search for it. Of course, there is much more to it.

Felix Klein [12], in his Erlanger program, proved that geometries can be associated with discrete groups of symmetry operations that leave certain characteristic properties of a space unchanged (e. g., affinities, linearities, and metrics). A little later, Sophus Lie [13] showed us how continuous groups could be associated with the differential geometry of manifolds. Still later, Emmy Noether [14] showed us how the physical conservations (e.g., energy, linear and angular momentum) can, following variational principles, be identified with the Lie group symmetries associated with space-time geometry. Thus we have, through these brilliant efforts, a bridge connecting the symmetries of a space (e.g., space-time) with the group of operations that might be performed on objects (e.g., systems) in that space; and these, in turn, are connected with the conservations of physical quantities. More recently, Wigner [15] made the remarkable observation that there is a structure in the laws of nature called the laws of symmetry. This structure is so far reaching that, in some cases, the laws of nature were guessed on the basis of the postulate that they must fit into this symmetry structure.

For disciplines like psychology that must deal with extremely complicated systems, cultivating an unshakeable existential commitment to this symmetry postulate may be the only guide through the morass of data that seems to increase exponentially each year. We can not easily assume that an unmotivated and unsystematic search through this ever-increasing welter of fact and fiction will somehow magically coalesce into summaries from which fundamental laws can be induced. We prefer to place our bets on the more deductive approach identified by Wigner, and conduct an active search for the symmetry structure by which the laws of psychology might then be guessed.

Indeed, to search for the symmetries among systems that interact might well be taken as the hallmark of a mature scientific program, for it is nothing less than the search for first principles. Psychologists, therefore, no less than other scientists, have an obligation to conduct such a search. We submit that the search will be well worth the "candle" for the reasons discussed next. This paper should be read as an attempt to spell out briefly guidelines for conducting such a search for the symmetry structure of laws into which first principles of psychological science must fit along with those of the physical and biological sciences.

1.5 Symmetry and Antisymmetry in the Coupling of First Principles

Natural kinds of things are complex; they have many properties. An operation that preserves one property usually does not preserve all properties of the thing to which it applies (unless it is a trivial "do nothing" identity operation). Hence nontrivial operations are Janus-faced: they both preserve and destroy properties. If you translate a book in Euclidean space, shape is left invariant but location is not; if you rotate it, again the shape remains invariant but its orientation does not. At a more complex level, if a face grows, inter alia, its identity, species, gender are left unchanged but its size and shape are not.

In nature operations cannot in some sense be *symmetry-preserving* without in some other sense being *symmetry-breaking*. You cannot have one without the other. To make an omlette, you must break the eggs. Thus we recognize that some things are conserved and some things are not. It is always an empirical question to determine which is which under a given operation. Most generally described, dynamics is the consequence of the Second Law breaking the symmetry of a proper subset of properties identified with a given structure while the First Law leaves another proper subset of properties symmetrical under the change. Simple system dynamics coordinates the preserving and breaking of symmetry between systems without complex interiors (e.g., particles and homogeneous objects). Complex system dynamics does so between systems with complex interiors.

It is important that we clarify operationally what is meant by the concept of "coupling" and how it relates the Laws during dynamical interactions between systems, whether they be simple or complex. In the next section, we show how the coupling of the Laws in both a side-side physics and an up-down physics is itself lawful.

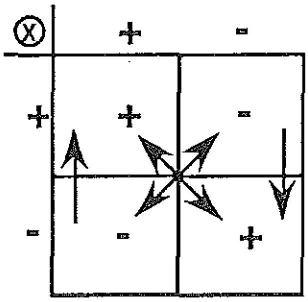
Coupling as a Generalized Version of Newton's Third Law. Newton's third law can be construed as the First and the Second Laws taken as a single corporate entity. By asserting that for every action there is an equal and opposite reaction, this law recognizes the mutuality and reciprocity of all systems' interactions. It is one of the most profound empirical results about nature that no exception has ever been found to this law of mutuality (symmetry-preserving) and reciprocity (symmetry-breaking) in systems' interactions [15]. The program we are promoting draws all authority for its methodology from an existential commitment to this fact. By this approach we can generalize Newton's third law to *up-down* as well as *side-side* theorizing about complex systems' interactions. We show how this might be done next.

Interactions as Mutual and Reciprocal Operations. Interactions of dynamical systems are mutual and reciprocal operations. Let a be the operation of system A on system B and b the operation of system B on system A (i.e., action and reaction). Where the product of the operations is mutual, $ab - ba = 0$, then the interaction preserves proper (unordered) symmetry and is reversible. Where order is unimportant the operation is said to be *commutative*.

$$\begin{tikzcd}
A \arrow[r, leftrightarrow, "{[A, B]}"] & B \arrow[d, "{[B, a]}"] \\
b \arrow[u, "{[b, A]}"] \arrow[r, leftrightarrow, "{[b, a]}"'] & a
\end{tikzcd}$$

Let pairs of upper case and lower case letters represent *same-scale* interactions at the macro-scale and micro-scale, respectively. Mixed case letter pairs represent *cross-scale* interactions. Letters A and a denote potential mode and B and b the kinetic mode.

Bracket products with same case letters are *even-valued* (commutative), those with mixed cases are *odd-valued* (anticommutative).



The diagonal (solid) arrows indicate *same-scale* interactions, while the vertical (dashed) arrows indicate *cross-scale* interactions in both the (upper) digraph and the (lower) table.

These are alternative but equivalent representations of the supersymmetry algebra entailed by systems that interact according to the First and Second Laws.

Figure 2: Supersymmetry Algebra for Coupling of the First and Second Laws

Conversely, where the product is reciprocal, ab + ba = 0, then it is symmetry-breaking in the sense of only preserving an improper (reflective) symmetry, or anti-symmetry. Where order is important, the operation is said to be *anticommutative.* Interactions that are both mutual (commutative) and reciprocal (anticommutative) are not simply symmetrical but symmetrical and antisymmetrical at the same time. Such systems with operations that are both commutative and anticommutative are sometimes said to be supersymmetrical (Freund, 1986). (Contrast this with vector analysis which has two distinct operations—the commutative dot (scalar) product and the anticommutative cross (vector) product).

More specifically, to any pair of macro-modes, A and B, or micro-modes, a and b, interacting at the same scale according to the First Law, we associate an anticommutative bracket product [a, b]= – [b, a] of their commutative operations, [a, b] = ab – ba = – (ba - ab) = – [b, a], to stand for the symmetrical, or mutual dimension of their interaction. Likewise, for modes operating across different scales according to the Second Law, we associate a commutative bracket product of their anticommutative operations, ab + ba = [a, b] = + [b, a] = ba + ab, to stand for the antisymmetrical, or reciprocal dimension of their interaction. The signature for the (Grassmann) algebra of this interaction within and between scales can then be represented as a 2 x 2 array with symmetric rows and antisymmetric columns. Notice that Figure 2 is just a more abstract rendition of the coupling of the First and Second Laws represented dynamically in Figure 1. The common ingredients across these different representations are symmetry-preserving and symmetry-breaking processes acting at the *same-scale* and *cross-scales*, respectively.

A final claim we wish to make, to which the *cross-scale* methodology is committed, has to do with linearity of the coupling of the first principles. If the First and Second Laws are coupled by supersymmetries, are they linearly or nonlinearly coupled? If linearly coupled, then the systems they govern will be ultrastable and not evolve. On the other hand, if they are nonlinearly coupled, then they will not be perfectly equilibrated but will be only metastable, which means they can drift, be nudged, or shocked into instability. Ultrastable systems undergo no catastrophes, and exhibit no chaotic dynamics.

Hence we conclude that the laws must not be linearly coupled. If so, then although the physical ramifications of this fact may be enormously complicated, its abstract expression might nevertheless be simple. One would expect this to show up as the supersymmetry brackets for interactions between complex systems being equal to something other than zero. We will not, however, pursue this point any further at this time.

2. Gradient Sets and Critical sets

If two or more systems interact, then there is a subset of solutions from the union of their total solution sets (including the null set) that can be used to understand the outcomes of their dynamical relationship (i.e., their differential equation set). This subset is called *the critical solution set* and defines an important set of symmetries existing between the two solutions (Figure 3). These symmetries consists of all of their shared solutions (i.e., of the roots of their first-order equations that remain invariant under some symmetry operation, say as expressed by a Lie group).

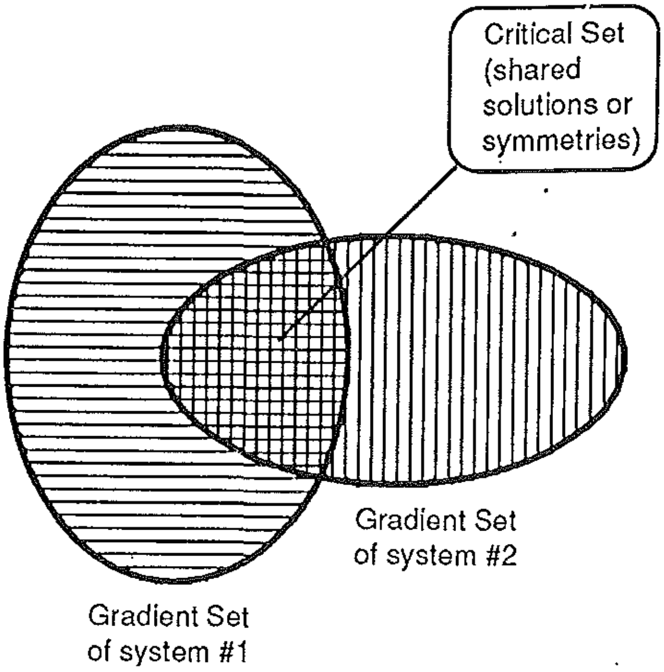


Figure 3: The Critical Set of Symmetries (shared solutions) Between the Gradient Sets of Two Systems. Given all possible measurements or observations on two or more related systems, then some of the functions defined on their properties will covary (represented by the intersection). This set of symmetrical functions defines the *critical set* of properties that contains all common solutions shared by the systems' gradient sets. The critical set of covariant properties (symmetries) defines a prototypical unit of analysis for any system that is not isolated but has a nontrivial environment , i.e., one in which it necessarily interacts with other systems. To emphasize this point, let us refer to non-isolated systems as belonging to an *ecosystem.* (More will be made of this point later).

A Mathematical Note on Gradient Sets. Traditionally, sets of vectors have been confused with their associated gradient sets [16]. The vectors that interest us are never free but are always *bound* to an intrinsic, physically interpretable geometry by a dual construct called their *covector*, or 1-form. The notion of a *gradient set* (as we use it) derives it name from the gradient, or 1-form, of a directional derivative rather than from the concept of force. Force is sometimes misleadingly defined as *the gradient of a potential* with which it is associated but to which it is not identical. A *force on a system* is a vector defined as the *(time) derivative of the rate of change of momentum* of the system. A vector is orthogonal to its gradient, or covector and, hence, cannot be identical to it. Classically, however, these two concepts have been confused because they always have the same scalar value.

A *gradient set*, then, is the set of differential (Pfaffian) forms that give direction to the tangent vector field on which the set of all the orbits or trajectories of the system is defined. With this distinction made, let us apply it to the clarify the concept of a critical set as the set of shared solutions between two or more gradient sets.

The intrinsic approach requires that the notions of vector—a derivative concept—and covector (or, 1-form)—a gradient concept—are kept separate and treated as having an analogous dual relationship to one another. This mathematical duality (a kind of reciprocal, nontransitive, isomorphic mapping) holds for vector and covector as it does for their higher-order field theoretic cousins, *streamlines* (i.e., differential lines of force—a vectorial concept) and equipotential *curves* (i.e., lines of equal force—a gradient concept). These curves need not be functions of potential energy but may be defined on total energy, adiabatic invariants, or anything else that remains unchanged during a period of observation. Thus we need to generalize these concepts to non-potential fields. Kay [17] puts it this way: "Dynamical systems theory allows us to consider attractors (i.e., critical sets) due to any time-varying system, even ones where the concept of energy is fundamentally meaningless ..."[18]. Thus we have done so by generalizing equipotential curves to *iso-similarity contours* and streamlines, or lines of force, to *first order integral curves of the relevant dynamical invariant.* (In Figure 1 these integral curves are designated as values along the vertical axis).

