

Perceiving Persistence Under Change and Over Structure

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We explored the hypothesis that to perceive an event is to detect an invariant specific to the style of change and an invariant specific to the object undergoing the style of change—the reciprocal invariants of transformation and structure, respectively. Events were defined by seven geometric transformations that produced a variety of styles of change over different objects. In Experiment 1, we examined the generative and abstract aspects of transformational invariants. An event was presented as a sequence of static samples and examined in interpolation and extrapolation tasks. The transformational invariant proved sufficiently generative and abstract to specify a unique style of change. In Experiment 2, we found that types of change and types of objects interacted, which implies that certain styles of change are more compatible with certain object structures. In Experiment 3, we found that the structural invariant was reliably detectable but that the degree to which structural properties were preserved depended on the kind and amount of transformation applied. Taken together, the experiments provide support for the hypothesis that perceiving an event is detecting the two reciprocal invariants of transformation and structure that together specify the event.

For most of us, the environment appears to be substantial and permanent, consisting, as it does, of objects such as ponds, mountains, cars, and animals. But nothing persists forever. Eventually, stagnant ponds dry up, mountains erode, cars rust, and children grow up. Things change. Change, on the other hand, is never random, but occurs in repeatable and consistent ways. Even as a ball changes its location on a moment-to-moment basis, it may trace out an invariant pattern, such as rolling. In other words, what looks orderly eventually

loses its order and becomes orderless, whereas what looks orderless nevertheless contains a certain order. As Gibson (1986) said, "The reality underlying the dimension of time is the sequential order of events, and the reality underlying the dimensions of space is the adjacent order of objects or surface parts" (p. 101). For Gibson, the conjoining of change and nonchange defines an event. Conversely, an event can be treated as having two components—change and nonchange. The change component of an event determines the particular style in which an event unfolds over space and time, for example, bouncing, bending, stretching, spinning, growing, or walking. The nonchange component, on the other hand, refers to that which undergoes the particular style of change. That is, without a ball to bounce, a card to bend, a rubber band to stretch, and so forth, an event of bouncing or bending or stretching would not be realized.

Although the environmental structure and dynamical processes just described are the sources of stimulation, Gibson (1986) contended that it is stimulus information that specifies those sources to a perceiver. In the case of vision, the optical structure at a point of observation, called an optic array, is structured lawfully in ways that correspond to permanent properties of the environment—that is, to the layout of surfaces and their material composition. The optic array is rarely static, but is constantly changing as the observer moves or as objects in the world move. Thus, like the events it specifies, information consists of a component of change and a component of nonchange. However, changes in the optic array are not arbitrary, but regular, reflecting the degree of regularity that exists in the environment. Environmental changes structure the changing array according to their underlying successive order. Similarly, environmental layout structures the ambient array according to its underlying adjacent order (Gibson, 1966; 1986). Together, change and structure determine a unique optic array at a moving point of observation.

In this sense, the optic array has been called "extended and enduring" (Gibson, 1982, p. 64). The extended component of the optic array specifies the permanent properties of the environment, and the enduring component specifies changing properties of the environment. These two components of event information have been termed a *structural invariant* (SI) and a *transformational invariant* (TI),¹ respectively (Pittenger & Shaw, 1975a; Shaw & Pittenger, 1977). Because this pair of informational invariants specifies an event, an observer can perceive—at the same time—the structure that undergoes change and the particular style of change inherent in the event (Gibson, 1975; 1986).

Change and nonchange, however, appear to be largely independent. Consider a rubber ball. Not only can a rubber ball bounce it can also rotate, stretch,

¹Our usage follows Lee (1976), who differentiated between optical variables (e.g., tau) that correspond to the visual information and physical events (time-to-contact) being specified. In our usage, the structural and transformational invariants correspond to information that specifies, respectively, the structure and transformation that compose a physical event. For alternative views of the issues surrounding this term, see Cutting (1983) and Shaw, Flascher, and Mace (in press).

and so on. Despite undergoing diverse changes, it is still recognizable as the same ball. Similarly, consider bouncing, a particular kind of change. Bouncing is not restricted to a particular ball—or even to balls in general. Many things (tires, springs) can be bounced. Despite enormous structural variation in the objects bounced, a common style of change—bouncing—can be recognized. In these examples, it appears that change and nonchange can vary independently.