Indeed, the dynamical geometries work just as well without a potential function if their distribution function supports a Lyapunov function with negative characteristic over some nontrivial space-time interval. Natural systems, of course, may also have chaotic ranges where the Lyapunov takes on positive characteristics. Hence we must leave room in this intrinsic approach for systems that operate in their state spaces near the transition to chaos. It is likely that in such near-chaotic regions, information for survival is most rich and organisms may profit from it and be most fit [19]. The nonlinear coupling of first principles allows for this fact.

2.1 Critical Sets as Observables

Figure 4 shows two boats in three different situations. The *gradient sets* for boats #1 and #2 exhibit three distinct *critical sets* of mutually exclusive shared solutions. Critical set (A) exhibits an invariant future point-solution (a *zero-order symmetry*)—a target point—with a shared solution represented by a pair of converging "lead" vectors; critical set (B) exhibits an on-going invariant line segment solution (a *first-order symmetry*)—a distance invariant—with a solution represented by parallel "lead" vectors; while critical set (C) exhibits an invariant past point-solution (also a *zero-order symmetry*)—an origin point—with a shared solution represented by a pair of "history" vectors. Let the pilot of one boat select a fixed line-of-sight from a marker on his/her own boat to a marker on the other boat. If the two markers remain

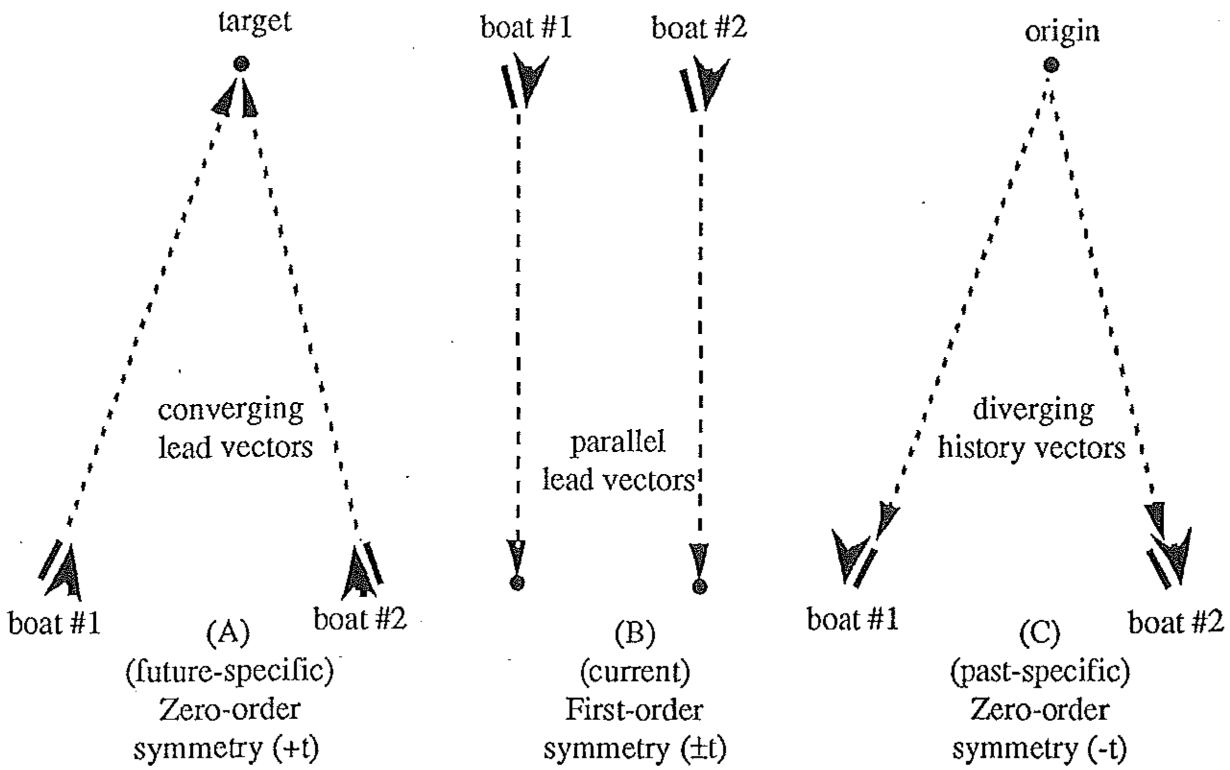


Figure 4: Critical Solution Sets. Gradient sets for pairs of trajectories with disjoint critical sets of shared solutions (assumed here to be extrinsically embedded in a 2-dimensional Euclidean space).

coincident on the same line-of-sight, then this specifies that the two boats share one of the three critical sets.

Thus all of the solution sets (past, present, and future) are directly specified by a spatio-temporal invariant that can be observed. Critical sets of shared symmetries, therefore, provide a unique class of observables that express most elegantly one system's interaction with another system. If the two systems are organisms and their environments, then critical sets must exist that likewise specify shared symmetries. James J. Gibson [20] has identified these critical sets for ecosystems and called them *affordances*. Ecological psychology is the discipline that has elected to study this important class of observables.

From this example, we see that the symmetry of the shared dynamical solutions of two systems, such as two boats navigating at sea, enters at different orders: For example, to move on a collision or rendezvous course the boats must share a *zero-order symmetry*—a point-solution. Hence a collision will be avoided if and only if there is no point-solution shared by the projection of their lead vectors. A collision requires a time-positive, zero-order dynamical symmetry so that neither boat gets to the shared-point too early. Other possibilities are to move on parallel or diverging courses. To move on parallel courses, the two boats must share a *first-order symmetry* relation—a fixed distance must be maintained between the two boats. On the other hand to move on (rectilinear) diverging courses, they must share no more than a *zero-order symmetry*, defined in the time-negative direction by projecting their history vectors to some hypothetical point of origin.

In Figure 5 we see that the geometric singularities and geodesics of the manifold coincide with the shared solutions of each of the three pairs of trajectories. This solution is intrinsically motivated where the solutions in the earlier Cartesian case were extrinsically motivated (Figure

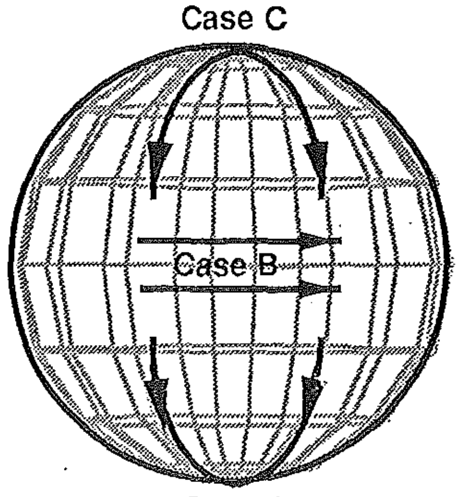


Figure 5: Gradient Sets for the Same Pairs of Trajectories Defined Intrinsically on a Spherical Manifold

4). By placing this spherical manifold in a work space or a gravitational field, one obtains a consistent physical interpretation for the gradient sets and hence for their critical sets. (For instance, instead of two boats under their own power—a work space—imagine that the trajectories belong to water droplets adhering to and running down a ball—a gravity field). These geodesics would correspond to minimal work paths, and their critical sets of shared solutions could then be given a consistent physical interpretation. Contrast this with the previous flat Euclidean space where no specific physical interpretation was given for the critical sets of shared solutions. Our interest throughout will be on gradient sets whose symmetries are determined by physically consistent, intrinsic geometries rather than physically arbitrary, extrinsically defined ones.

Thus we see that dynamical outcomes of two systems interacting may be predicted if the order of symmetry of their shared solutions between their gradient sets can be determined. These ordered symmetries comprise their critical set of shared solutions between their gradient sets. Searching for the critical set of shared solutions, as the logical intersection of two gradient sets, is a useful general strategy for predicting and classifying system interactions. Given all possible trajectories of the systems in question, then the symmetries across their gradient sets specify the total set of shared solutions of all possible orders. This is always the smallest and most informative set of observables.

The critical set, being symmetries, also has the most epistemic directness because their description requires the most "shallow" logic of description possible. Symmetries are directly specified by the property conserved, and are not symbolic or mediated by any other property. For instance, as paradigmatic of direct specification, recall the invariant mapping of the two markers on ones own boat and the target boat. The invariance (coincidence over time) of these markers, defined within a line-of-sight geometry, provides direct specification (in spatial degrees of freedom) that is invariant over time for non-local space-time events (parallel courses, future collision point, or past common orgin). The fundamental importance of critical sets is their ability provide the lowest-order available information (as geometric or kinematic invariants) for kinetic events that are spatio-temporally remote or extended.

2.2 Same-scale and Cross-scale Critical Sets

Cross-scale analysis necessarily requires the interaction of two systems not of the same kind. Any mutual compatibility upon which cooperation is to be based, or that competition might destroy, requires shared solution sets. What is the origin of the dynamical effects for which solutions might be shared between systems? Throughout we will argue that critical sets that are intrinsically specific to a system's dynamics arise as a consequence of the dominance of the *up-down* (*cross-scale*: macro<=>micro) dynamics over the *side-side* (*same-scale*: micro <=>micro or macro<=> macro) dynamics.

Let a system be represented by a set of trajectories within phase-space—where each trajectory is a member of a one-parameter family of solutions to the dynamical equations of the system. The fundamental insight of the *up-down* approach that we will draw upon repeatedly is that irreversible transport processes originate from a nonlinear coupling across scales.

(1) Linear critical sets (Figure 6). The shared solution sets for classical dynamical systems exhibit only symmetries that are linearly defined at the *same-scale*. These are point-to-point (or component-to-component) relationships. In such geometries there are singularities but no attractors, no cross-trajectory symmetries (expansions or contractions), and no *cross-scale* symmetries. The geometry that organizes the field of such systems assumes a flat space where

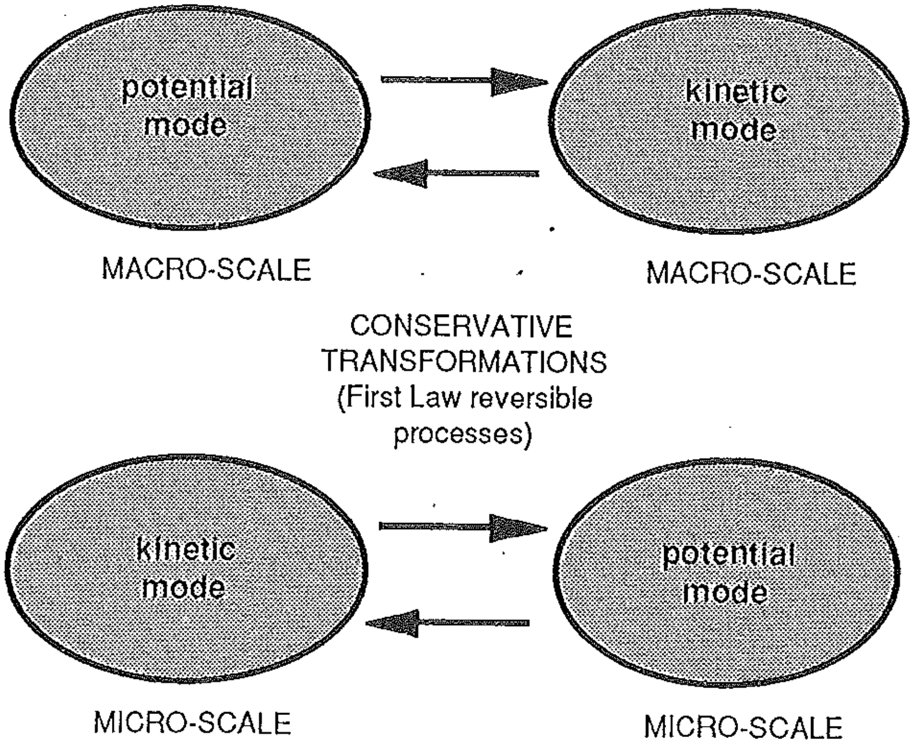


Figure 6: Same-scale Interactions at Two Different Scales. Interactions across the same scale are, in spite of the symmetry-breaking that increases the order of modes, inherently reversible. Recall that in conservative (Hamiltonian) systems the Second Law determines potential energy flow, while it is only the total energy (energy in both the potential and kinetic modes) that is conserved. Hence the First Law applies globally while the Second Law applies only locally (in space-time). This means that none of the gradients across *same-scale* modes are irreducible, and that interactions are only temporarily dominated by the Second Law, being both initially and finally dominated by the First Law. Hence *same-scale* conservative dynamics implies the linear independence of the first principle coupling.

time is treated as an extrinsic rather than an intrinsic variable. Such interactions, therefore, take place against the background of space-time rather than being intrinsically assimilated within its structure.