If persisting structures (such as balls) under change and persisting patterns of change (such as bouncing) over different structures are to be perceived, then the corresponding stimulus information must be highly *abstract*. In other words, the information contained in the optic array must be made available, not in terms of surfaces and objects or of change, but in terms of the underlying adjacent order of the structure and the underlying successive order of the change, respectively. This is because the underlying structure of the ball remains the same despite the various styles of change it undergoes. It is this underlying order contained in the optic array in the form of the structural invariant that specifies an object we recognize as a ball. Similarly, the underlying pattern that defines bouncing remains the same despite the various structures to which bouncing is applied. It is this underlying pattern of change contained in the changing optic array in the form of the transformational invariant that specifies a particular style of change we call bouncing.

The observed independence of event components (i.e., structure and change) has been identified as a fundamental characteristic of event perception that any adequate theory must address (Mark, 1979; Mark & Todd, 1985). Other observations, however, argue against our assuming the complete independence of structure and change. Clearly, not all physical objects have the material properties to support any style of change. For example, consider rocks and ponds. Neither rocks nor ponds have the necessary material properties to support burning. Consequently, burning may not be apparent if applied to rocks or ponds. It is also the case that under certain styles of change physical objects lose their material properties, which eventually leads to the loss of their identities. For example, the structural invariant specifying “ballness” may not be revealed when a snowball melts. Taken together, these observations suggest that the two components of an event (structure and change) may not be completely independent of each other; but instead each may be mutual and reciprocal relative to its counterpart. Similarly, each component of event information (that is, the structural or transformational invariant) may not be absolutely abstract; instead, each may be relative to its counterpart. Hence, an adequate theory of event perception must not only address whether each component of event information (i.e., the transformational and structural invariant) specifies its corresponding event component (structure and change), it must also address the degree to which event information must be abstract in order to support event perception.

Another aspect of event perception that must be addressed is that information available in the optic array is continuous and infinite, whereas organisms operate discretely and finitely. That is, an organism cannot sample the un-

bounded array continuously but can only sample a finite region discretely. In spite of this limitation, observers still can perceive the properties of an event because as the eye successively samples the changing array over time, the observer detects the successive order of change from the array. Similarly, as the eye scans the ambient array, the observer detects the adjacent order of structure contained in the array (Gibson, 1966). Although discrete and partial in nature, this sampling and scanning process presumably allows the observer to detect information that is sufficiently generative of both change and structure to specify an entire event.

In this sense, the information for perception is said to be both timeless and formless (Gibson, 1986). It is timeless because past memories or future predictions are not necessary to perceive change. It is formless because object perception does not reduce to frozen perspective glimpses (Haber, 1983). The enduring component of the optic array, the transformational invariant, specifies not only perspective invariant aspects of the event, but retrospectively and prospectively invariant aspects as well (Gibson, 1975; for further discussion of issues regarding the generativity of event information, see Shaw, Flascher, & Mace, *in press*). The extended component of the optic array—the structural invariant—specifies, not only perspective appearances but also the unbounded and stable properties of the environment (Gibson, 1966; 1986). Hence, information in Gibson's sense is not only abstract but also generative of the respective orders of change and structure.

Our study was designed to test specifically the hypothesis that event perception involves detecting an invariant specific to the style of change and an invariant specific to the object undergoing the style of change—the transformational and structural invariants, respectively. The experiments were designed to investigate the extent to which the two components of event information (the structural and transformational invariants) are abstract and generative. To this end, stimuli were created by applying a variety of geometric transformations to different objects to produce a variety of events. Previous research has shown that geometric transformations are often perceived as distinct styles of change when applied to a variety of objects (Mark, Todd, & Shaw, 1981; Mark & Todd, 1985; Pittenger, Shaw, & Mark, 1979; Pittenger & Todd, 1983). The application of various growth and nongrowth transformations to facial profiles and geometric objects resulted in various events, of which growth events were only one type. The variety of events simulated allowed us to address more general questions about event perception, thereby extending the scope of the previous studies that were designed primarily to describe the information that supports the perception of growing faces and to validate a model of craniofacial growth.

The separation of transformation and structure allowed us to isolate and examine independently the two components of an event, that is, permanence and change, and the abstract and generative nature of their corresponding event information. For example, in one condition, a transformation was applied to a variety of objects. It was hypothesized that, if event information (specifically, the transformational invariant) is indeed abstract, participants should be able to

recognize a particular style of change, even when that change occurs over different structures. In the reciprocal condition, one object was subjected to a variety of transformations. Similarly, it was hypothesized that if event information (i.e., the structural invariant) is indeed abstract, an object's structure should be easily recognizable even when it undergoes different styles of change. If, however, either invariant is not recognizable with equal facility throughout the entire range of its counterpart, this will suggest that the invariant is not recognizable to the same degree under the varying conditions and, therefore, that the two event components are not independent, but are to some extent mutual and reciprocal. Moreover, the degree to which each invariant is revealed in relation to its counterpart will provide some indication of the invariant's degree of abstractness.