It should also be noted that different initial conditions never share common solutions. This is reflected by the fact that they are represented by non-intersecting closed orbits in phase space; hence they can share no solution which would appear as an intersection of orbits.

(2) Nonlinear Same-scale critical sets. Before *cross-scale* dynamics can emerge, there must be within-scale variation between the trajectories of phase-space; that is, there must be a gradient set defined on some conserved quantity. Two symmetries that are antisymmetrically related are possible under such between-trajectory variation.

This situation is represented in Figures 4 and 5. For the boats useful (macro-scale) work is done by them in the direction of a target (a nonlocal constraint) against the (micro-scale) retarding force of friction (a local constraint). Similarly, for the water droplets work is done on them by gravity, in the downward direction, against both the adhesive force and the frictional retarding force. (Without the adhesive force the droplets would simply drop off of the sphere at its equator). The relationship between the macro-scale (work) interactions and micro-scale (adhesion and friction) interactions is nonlinear, but they are treated classically as if they were at the *same-scale*. This is done by representing the micro-scale interactions as coefficients of constraint in the macro-scale equations governing the behavior of the systems in their respective work spaces.

Alternatively, the first step toward understanding the nonlinear coupling of the First and Second Laws is to recognize the need for intrinsically defined *cross-scale* interactions (Figure 7). This must precede the next step of understanding how they might be coupled into the same dynamical regime. Phase space descriptions of nonlinearly related *cross-scale* interactions provides an important tool for such intrinsic analyses. In phase space, the macro-scale interactions are exhibited by expansion of the orbits (*escapement*) or alternatively as contraction of the orbits (*dissipation*). But in the classical analysis these would be represented alone, as transformation of the trajectories toward one of either extrema (maxima and minima, respectively).

(a) *The Critical Set of a Contraction Gradient.* The systems' trajectories may agree in direction by diminishing (through dissipation) to a point-equilibrium (a minimum) as time goes to infinity. At infinity the micro-scale and macro-scale share the same solution. Here a singular value, a point, is the critical (shared solution) set for all trajectories. It is called a *point-attractor*. Hence (as in the boat case) we recognize this as a dynamical geometry where the behavior of a family of systems (i.e., their gradient set of trajectories) share a *zero-order* symmetry. This provides one possible interpretation for Figure 1b.

In order to reach this symmetry, the gradient must collapse so that there is no more transport (distribution) of the conserved quantity. This happens when time is taken to infinity. To avoid having to suppose that time goes to infinity, thresholds on the the macro- and micro -scales might be set. Extrinsically determined thresholds provide only an arbitrary short-cut to finite solutions, and are a ploy of *side-side* thinking. A less arbitrary approach is to treat the system fractally so that when it is rescaled, the limits fall out as a natural property of the intrinsic geometry (Figure 16).

(b) *The Critical set of an Expansion Gradient.* Variation of the systems' trajectories may again agree in direction. But in contrast to the contraction dynamic, the common behavior is to expand (through an escapement) from a common center. Here, under indefinite time-backward integration, this point is also a critical set for the gradient sets of trajectories. It is called a

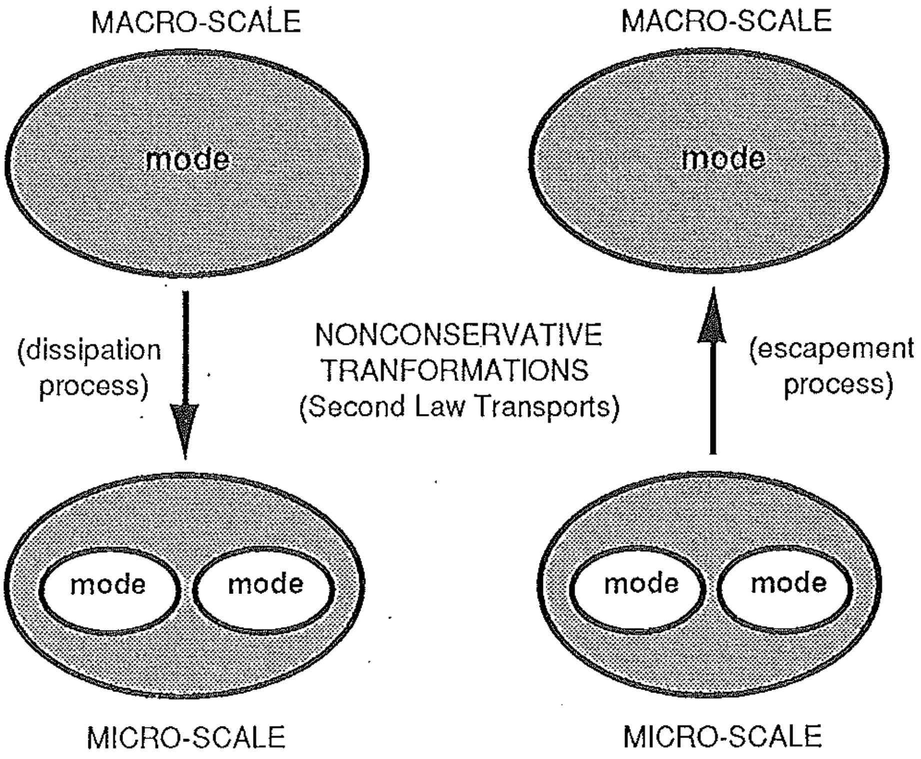


Figure 7: Cross-scale Interactions without Macro- or Micro-same-scale Interactions. The Second Law dominates *cross-scale* interactions in either of two forms of gradient sets: the dissipative and escapement. In its *dissipative* form, ordered macro-modes are degraded into those micro-modes that have the least order among the available micro-modes. In its *escapement* form, micro-modes of low order are upgraded into macro-modes.

point-repellor. Again we recognize this solution as belonging to an intrinsic dynamical geometry where the behavior of a family of systems share a *zero-order* symmetry. This provides a possible interpretation for Figure 1c.

Taken alone, these unidirectional gradients may only become *cross-scale* at limit: On the one hand, the contracting gradient set of phase space trajectories reaches down, asymptotically, from the dissipative macro-scale as if seeking the critical-point solution (the point-attractor) at the micro-scale but never reaches it in finite time. On the other hand, the expanding gradient set of phase space trajectories reaches up, asymptotically, from the escapement-driven micro-scale as if to break away from the push of the critical-point solution (the point-repellor) at the macro-scale but never escapes its push in finite time.

These single optimality principles, one seeking the minimum the other seeking the maximum, are each held captive by a *same-scale* dynamic—escaping to the macro-scale and dissipating to the micro-scale, respectively. By balancing these opposing dynamics, we obtain a *mini-max* optimization principle that determines a new dynamic operating reciprocally across the two scales.

(3) Nonlinear Cross-scale critical sets: Limit Cycles (Figure 8). Although systems governed by an attractor or a repellor exhibit dynamical geometric symmetries, they are only of the lowest order. There are no *cross-scale* critical sets (symmetries) until we have a system governed by a higher-order dynamical geometry. *Cross-scale* critical sets require a balance between opposing attempts to reach the micro-scale and the macro-scale—a tension that coordinates lower-order antisymmetrical gradient sets. From this tension between a pair of competing

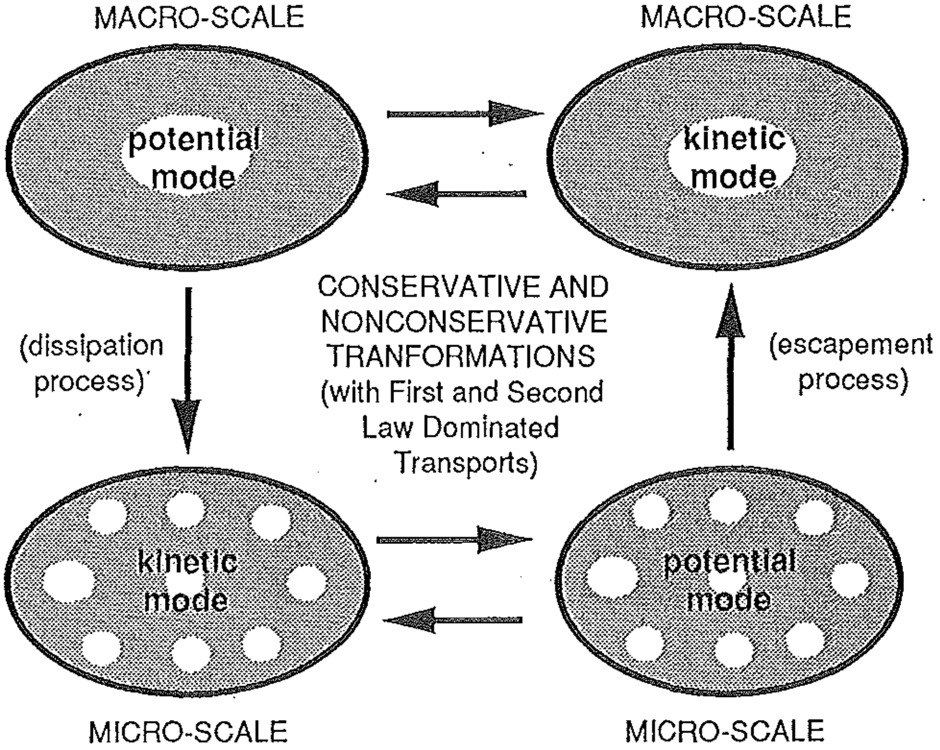


Figure 8: The Limit Cycle: Cross-scale Interactions Coordinating with Same-scale Interactions. By a balanced nonlinear coupling of dissipative and escapement interactions across the macro- and micro-modes a new dynamical regime arises in nature. In its *dissipative* form, ordered macro-modes are still degraded into those micro-modes that have the least order among the *available* micro-modes, but now only micro-modes of non-zero order are available. The range of the dissipative variable is no longer independent of the range of the escapement variable. This constraint is the source of the nonlinearity in their coupled interaction. In its *escapement* form, micro-modes of low order are upgraded into macro-modes with the degree of order satisfying the *mini-max* balance existing between the expansion (repellor) and contraction (attractor) dynamics. When confined to the dimensionality of the phase plane, this solution shared by the two point-dynamics determines the well-known critical set called the *limit cycle.*

lower-order symmetries (the point-solutions) arises a higher-order symmetry. This higher-*cross-scale* symmetry is not a product of an attempt at ideal optimization, but of a mini-max strategy, where the system's gains are maximized rather than optimized.

Minimally, the lowest-order system that can exhibit nonlinear critical sets must be at least of the first-order. Such a system must impose a geometry capable of composing two zero-order symmetries into a first-order symmetry. This can only happen if the system is sufficiently complex to dynamically coordinate an attractor dynamics with a repellor dynamics. When this is done successfully so that the two gradient sets now exhibit common solutions in terms of trajectories rather than points, then we are said to have *a limit-cycle solution* as the relevant critical set. Limit cycles provide a nonlinear cross-scale invariant analogous to the linear *same-scale* symmetry depicted in Figure 1a.

Systems with limit cycles are capable of higher-order critical sets. This takes us closer to the tools needed to study complex systems. Furthermore, when the system is sufficiently complex to have multiple potential wells, then saddle-points exist between each pair of wells. Furthermore, because the wells have competing attractor dynamics, the system can exhibit, in addition to periodic orbits of the limit cycle type, the erratic aperiodic behavior popularly labelled as "chaotic" dynamics.

2.3 Hyper-Critical sets for Systems Governed by Irreducible Dynamics.

Of course, we might continue to extrapolate from lower-order to still higher-order critical sets if the corresponding higher-order symmetries can be recognized in the behavior of systems. For instance, still higher-order systems might coordinate gradient sets consisting of limit cycles into a *ring* of such cycles, and therefore, be governed by a set of critical sets whose shared solutions exhibit correspondingly higher-order symmetries. As we shall illustrate next, it is our contention that the critical sets governing the interactions of natural systems that perceive, act, and exhibit intentional behaviors are the result of just such evolutionary extrapolations to higher-order critical sets. The shallow supersymmetry logic of such systems, regardless of order, is expressed by the digraph and table of functors shown in Figure 2.

It is important to note that not all contemporary approaches to complex systems use this *cross-scale* strategy for modelling as proposed here. Indeed, most never stray from a single scale of analysis. For instance, the *parallel-distributed-processing* approach seems quite content with *side-side* reasoning. It seeks no such higher-order critical sets but confines its search strategies (e.g., simulated annealing) to a single potential manifold, letting the search strategies relax onto a zero-order (point) solution set.

Each critical set is a *mode* into which a conserved quantity is organized. Higher-order critical sets, that is, *a critical set of critical sets*, posit higher-order modes into which a system (of conserved quantities) might be organized. Consider: To stay alive, any organism must conserve gases (breath), liquids (blood), and solids (tissues) or else its gradient sets (respiration, circulation, metabolism) will be reversed and become disordered under the Second Law.

Gibson [20] has defined each life-supporting property of the environment common to a kind of organism (termed an "affordance") as "an invariant of invariants." This is identical with our notion of a *critical set of critical sets* (a symmetry among symmetries) denoting *a shared solution among the solutions shared by two or more systems.* If we add to this that such higher-order solutions that must be sustained by the tension set up by the Second Law competing with itself over the micro- and macro-scales, then we have the possibility of systems with irreducible dynamics because their gradients are irreversible. Such systems live because their geometries are constantly under tension from forces (the gradients of their potentials are not zero).

Contrast this *cross-scale* approach to *same-scale* approaches. Whereas the former is characterized by irreducible multi-modal interactions, the latter are characterized by interactions ultimately reducible to a single-mode. Hence they are ultimately governed by the First Law alone rather than the coupling of the two laws as equal partners. They allow action by the Second Law only locally, that is, only on a portion of the mode. Any actions that globally relax onto the entire mode necessarily destroy all gradient sets by rendering them maximally symmetrical, or flat.

With all of the potential degraded to the lowest-order available micro modes, there can be no attempt of any mode to redistribute; hence stagnant puddling occurs under the aegis of the First Law alone. This yields only reversible gradients because, at limit, they collapse onto the symmetrical distribution of the conservation. Thus each single scale approach ends by killing off its own dynamics, resulting in a dead geometry, that is, a Euclidean flat manifold. Notice that this constitutes a violation of the supersymmetry constraint because there is no anticommutator.

In the next section we sketch a system capable of *autocatalytic* dynamics, that is, a dynamic which feeds off itself and bootstraps from lower order modes of organization to higher order ones, through a regime where escapement processes dominate dissipative processes.

2.4 Thermodynamic and Epistemic Engines

Every living system depends upon shared solutions between its modes of energy and information organization and those of other systems in its environment (e.g., the shared solutions between the chlorophyll limit cycle of plants and glycolysis limit cycle of animals which allows them to feed synergistically off of each other). But what is the scale of analysis that guarantees the dynamics required to support the higher-order shared solutions between an organism and its environment? We call any system that supplies the dynamics for another system, an *engine* for the system it supplies.

There are two kinds of engines—*thermodynamic* and *epistemic engines* [9]. Tradition has it that *same-scale* physics is the study of thermodynamic engines, while *same-scale* psychology is the study of epistemic engines. *Same-scale* biology, under the traditional scheme, is the study of their coupling. This leads to the inevitable paradoxes of *mind-body dualism* and *mind-matter dualism*, and the temptation to offer, therefore, a *mind-body identity* or psychological, biological, physical *reductionism* to escape the perplexities of dead-end theorizing.

Cross-scale theorizing offers an alternative. We propose that *cross-scale physics* be the study of the coupling of the epistemic engine with the thermodynamic engine across the scale of critical sets of solutions shared by interacting systems *without* complex interiors, that *cross-scale biology* be the study of their coupling with reference to the shared solutions of systems *with* complex interiors that interact with those *without* complex interiors, and that *cross-scale psychology* be the study of their coupling with reference to interactions among systems *with* complex interiors. Finally, since the macro-scale space necessarily provides the environment of the micro-scale events, its seems to us appropriate to call any *cross-scale* science—an ecological science. Hence these job descriptions apply to ecological physics, ecological biology, and ecological psychology, respectively.

It is for this reason that we believe all sciences are properly carried out as multimodal *cross-scale* investigations of nature, and yet that they are distinguished by complementary methods and aims. Under this view, there can be no dualism nor reductionism. In the next section, we illustrate the *cross-scale* approach to science by examples that demand analysis at all scales of complexity.

3. Self Organization in the Far-From-Equilibrium Region

The dynamics of a system is assembled out of competing tendencies between the *cross-scale* dynamics and *same-scale* dynamics. At equilibrium all *cross-scale* transport processes vanish. The system is in a state defined by *reversible (within-scale) transports* that alternate between time-dependent (kinetic energy) and space-dependent (potential energy) storage modes (Figures 1a and 6). There are no cross scale transports at (global) equilibrium. In near equilibrium conditions there are *cross-scale* interactions. The most notorious *cross-scale* transports are the *dissipative (frictional) processes* that transport energy from macro storage modes (motions in a macro frame) into micro storage modes (motions in a micro frame, such as heat) (Figure 7—left). The less heralded transport processes involve the movement of energy from micro storage modes to macro storage modes. These transports are termed *escapement processes*. A micro—>macro transport process constitutes a thermodynamic energy pump (Figure 7—right).

The Second Law is a statement about the temporal stability associated with the storage modes of a system. It states: In an isolated system the most stable storage mode is the most micro-scale available under existing boundary conditions. With time (as time goes to infinity) the system will exhibit a natural tendency to evolve onto the most micro available storage mode. Put differently, structure, by definition, refers to time-independent storage modes, and function to time-dependent storage modes. All macro storage modes will tend to be torn down with time. Macro storage modes can be assembled and maintained if and only if energy and matter are 'pumped' into the macro modes to offset the macro—>micro transports driven by the Second Law. If the system is to exhibit a stable macro storage mode, then the macro—> micro (dissipative) transport must be of a higher order then the micro—>macro (escapement) process. The only additional requirement for the emergence of a stable macro storage mode is that at least one of the *cross-scale* transport processes must exhibit a nonlinear force-flow relationship. No stable macro storage modes can emerge, and be sustained (structurally stable), if all the *cross-scale* transport processes are linear. Moreover, as noted above, if only one of the transport processes is nonlinear (higher than first order), then that process has to be the dissipative process.

Whereas traditional mechanical and thermodynamic models invoke predictions about forces and entropy, the new models invoke predictions about qualitative similarities within, and between, behavioral trajectories (i.e., the evolution of system states). The new predictions identify generic ways in which behavioral trajectories qualitatively unfold towards states of increasing order, i.e., the forms they embody as they self-organize. Accordingly, this new physics is sometimes referred to as a physics of self-organizing systems.

In contrast to the existing assumptions, Prigogine and his colleagues found that when their chemical systems were displaced far from (global) equilibrium, the linear relaxation dynamic broke down and was replaced by a nonlinear dynamic that drove the system further away from equilibrium. They found that in the far-from-equilibrium region a new thermodynamic path (branch) existed that yielded constructive effects as a by-product of the dissipative processes associated with the Second Law. In the far from equilibrium domain systems suddenly exhibited intrinsic tendencies to spontaneously self-organize. In recognition of the central role played by dissipative processes in self-organization, Prigogine [5] termed these open systems "dissipative structures."

3.1 Intentional Dynamics: A Social Insect Example

A model of nest construction by social insects is presented here to illustrate the principles of self-organization in a complex system. This model incorporates the paradigmatic properties of *cross-scale* interactions as well as the indispensable *same-scale* interactions minimally required for a system to carry out an intentional behavior. Among the required interactions are the *cross-scale* coupling of a thermodynamic and an epistemic engine. The specific feature of this *cross-scale* coupling is the *autocatalytic* mechanism by which the goal-directed work (e.g., nest-building) of the ecosystem (termite population + environment) is carried out. If we are correct in our hypothesis, then the paradigmatic properties of this socially complex system exploit externally, and thus make visible, the same principles that govern the less visible, inner workings of individual systems with complex interiors (i.e., with central nervous systems).

The insects of interest are African termites, who periodically cooperate to build nests that stand more than 15 feet in height, weigh more than 10 tons, and persist in excess of 300 years. The feat is made even more remarkable by the fact that each termite works independently of each other termite, being locally controlled by pheromone (molecular) distributions that arise from materials excreted by the termites themselves and then strewn by them around the building site, at first randomly, and then in increasingly more regular ways. The pheromone-laden, excreted building material dictates the patterning of the collective insect activity which, in turn, determines the remarkably novel architectural structure that ultimately arises from this dynamically improvised plan.

The Attractor Dynamics of the Intentional (but Unintended) Plan. This construction process involves the coordination of more than 5 million insects, and results in the recursive evolution of a set of macroscopic building modes: random depositing—>pillar construction—>arch construction—> dome construction—>random depositing—>... and so on. Each mode is separated temporally from the previous mode by a change in the qualitative structure of a pheromone field that relates insect motion to spatial coordinates of the building site. The qualitative structure refers to the global diffusion pattern of pheromone flows.

The qualitative structure of the diffusion field can be classified by the layout of local regions in the pheromone field where the gradient vanishes. These regions are technically termed *attractors*: An attractor is a solution shared by multiple trajectories originating from different initial conditions. The attractor is a global symmetry that relates local trajectories. The attractor defines a set of solutions shared by all trajectories in the local neighborhood of the attractor. The local trajectories either converge or diverge from the attractor.

In physical fields, the attractors define local regions in which the potential energy gradient degenerates (goes to zero). The region surrounding the attractor is the *basin* of attraction. The basin is defined by all the gradient flows that converge or diverge from the attractor. The attractor defines an invariant solution for all initial conditions started within the attractor's basin, as time goes to infinity. These attractors define global organizing centers for local trajectories. The flow pattern is globally organized by the layout of attractors in the work space. If the layout remains constant under an action (i.e., a transformation) on the system, the pattern defining the flows is also stable. If an attractor is created or annihilated, however, the pattern defining the flow will become unstable (the pattern defining the flow will change topologically). Instability in the flow field is a function of the creation and/or annihilation of one or more attractors.

Graded Modes of Same-scale and Cross-scale Interactions.

Space. From the brief sketch of this intentional activity, we can begin to appreciate the nested complexity that a system with intentional dynamics must have. For example, a macro-spatial structure (the building site) and a micro-spatial structure (the excreted building materials) are lawfully coupled across scales. But what does the coupling?

Geometry. They are coupled by the geometry of stationary properties of the pheromone diffusion process. The macro-scale pheromone diffusion geometry both regulates and is regulated by the micro-scale geometry comprised by the individual trajectories of the insect population. But what couples the interactions of these two dynamical geometries across scales?

Fields. These geometric modes of organization are coupled by the macro-scale of the pheromone diffusion field which supplies the information field that guides the interior (micro) field processes of each individual in the insect population. But what couples these two fields across scales? Fields can only be lawfully coupled by an engine. What kind of engine?

Engines. The engine must be complex. There must be a *thermodynamic engine* which does the work (e.g., flying, excreting) on the macro-mode field but which is sustained by a micro-mode field (metabolically complex interior)—in the sense of receiving a negentropic boost. Additionally, there must be an informationally tuned, or *epistemic engine* which is guided by the micro-mode field (pheromone molecules) but is sustained by a macro-mode field (e.g., food stuff that is later excreted). The running of these two engines determines the perceiving-acting cycle; the cycling gets the job of nest-building done. But what couples the two engines across scales?

Systems. Engines can only be coupled by systems that support intentions, say, the intention to build-nests. This highest mode of organization that lawfully couples the two engines across scales is called an ecosystem. Ecosystems exist as micro-modes within the broader context of evolutionary physics which produces chemistry, biology, and psychology it its wake.

Thus geometries couple spaces just as fields couple geometries, engines couple fields, and systems couple engines. Our thesis is that this hierarchy (or lattice) of modes of organization over spatial and temporal degrees of freedom is a cosmologically defined ladder that physical, biological, and psychological systems have climbed over eons of evolution. Each next step is but another coupling of first principles that produces a higher-order mode.

In the following elaboration of the selected goal-directed activity, this implicit hierarchy of first principle couplings, as expressed at the spatial, geometric, field, engine, and system scales of organization, will be made explicit through diagrams that show the micro-macro- modal coupling of the two laws. Notice, therefore, that each coupling makes possible the lawful interactions of modes both across-scales and within the *same-scale* It is also important to emphasize that the "mechanism" by which coupling is achieved is autocatalytic or self-assembling, self-regulating, and self-sustaining over the spatio-temporal period required to fulfill a stipulated intention at the ecological scale.

It should be noted that for an *intention at the ecological scale* to be fulfilled does not require that an individual component of a system (e.g., the organismic engine) necessarily be aware of the global consequences of its actions, but only that the intention be globally manifest in the long range coupling of the laws that produce the appropriate action of the ecosystem, such as nest-building. Thus, this is not a call for an anthopomorphic theory but for an intentional dynamics. Indeed, in the sense used here, an action can be *intentional* without being *intended* [10; 21].

3.2 Insect Nest Construction: Emergence of New Cross-scale Symmetries

Each spring termites develop a sensitivity to a pheromone secretion in their waste. Once this waste has been deposited atmospheric diffusion of the pheromone creates a gradient field that can 'orient' nearby insects. The recent deposit lies at the center of the diffusion field; technically it can be referred to as an equilibrium point—a point in a gradient field where the gradient vanishes. Each deposit temporarily defines the spatial location of an equilibrium point relative to the pheromone diffusion field. In our current terms, the pheromonal diffusion determines gradient sets whose critical set of shared solutions consists of zero-order symmetries (point-sets, or attractors), first-order symmetries (line sets, or streamlines), second-order symmetries (equipotential shells enclosing the singularity), and so forth.

The diffusion field spreads out in accordance with Fick's law, which relates the rate of flow to the gradient of the field. As time passes the amount of pheromone at the equilibrium point decreases (a dissipative process), scaling the field gradient accordingly. Eventually the concentration of pheromone at the equilibrium point approaches that of all points in the gradient field at which time the system reaches equilibrium. At equilibrium there are no gradients (grad = 0 for the entire field) and, therefore, no local equilibrium points. If only a few insects participate in nest-building the depositing is so infrequent that the pheromone field of recent deposits goes to global equilibrium before another insect can be influenced by the deposit. Hence *use-*

ful information exists to specify the dynamical plan of the architecture only if the field is stationary relative to the density and rate of travel of the insect population.

Perceptual Tolerances. The behavior of insects during nest construction is organized by an evolution of relatively stationary attractors in the pheromone field. A change in the attractor layout induces an instability in the pheromone flow pattern. The instability drives the system to a state of greater order, as instability begets self-organization. This is the autocatalytic action of the perceiving-acting cycle which defines the running of the epistemic engine.

As noted above, the insects relate to the pheromone field through a perceptual coupling. The perceptual coupling only links the insects to the pheromone field in regions of the building site where the pheromone concentration exceeds a critical activation threshold for their perceptual system. Once insects enter an activation region they follow paths mapping the streamlines of the pheromone gradient. The insects' journey up the gradient terminates ultimately at the region of maximum concentration—the equilibrium point. On arriving at the equilibrium point the insects deposit their waste. With the loss of their waste material, the insects lose their pheromone affinity and cease to be oriented by the pheromone field. The pheromone affinity returns with the build-up of new waste material in the insect.

Random Deposits: An Equilibrium mode. In the first phase of nest building the motion of insects is only weakly coupled to the motion of pheromone molecules since only very small localized regions contain enough pheromone to exceed an insect's perceptual limit. The result is a random depositing mode of nest building (see Figure 9). The motion of the insects is essentially independent of the motion of the pheromone molecules. In the absence of regions of high concentrations of pheromone, the depositing pattern is dominated by random fluctuations. Gradient dynamics on the pheromone field play no role in the organization of insect motion. The motion of the insects is at equilibrium with respect to the pheromone field when the pheromone gradient is uniform. In the equilibrium mode the motion of each insect is independent of every other insect; no preferred depósit sites orient insect flight patterns (i.e., no local equilibrium points organize the field dynamic). Random depositing persists as long as the number of insects participating in nest building is small.

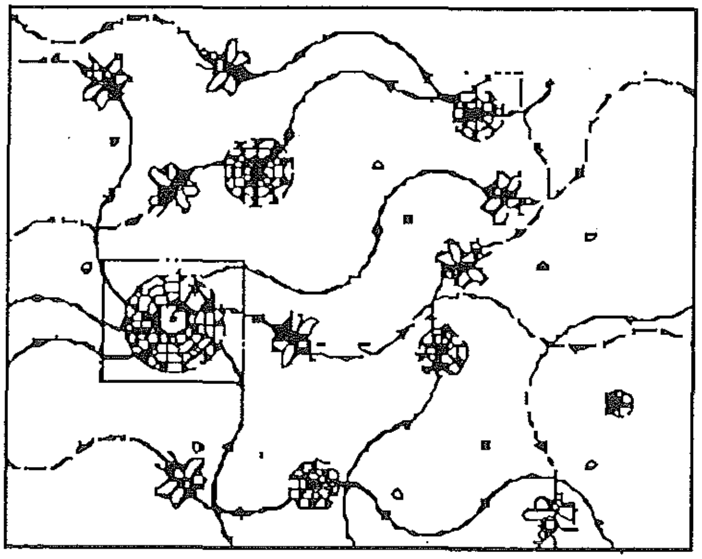


Figure 9: Termite Zeroing in on Random Deposit Sites

Pillar Construction: A Near-Equilibrium mode. As more insects participate the likelihood of an insect passing an active site increases. Beyond a critical number of participating insects, the equilibrium condition of the flight pattern breaks down, and a small number of preferred deposit sites begin to emerge (see Figure 10). Increases in the rate of depositing on preferred sites increases the size of the gradient field that attracts the insects, which, in turn, increases the size of the gradient field, ... and so on. As the size of a deposit site grows, long range coordination patterns begin to develop among the flight patterns of insects, as more and more insects begin to orient their motion to the pheromone field. The result is an autocatalytic reaction resulting in a rapid amplification of material deposits at points of highest pheromone concentration (equilibrium points).

As the autocatalytic reaction continues, a pillar begins to be shaped out of the waste deposit (see Figure10). The pillar is constructed at the location of the equilibrium point, with only the top of the pillar remaining an active deposit site. As the pillar develops the inactive portion of the pillar becomes a material instantiation of the equilibrium point that was previously instantiated in flow-field dynamics. The pillars that persist are "memories" of the equilibrium points that were dynamically embodied in the pheromone field. The pillars act as constraints on the insects flight patterns long after the pheromone field has gone to equilibrium.

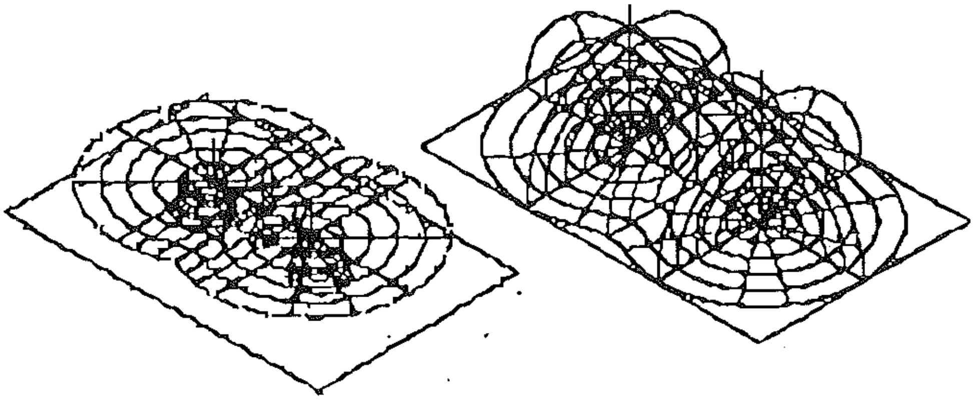


Figure 10: Pheromone Diffusion Gradient Surrounding Two Pillars. Because the two pillars act as competing attractors for the termites, a *saddle-point* is created between them. Here we see a 2-D (left) and 3-D field of equipotential curves radiating out from the deposit sites soon to become pillars.

Arch Construction: A Far-from-Equilibrium mode. During pillar construction, active deposit sites contain only one equilibrium point. While there can exist multiple active deposit sites, none of these sites contains multiple equilibrium points. Put differently, there are no pheromone gradients that are constructed out of competitions among multiple equilibrium points. As the size of the active gradient regions enlarges, competitions begin to develop between gradients generated by neighboring equilibrium points (pillar sites). This competition occurs when the active portions of the gradient fields begin to overlap. Out of this interactive competition are born saddlepoints that organize the interface boundary separating the two gradient fields. As the system is displaced further from equilibrium, the linear dynamic (associated with relaxations to a single equilibrium point) breaks down as competitions begin to develop between neighboring equilibrium points. In this far from equilibrium region, multiple equilibrium points begin to compete for local control over insect trajectories. The linear dynamic (linear flow-force relations) is replaced by nonlinear dynamics (nonlinear flow-force relations).

The saddlepoint breaks the symmetry of the location of deposits by introducing an inward bias in the direction of the competing equilibrium points. The addition of this bias adds a curvature to the pillar that results in the construction of an arch (Figure 11). While the saddlepoint defines a local symmetry-breaking transformation in the depositing activity at the pillars, it also defines a more global symmetry-preserving transformation that relates the gradient fields of the two competing equilibrium points. The saddlepoint defines an invariant solution that satisfies simultaneously the local gradient field constraints of both pillars. The saddlepoint is a higher-order attractor defining a symmetry that is invariant over the two competing gradient basins. The construction of the arch emerges out of the more global symmetry of the saddle-point. The saddlepoint symmetry is used to coordinate the unfolding trajectories of the two local attractors defined at the tops of the pillars.

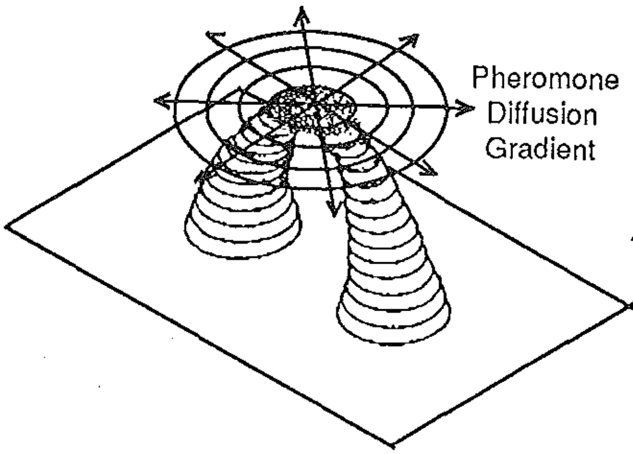


Figure 11: Arch Formation

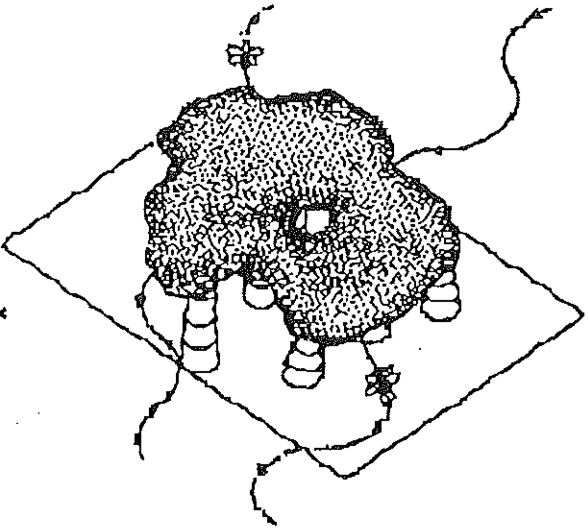


Figure 12: Dome Construction

Dome Construction: A Return to the Equilibrium mode. The completion of the arch is associated with the coalescing of the two pillar equilibrium points with the saddlepoint, resulting in the annihilation of the saddlepoint and the emergence of a single equilibrium point at the top of the arch (Figure 11). Gradient flows emanating from the new equilibrium point interact with neighboring gradient flows, resulting in the emergence of an intricate pattern of new saddlepoints. These saddlepoints organize a new gradient layout that, in turn, provides new constraints which coordinate the construction of a "dome" (Figure 12).

Upon completion of the dome the far-from-equilibrium condition is annihilated; this results in a return to the equilibrium mode. A new construction cycle then begins, starting with the random deposit phase on the surface of the dome. The system begins another cycle through the sequence of construction modes (random deposit—>pillar construction—>arch construction—>dome construction—>random deposits... and so on.

3.3 Self-assembly of an Epistemic Engine

The insect behavior both contributes to the structure of the pheromone field and is oriented by the structure of the pheromone field. Insects contribute to the pheromone field through their frequent deposits. The collective depositing onto a small set of localized regions defines an escapement process. The escapement is the transport process that moves energy from the micro storage mode associated with the coordinates of motion defined by individual insects into the macro storage mode defined by the localized deposit site. This transport process assembles localized potential reservoirs that then provide the requisite forces that drive the diffusion processes. The insect transport of pheromone from many spatial coordinates into a few spatial coordinates defines a thermodynamic pump. The insects' transport processes create potential reservoirs that sustain the pheromone gradients. The thermodynamic pumping up of potential in the work space is the replenishing phase of a thermodynamic cycle.

The diffusional flows that transport the pheromone from the macro storage modes into micro storage modes define the tearing down phase of the thermodynamic cycle. The pheromone gradients, powered by the macro potential deposit sites, provide the constraints that orient the insects' depository activity. In this regard the nest building system is exemplary of a self-reading and self-writing system. The circular linking of the replenishing and tearing down phases through a perceptual coupling forms a closed thermodynamic-epistemic engine cycle (Figure 13). This engine is driven by a reciprocal causality of the following kind: force field (muscular activity)—> flow field (pheromone control constraints)—> force field (muscular activity)—> flow field (pheromone control constraints) ... and so on. which can be described alternatively as an action—> perception—> action—>...cycle.

While the cycle is closed in terms of the reciprocity relation between forces (kinetics) and flows (kinematics), it is open in terms of properties that constitute the self-descriptors. New properties (self-descriptions) can emerge out of competitions between equilibrium points (attractors) in the closed force—>flow—>force loop. The closed loop of transport processes exemplifies an open, thermodynamic-epistemic engine that is functionally organized out of couplings involving transports of both kinematics (spatial and temporal patterns) and kinetics (energy and momentum). The loops define a system satisfying requirements for both a perceptual realism and a source of novelty: The reciprocal interlocking of the force—>flow—>force loop guarantees the preservation of realism; remaining open to the generation of new informational primitives within the loop preserves the possibility for novelty.

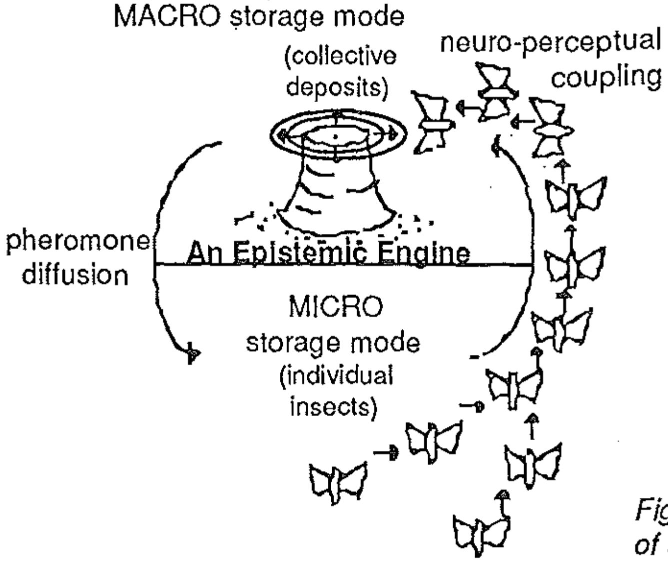


Figure13: The Autocatalytic Cycle of a Nest-building Epistemic Engine

3.4 Nest-building as a Self-organizing system with Intentional Dynamics

Insect nest-building exemplifies a self-organizing system in which new order emerges out of a competition between replenishing (escapement) processes and dissipative processes. The proposed mechanism reveals dissipation as an active contributor to this ordering process. Whereas the Second Law has been viewed as a destructive agency that tears down order, a new view of the Second Law has emerged recently that views it as an active participant in constructive processes. This conclusion has followed from the theoretical and experimental findings indicating that systems open to the import of high grade energy (replenishing processes) and the export of low grade energy (dissipative processes) can develop new symmetries that lead to new intentions exhibited as attractor states. The new attractors emerge out of the competition between the dissipative and escapement processes. The attractors are invariant solutions (symmetries) that relate the micro states of a system to macro states. The construction process follows from the nonlinear coupling of the First and Second Laws, and is fundamentally organized by this critical set of *cross-scale* (macro-micro) symmetries. Figure 13 provides a concrete instance of Figure 2 defined over an informational escapement and a molecular diffusion dissipation.

4. Perceptual Information as the Critical Set of Morphological and Growth Gradient Sets

The intrinsic approach assumes that geometrically defined gradient sets, namely, those that are defined over observable differences in size, shape, or distance, will often share solutions with those of physical fields, for example, geometric gradient specifying age-level of a face and the gradient sets of craniofacial growth, as determined by genetic and epigenetic constraints (e.g., gravity). A line of research that one of us (Shaw et al. [22]) began nearly two decades ago was founded on this as its methodological axiom. The truth of something like Packard's conjecture, namely, that information is potentially richest near regions of impending chaos, was also one that implicitly motivated the more recent stages of this research project.

For instance, orthopaedic and orthodontic treatment plans typically capitalize on the chief property of a chaotic dynamic—that small changes in initial conditions over short durations can have major effects over longer durations. Assume that a pre-adolescent child falls and damages the condyle (the TMJ region where the mandible inserts into the skull). If left untreated, this brief traumatic incident can cause arrested growth at this site, so that during the growth spurt the jaw remains juvenile while the rest of the craniofacial complex matures rapidly

Luckily, stress can be a direct stimulus to growth as well (Wolff's Law). The application of forces by engineered appliances, such as braces or stress-frames, can minimize treatment costs (e.g., money, time, and suffering) and maximize beneficial outcomes. Treatment, a controlled trauma, can also apply corrective stresses to direct the growth processes just before the onset of adolescent growth spurts—a range presumed to be near the transition to a temporary period of metabolic chaos. For the past decade, the Growth Project at the University of Connecticut has been working to develop intrinsic dynamical geometries to model the craniofacial growth processes and the interaction of treatment with it [22; 23; 24; 25; 26].

Over the years, we have become increasingly more committed to the *cross-scale* approach to this problem because psychology, biology, and physics all play indispensable roles in the construction of such growth models. As psychologists, we began with the question of how one perceives age-level and, to our surprise, found that the primary source of such information is

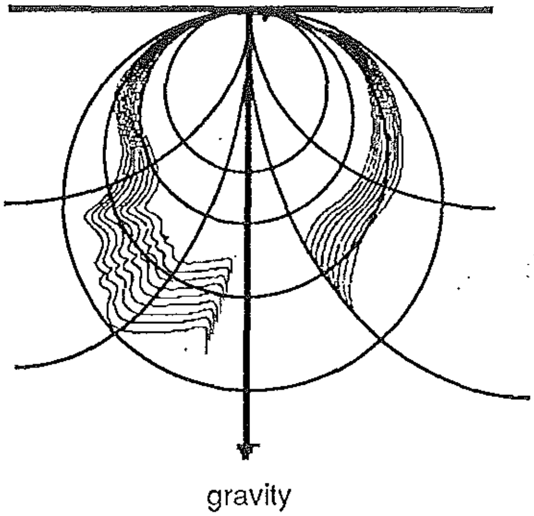


Figure 14: Gravity as an Epigenetic Constraint on Craniofacial Growth

gravity (Figure 14). Thus we could no more avoid the physics of the problem than we could the psychology or biology. It is instructive and illustrative of the *cross-scale* approach to see why this was so.

As the only tidal force acting asymmetrically on the head (the other tidal force—atmospheric pressure acts symmetrically), gravity induces the stresses on hard tissues that induce a piezo-electric field effect by which ionized salt deposits are attracted to their boney matrix sites. This may be the epigenetic director of genetically produced growth processes. Evolutionary design of a structurally sound head is one that will withstand the frequent enormous mechanical forces of mastication, the gentler but unrelenting hydrodynamical forces of breathing and swallowing, and so forth. It seems reasonable to assume that through natural selection the morphological design would have optimized the epigenetic constraints for a functionally fit structure.

Likewise, we have found evidence that leads us to believe that perceived attractiveness of faces, by which growth treatment plans are set, is due to growth trajectories remaining geodetic in the same models. This has important implications for the dynamics of intentional systems. We will return to this issue after introducing the important concept of intentional dynamics.

Intentional Dynamics of Treatment Planning. D'Arcy Thompson [26] suggested that *the policy of growth is to be symmetrical.* But all growth is not symmetrical, as the asymmetry of faces, fruit, and other quasi-bilaterally symmetric objects clearly shows. One interpretation is that the policy of growth is to solve the cohomology problem so morphology is remodelled over time to fit certain nonlocal boundary conditions. Asymmetrical, or, better, noncohomological, growth is then deformity—even when it remains within a normal range. When the symmetry-breaking of the nonlocal constraints is too great, then aesthetic form (e.g., facial attractiveness) and function suffers. Because growth must tessellate space-time in non-Euclidean ways, then there is a non-trivial cohomology problem to be solved by growth processes. By this policy, the intention of growth to remain symmetrical requires that local epigenetic (Second Law) processes be nonlinearly guided by nonlocal genetic constraints (First Law). The nonlinear coupling of these first principles normally achieve this intentional dynamic without extrinsic help.

Wolff's "law" that *stress is a direct stimulus to growth*, and Thompson's principle that *the policy of growth is to be symmetrical*, as we interpret them, are nothing but expressions of the

Second and First Laws, respectively, at the biological scale of analysis. The successful coupling under normal intention of these first principles produces no tissue excesses or deficiencies), but conserves biological form. In doing so, this also conserves related psycho-social functions—one of which is to be sufficiently attractive to the opposite sex so as to appear fit for mating. When the final stage of growth is reached (for the most part, around age twenty), then the Second Law collapses onto the First Law at the equilibrium point, that is, when the growth potential is exhausted.

But what about the cases when the cohomological problem is not solved by growth? When this happens, the intentional direction of growth dynamics is lost, and deformity creeps in. Under these circumstances, treatment must intercede if biological form is to remain conserved within normal bounds. To the extent that the covariant derivative of growth trajectories does not remain symmetric, growth is deformed. The *cross-scale* assumption of cohomologically conditioned intention, in fact, allows us to compare the deformed growth geometry of a patient against the ideal normal growth geometry [27; 28]. The treatment appliances define new transport processes that restore the lost symmetry by getting rid of unwanted symmetry-breaking brought on by deforming effects of trauma and disease. Here, the linear deficiency exhibited (say, by the covariant derivative being asymmetrical), provides evidence that the laws at the scale of growth processes are nonlinearly coupled as assumed.

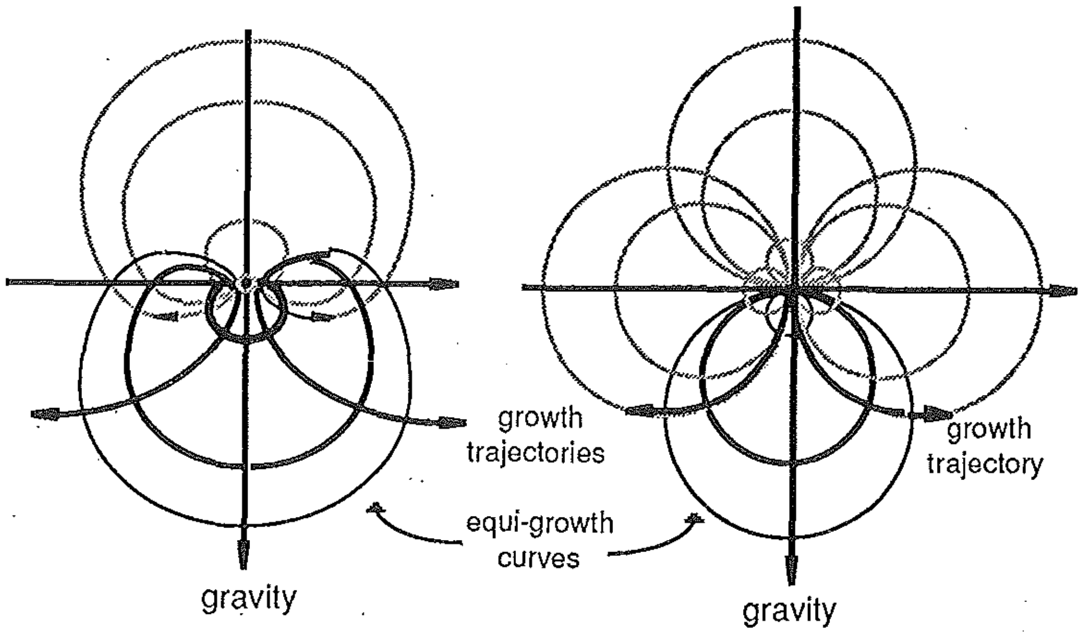
Nor can one doubt, since treatment must be designed to conserve the boundary conditions for normal growth, that the First Law operates locally through intentional application of the Second Law. If treatment can specify and pursue an intended goal, then the genetic policy of growth can specify and likewise pursue an unintended goal. In both cases, there is a symmetry of purpose that comprises the common critical set of the treatment and growth gradient sets. The manner in which either solves the cohomological problem is the content of that intentional act.

Treatment goals, like genetic policy, pose nonlinear constraints on the local application of the Second Law—the treatment forces. If the local epigenetic forces are not properly tuned by genetics to nonlocal constraints, then they must be so tuned locally by the orthodontist, who monitors their current effects, in an anticipatory fashion, against the boundary conditions (morphological goals) to be conserved. For either of these intentional dynamics to succeed, the space-time manifold of growth must be tessellated by naturally or artificially imposed treatment steps (in the manner discussed by [21]). This is a cohomological problem of the highest order, and poses a major challenge to the *cross-scale* approach to intentional dynamics.

The first step toward confrontation with this challenge is to discover the manifold for craniofacial growth for which a solution to the cohomological problem must be resolved. Currently, there are two competing models that are under investigation: what we call the *nodal point model* and the *hydrodynamic model.*

4.1 Competing Growth Models

It is clear that streamlines are directions of change, or symmetry-breaking, defined over iso-similarity contours which are themselves the stationary states of conservation-like quantities wherein symmetries are preserved. Since some abstract transport process must be defined by a style of symmetry-breaking—call it a *generalized equation of change*, then this is the point of entry of the Second Law into intrinsic dynamical geometries. Likewise, and dually, since some abstract quantity must remain in equilibrium, in balance, or be conserved if the gradient set is to be available in order to give a meaning to the direction of change—call it *a gen-*



(A) APPLE MANIFOLD (B) ORANGE MANIFOLD

Figure 15: The Dynamical Intrinsic (orthogonal) Geometries for Two Classes of Growth Systems. The manifolds (A) and (B) are but two of a family of geometries that might permit orthogonal instantiations of both the local (epigenetic) and nonlocal (genetic) constraints of a tolerably sub-optimal growth process. Geometry (A) is a nodal point growth model while geometry (B) is a hydrodynamic growth model. Although we are investigating the appropriateness of these two growth models for human heads, the issue of *cross-scale* modelling can be addressed more simply by considering their application to modelling the growth of fruit. Because the manifolds abstractly carry all the effects of the first principles, it does not matter to what the models are applied. Different objects call for only a change in the boundary conditions on the model.

eralized equation of state—then this is the point of entry of the First Law into these geometries.

To illustrate this point, consider the way hanging fruit grows. If, like an apple or orange, it grows from a nodal point at the end of a twig under the influence of a gravity, then its orthogonal geometric characterization is intrinsically specified (Figure 15). Note that the apple is invaginated at the top while the orange is not. Hence since these two fruit have related but distinct morphologies, then their *equations of state* must differ. (That is, their topologies differ in a fundamental way: the apple is a "puckered" sphere—a nonconvex manifold—while an orange is a convex sphere.) But due to the common action of gravity on them, their growth processes, or equations of change, must be similar.

Figure 15 shows two geometries whose manifolds may have the same generators as the two fruit. Hence the geometry of the apple and orange manifolds should be orthogonal, in the sense discussed earlier, so that their coordinate systems are intrinsically related to their dynamics vis a vis the nonlinearly coupled first principles. Here the craniofacial growth curves are specific to the application of the Second Law, while the morphological curves are specific to the First Law. At an abstract level, where the two geometries are equivalent under a Lie group of coordinate transformations, we find that although their structural symmetries (equations of state) differ, they share styles of symmetry-breaking (equations of change) across their respective iso-similarity (facial) contours. This transformational invariant of growth in a gravitational field comprises a critical set of common solutions (generators of the higher-order group) for the lower-order structural variants. The existence of this higher-order critical set provides the foundations for attempting a general theory of epigenetically controlled growth.

4.2 Intentional Dynamics as Illustrated by the Cohomology Problem

The insect nest-building and the treatment of cranio-facial growth anomalies are expression of systems with intentional dynamics. Intention acts as a hidden nonlocal singularity which constrains the boundary conditions on both processes. The setting up of an intentional dynamic requires the application of the Second Law, while the conserving of the intention during action naturally falls under the First Law. But what exactly are nonlocal constraints and how do they work? What sort of critical sets do they implicate? Some hints are provided by the problem of cohomology.

The Cohomology Problem. The cohomology problem is related to tiling in two dimensions or packing in three dimensions so that the manifold is covered exactly. More precisely, the problem is how do modular quantities, when distributed under only local constraints, fit together globally over the manifold that they attempt to cover. Do they leave gaps or overlap? If they do, then this gives rise to an associated question: How might the curvature (shape) of the manifold be remodelled so that the previously ill-fitting modules now perfectly tessellate it?

A simple self-similarity rule (like fractal rescaling) can be used to solve the cohomology problem exactly if the objects used have an exact relationship to the manifold to be covered. For example, for finite problems an easy solution is found if the modules are of a size, shape, and dimensionality that makes them even fractions of the boundaries to be filled. Under these homogeneous boundary conditions, the work may be carried out at any site using any number of tiles without regard to the size and shape of the containing space. All that is required is that no tiles be added or lost in the process. This is the conservation law stripped of its energetic content.

Furthermore, if we introduce timing as a new boundary condition so that the work must be completed on schedule, then we place greater demands on the required distribution function, for it must fill exactly both the available spatial and temporal degrees of freedom of a space-time manifold. This is the Second Law stripped of its momenta content. It is these versions of the first principles that are needed for *cross-scale* analyses. Whatever physical, biological, or psychological content these laws have must be added intrinsically at the scale of system organization to which they are applied. Hence any distribution function (a local Second Law constraint) will work, so long as the boundary conditions are conserved (a nonlocal First Law constraint). It is not always possible for local distribution functions to remain symmetrical with the boundary conditions, and in that sense, to conserve them.

There are two cases where nonlinearities (errors) arise because of local symmetry-breaking of global constraints. The discrete case is illustrated by the breakdown of self-similarity under fractal rescaling (Figure 16). Likewise, there are continuous cases where the Second Law fails to cooperate with the First Law. These cases of nonlinear coupling of first principles produce inter alia a change in dimensionality and curvature of the gradient sets of the manifold which in turn can give rise to new critical sets (singularities) than were originally present. Hence boundary conditions set by the First Law may be satisfied or violated by the Second Law only to give birth to new and sometimes totally unexpected boundary conditions. Strange attractors that lead the distribution function toward or into regions of chaos may be a natural outcome of a failure of first principles to cooperate linearly.

4.3 The Orthogonality Condition

Because the Euclidean coordinate systems for a flat (gradient free) manifold is undoubtedly the simplest conceptual description (e.g., consider the simplicity of its Laplacian), one may well wonder why it cannot be used at all scales of system organization. One reason is that, for a physically interpreted system, whose structure matches that of the geometric manifold, then they have as critical sets the same symmetries—the dimensionless (Lie) group generators.

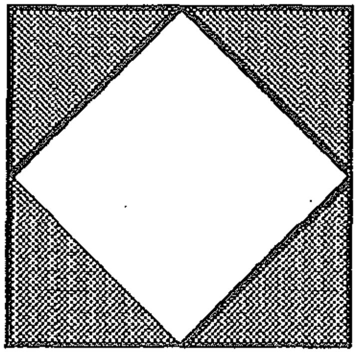
Hence all dynamical equations (e.g., the Helmholtz equation) are most easily solved in the appropriate orthogonal coordinate system where the most appropriate is the one that intrinsically describes the vectorial curves of change and the co-vectorial curves of nonchange. Finding these curves for four-dimensional curved space-time manifolds in which systems self-organize and act is the central (cohomological) problem to be addressed. To gain an intuitive appreciation of the importance of this problem, let us begin by considering a finite version of this problem in flat Euclidean space using discrete tiles.

Consider the problem of tiling a floor, as a lower-order two dimensional cohomological problem. We shall treat this as a generic case for *intentional dynamics* to study because we believe it reveals the logical prototype for an intention. The *cross-scale* approach requires that macro-scale (nonlocal) constraints, as governed by the First Law, nonlinearly constrain the application of the micro-scale constraints, as governed by the Second Law. Assuming square tiles for a square room (i.e., the orthogonality condition), then the tiles represent the quantity to be conserved, and the room the boundary values of the conservation, that is, the number of tiles to be used. The cohomology problem is how to find a distribution function by which the tiles will exactly fit the room without being added to or subtracted from. Therefore the tiles may not be cut or pasted onto. Overlapping of tiles (space-time cells), however, is permitted if all else fails, for reasons that will become clear in a moment.

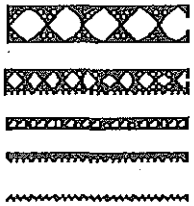
Assume that the tiles only fit the walls exactly if you start in the center of the room. All other tiles must be added to this properly placed "garden of Eden" cell; otherwise there will be overlapping on some edge. Hence having the proper initial condition is constrained by what the final condition must be. The moral of this example is that the application of a local distribution function is logically conditioned by the nonlocal final condition. Unless the local distribution law is anticipatory, conservation of the boundary conditions breaks down. This says that the First and Second Laws when cohomologically coupled, express the prototype of an intention treated here as a nonlocal constraint. Of course, spaces, geometries, fields, and engines do not have intentions—only systems do. They do share, however, under the orthogonality condition, generators at some level of abstraction that are sensitive to nonlocal constraints. It is this property that makes intentional dynamics possible at the systems level, otherwise all intentions would have extrinsic origins. They would have to borrow intentions from some already formed, unexplained agent. To have an intrinsically designed-in intention (a nonlocal constraint sensitivity) from the ground up, so to speak, makes such systems "smart" mechanisms [29].

Thus, a *system with intentional dynamics* is simply one with an operator by which local applications of the Second Law must be cohomologically conditioned by nonlocal constraints which preserve the integrity of the First Law. Structural homologies of space are inherited by the geometries that select them, just as the geometric homologies are inherited by the fields that select them, and so forth, on up the ladder from fields to engines to systems.

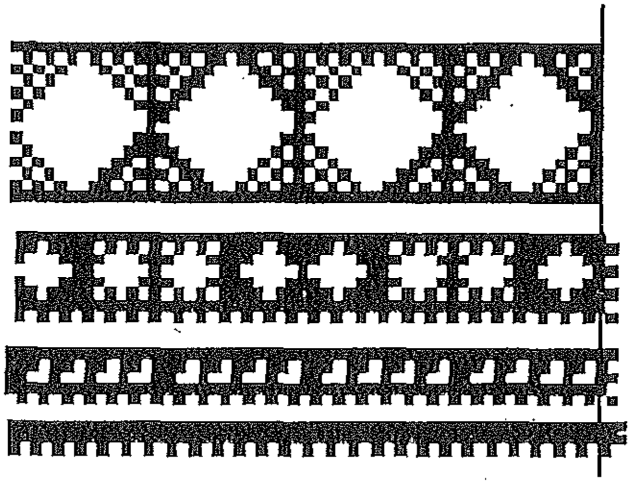
Let us consider a simple example of the cohomological problem where a system fails to inherit the homological structure of a geometry imposed on the manifold of its state-space. The



(A) Original Cell



(B) Original cell reduced and iterated at five scales



(C) Last four fractal sequences magnified 400%

Figure 16: The Break-down of Self-similarity Under Fractal Rescaling. Here the system is a MacIntosh SE microcomputer and the geometry is a fractal tessellation. By programming the computer to generate the fractal pattern, we find that the homological structure of the gradient set of the fractal geometry and the gradient set of the computer's state-space fail to share a cohomological symmetry as their critical set.

patterns in Figures 16b and 16c are derived by iterating and rescaling a single fractal unit shown in Figure 16a. Even though minification and magnification are linear transformations and should, therefore, preserve self-similarity under rescaling (i.e., remain cohomologically similar) they do not when mapped into the state-space of the (computer) system. Thus nonlinearities are created. This is because there is greater noise (rounding error) at micro-scales than at macro-scales in the computer than in the complex number space of the fractal geometry. This can be especially seen in (c) of Figure16 where the last four patterns in (b) are each magnified 400%.

Notice that not only has the geometry of the original tessellation cell been lost, but the sequence has (1) lost its symmetry at smaller scales, and (2) changed its length under rescaling—developing an overshoot. This last broken-symmetry is an alteration in the distance metric of the geometry. Hence this linear deficiency specifies the change of curvature that the space would have to undergo if self-similarity under fractal rescaling of its contents were to solve the cohomology problem. In general relativity theory, forces are associated with the change in curvature.

Such geometro-dynamical effects leach up into systems from the lower modes they organize (i.e., from their underlying engines, fields, and geometries). However, they have their ultimate origins in the recesses of the base-spaces whose homologies are inherited and organized into these higher modes. The failure for the modes to remain homologous across scales is a failure for cohomology (the meshing of homologies) to have a *cross-scale* solution. Under the *cross-scale* approach, we are not surprised that physical forces, biological processes, and perceptual information share critical solution sets on some occasions, namely, when the cohomology problem is solved across scales, but not on others, namely, when this problem is not solved. Clearly, the failure of homologies to mesh across scales is to be expected since the higher modes (e.g., engines and systems) have fewer degrees of freedom (i.e., less symmetry) than the lower modes (geometries and fields).

Under this cohomological framework, forces show up as a linear deficiency in the system's equations of change; or if represented by a tensor field in an affine space, as an asymmetry in its covariant derivatives; or if represented by Lie groups, as Lie bracket operators that do not equal zero. In short, for (bio)physcial systems there is great advantage to assuming that the generators of the forces that redistribute the system in its state space are the same (in the sense of being dually isomorphic to) the generators that change the manifold intrinsic to that state space. This is the meaning of the commitment to do *cross-scale* ecological science. This commitment reveals itself in the same way in all sciences if, as we assume, they are but different modes of organization governed by the same nonlinearly coupled first principles. This is exemplified by craniofacial growth. When the cohomology problem is not solved by the system deformity and unattractiveness result. A successful treatment plan, like a successful termite nest-building "plan", is an epistemic engine. The epistemic engine runs the perceiving-acting cyle. By doing so, it regularly tesselates space-time in a manner dictated by the cohomology condition. An unsuccessful epistemic engine creates homological irregularities that show up as failures in action coordination or as anomalies in perceptual information.

5. Concluding Remarks

We would like to conclude with a few orienting remarks for those who find this cross-scale approach to their liking.

(i) If flat Euclidean space is not the intrinsic solution to all our problems, then how do we select the appropriate non-Euclidean geometry?. This is where overlapping of modules becomes relevant. To reiterate our earlier point: The overlapping of cells in a tessellation of a given manifold tells us that some curvature other than what we have is needed to solve the cohomology problem. A solution to the cohomology problem is logically coextensive with the nonlocal conditioning of local distribution functions by an intentional operator. In this way, intentional dynamics can be placed on equal footing with other scientific theories where the locality assumption will not work (e.g., Bell's theorem). The use of cohomology theory in this regard has been spurred on by Penrose's [30; 31] twistor program, whose philosophy we find congenial to our own.

(ii) Assume that for some given manifold and some system that has inherited its homologies, the orthogonality condition is satisfied. This is a nontrivial assertion since a system has more structure (less symmetry) than its manifold. Hence a many-to-one mapping has had to be satisfied. Satisfaction of the orthogonality condition means that the system's state space has the same generators as the manifold. Therefore, if there is a cohomology problem for the manifold, then it is inherited by the system by transmission over the lattice of coupled and coupling modes described earlier. In each case, the program of explicitly modelling this heritability of homological constraints to explain various aspects of intentional dynamics will require much theory-driven research.

(iii) A need to alter the curvature of the manifold tells us that some corresponding change must be made in the system if the first principles are to apply cooperatively to satisfy a shared

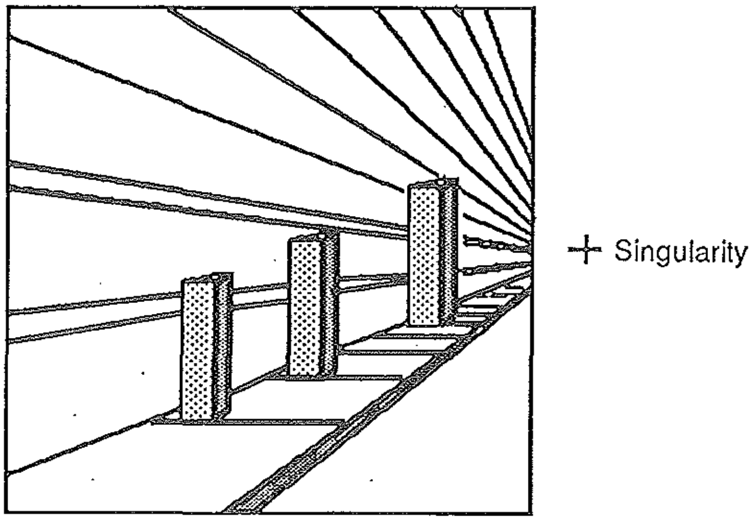


Figure 17: Warping of Distance Metric by a Nonlocal Attractor

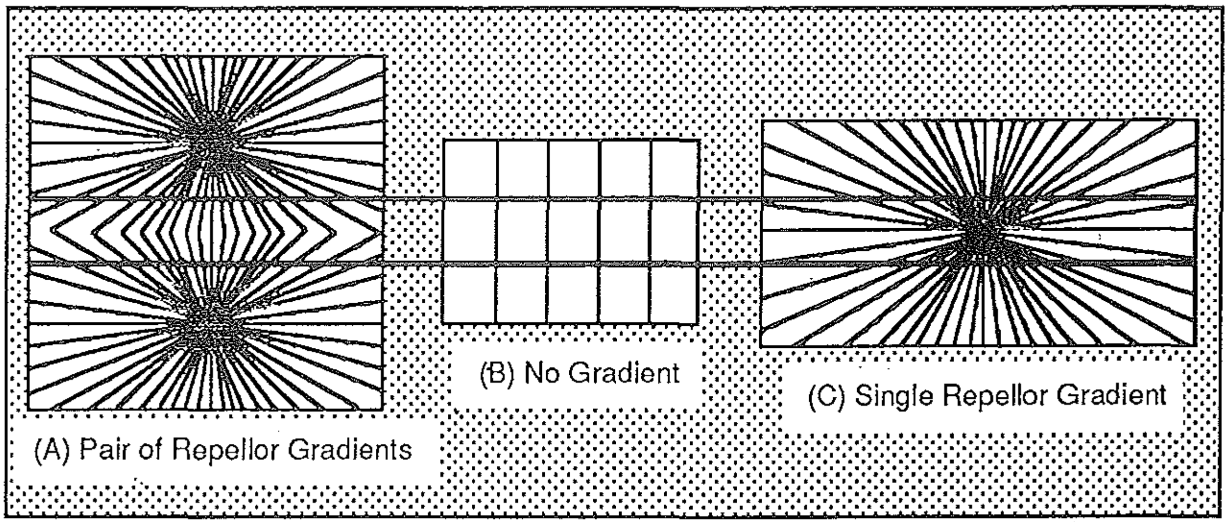


Figure 18: Geometro-dynamical Effect Manifested as Warping of Manifolds Due to Lack of Cohomological Critical Sets Across Scales. By tracking the equidistant, parallel lines depicted by the trivial gradient sets of a flat space (B) to the left (A) and to the right (C), we see what failure of our nervous systems to solve the cohomology problem means perceptually. The information for change in curvature of the lines is due to the failure of gradient sets (A), (B), and (C) to share a common homological solution. Hence the pair of lines conforms locally to the direction and distance metrics of the manifold to which they are most proximal. Our state space as observers is being warped by what it detects rather than causing the effect itself. The critical set properties have as much reality status as any other physical property, and more than most. Hence the lines are indeed curved, and they are not illusions!

solution (e.g., a goal). Curvature changes that solve the cohomological problem for the manifold produce changes in the system. Let us call this the *geometro-dynamic effect.* This effect is classically interpreted by *side-side* physics as an application of extrinsic "forces" to a system so as to nonlinearly warp its state space. From the point of view of *same-scale* psychology, geometro-dynamic effects are assumed to be nonphysical—a product of mind. For example, so-called "perceptual illusions" (as shown in Figures 17 and 18) are assumed to be self-induced alterations in one's "mental" state space—being either neurogenic anomalies or cognitive "errors" that distort perception of the world. Perhaps, there is another explanation that avoids this mind-body dualism.

(iv) Curvature of space itself, as we know from general relativity, can be the source of forces since it determines the gradient of a potential that is not zero. Recall that the gradient of a potential has the scalar quantity of a force and gives the force vector its direction. Up-down reasoning must argue that a perceived change in curvature of a graphic object, when moved from one geometric context to another, is nothing more than a natural outcome of the mismatch in homologous structures mapped across scales. If the state space of a MacIntosh SE can be warped by a geometric pattern, then so can the nervous system of a human perceiver. If so, then the so-called perceptual "illusions" are not illusions at all. Rather they are the lawful product of a geometro-dynamic effect and, therefore, as real as any other critical set shared by gradient sets.

(v) Let us pursue the analogy to general relativity, where the notion of geometro-dynamical effects was authored. If it is the curving of a space-time manifold that determines forces, then forces extrinsically applied or self-induced cannot explain the perceived change in curvature. Indeed, forces are such that they should not be reified. Einstein's equivalence principle identifies inertially produced pseudo-forces with gravitational attraction. However, it also recognizes that the latter satisfies the cohomological condition while the former does not. Here the gravitational frame, being universal, is a nonlocal constraint relative to the inertial frame, which is local. General relativity shows that gravitation effects across different space-time locales tessellates the manifold cohomologically, while inertial frames do not. Consider the inconsistency in the motion effects felt by two passengers in two adjacent outer-space elevators, accelerating in opposite directions. They both feel the acceleration as they would gravity but here it would be pulling them in contrary directions. This felt equivalence is not an illusion simply beacuse it is a locally generated effect. It is not a shared symmetry of the same general (i.e., nonlocal) caliber as universal gravity. Indeed, the cohomology approach dispels any attempt to treat the pseudo-force effect as a reified unknown force (e.g., a mental causation). The difference between pseudo and gravitational forces is simply the difference between something that fits into a critical set and something that does not—as any relativity-versed third observer outside of the two inertial frames (elevators) could determine. Cross-scale science provides a frame independent means for explaining the descrepancies—felt or perceived—between local and nonlocal constraints.

(vi) A solution to the cohomology problem requires that all of the available degrees of freedom in the space (or, space-time) be used up by the tessellation of cohomological modules. When this is so, then all of the causal efficacy of physical pseudo-forces or mental pseudo-forces is naturally assimilated into the curvature of the most fundamental space. Any leftover "force" effects must be residuals of the cohomological process, and will present themselves as nonlinearities, or hidden mental variables. If the process continues sufficiently deep into the micro-scale, regions near chaos are entered and, perhaps, crossed. When this happens, as was seen in the insect case, the old homologies of the base-space dissolve and new singularities (the ring of modes) emerge to reorganize the space into a higher-order. We see this forceless but informative geometro-dynamical effect most clearly in so-called perceptual illusions (see Figures 17 and 18).

This forceless (curvature-based) geometro-dynamical effect is of the same kind as the process by which mass singularities curve physical space-time. The major difference is that such effects may be induced through neuro-perceptual fields into the engines of thought and experience. These pockets of inconsistency are like local inertial frames; they show up as nonlinearities (usually wrongly called perceptual "errors") at the more exacting level of systems integration. Hence we see the illusions but we do not see them as being consistent over local frames. We only accept as real that which is cohomologically systematic across our experience.

This then is the distinction between the real in nature and the phenomenal. This is why science, being empirical, is justified in letting the consistency of its experimental epistemology drive its theory. This is also why physicists are usually realists (although some quantum theorists are not), while psychologists and philosophers, disrespectful of the cohomology criterion, often are not. These thinkers eschew frames of reference that conserve consistency over the sciences.

Rationalists are optimists, who believe that nature has solved the cohomology problem; dualists are pessimists on this issue; and solipsists are cynics. Phenomenalists are simply undecided. Pragmatists recognize that, although there is near consistency in nature, chaos lurks beneath the surface for those who delve too deeply into such matters. But so does the richest information about Nature.

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