To demonstrate the generative nature of event information, the stimuli in the studies reported in this article were presented in a series of static displays. Although an event is arguably more than the dynamical subsequence of event samples, we assumed that a subsequence of dynamically presented samples would be sufficiently generative to specify the successive order of the entire event (Jenkins, Wald, & Pittenger, 1986). Our rationale for using static displays can be justified by the unusual effectiveness of comic strips and graphically illustrated instructions for assembling complex objects. To be as effective as they are seems to demand that the successive scanning of these static samples must be functionally equivalent to the detection of continuous unfolding of a dynamic event. The effectiveness of static displays has been demonstrated empirically by Peterson (1974), who showed that orbiting or rotating events were equally well specified when the samples detected were presented at apparent motion rates of successive sampling or well below such rates where no apparent motion was seen. In both cases, perceptions seemed to be the same. From this, we hypothesized that if the transformational invariant is indeed generative, then participants should be able to recognize a spatiotemporally extended event from a sub-sequence of event samples. Similarly, if the structural invariant is truly generative, participants should be able to recognize the structural properties of the object undergoing changes from the partial glimpses of the object discrete samples allow.

Three experiments were conducted in this study. In Experiment 1, we examined the generative and abstract aspects of the transformational invariant; in Experiment 2, we addressed the issue of the relative persistence of a style of change *over* structure; and in Experiment 3, we addressed the question of the persistence of structure *under* various styles of change.²

²Our usage of the terms *under* and *over* is consistent with their usage in mathematics. For example, a geometric transformation refers to the functional relation among geometric objects. The usual functional notation to express this relation is $P' = T(P)$ where P is the independent variable, P' is the dependent variable, and T is the transformation. This can be read as P' is the image of P under T (Gans, 1969). On the other hand, a geometric object constitutes a set of points over which a transformation can be defined. Following this line of reasoning, we have chosen to refer to an event (that is, the application of a geometric transformation to an object structure) either as structural invariants preserved under a style of change (transformation) or as a style of change persisting over structure.

EXPERIMENT 1: THE ABSTRACTNESS AND GENERATIVITY OF THE TRANSFORMATIONAL INVARIANT

In Experiment 1, we investigated the abstract and generative nature of transformational invariants. To this end, an event was defined as the application of a geometric transformation to an object structure that was displayed on a computer screen as a sequence of static samples. One of the members was deleted from each sequence—either an intermediate member of the sequence or the last member of the sequence (see Jenkins et al., 1986, or Pittenger & Shaw, 1975b, for a similar manipulation of events). Participants were presented with a choice set of profiles from which they were to select the profile that best completed the sequence in terms of pattern of change and position in the sequence. The profiles used in the choice set may or may not have been of the same person shown in the target sequence. If the transformational invariant is indeed abstract, then, despite the fact that different individual profiles are used in the choice set than are used in the target sequence, participants should be able to select the profile that resulted from the same transformation as that which generated the target sequence.

If participants can accurately fill in the gap, this will also provide support for the generativity of the transformational invariant. As information about a style of change, samples of a transformational invariant are specific to the time evolution of the event. That is, at any given time in the unfolding of the event, the transformational invariant is the basis of information about how the event has unfolded retrospectively in the interval prior to it and how it will unfold prospectively in the interval after it.

For the sequence with the gap in the middle, participants must interpolate the missing member within the sequence—the *interpolation task*. For the sequence with the gap at the end, participants must extrapolate beyond the sequence shown—the *extrapolation task*. If the information specifying the style of change of an event is detectable over a static sequence and is indeed generative, then participants should be able to interpolate and extrapolate an event sequence with equal facility. However, to do so correctly, participants must distinguish one style of change from another as manifest in the static sub-sequences.

Method

Participants. Thirty-two undergraduates at the University of Connecticut participated in the experiment in partial fulfillment of a course requirement. All participants had normal or corrected-to-normal vision. Sixteen participants served in the interpolation condition and 16 served in the extrapolation condition.

Stimulus materials. Seven well-defined global mathematical transformations, that is, coordinate transformations, were used to produce a variety of styles of change. The seven coordinate transformations included four topological transformations, two affine shears, and a similarity transformation. The four topological transformations included a hydrodynamic transformation, a hydrostatic transformation, an orthogonal cardioidal strain transformation, and a cardioidal strain transformation. Their underlying geometries are shown in Figure 1 and their mathematical formulations are shown in Appendix A. The two affine shears were *x*-axis shear and *y*-axis shear. In addition, a null transformation was included as a control. See Appendix B for their mathematical formulations.

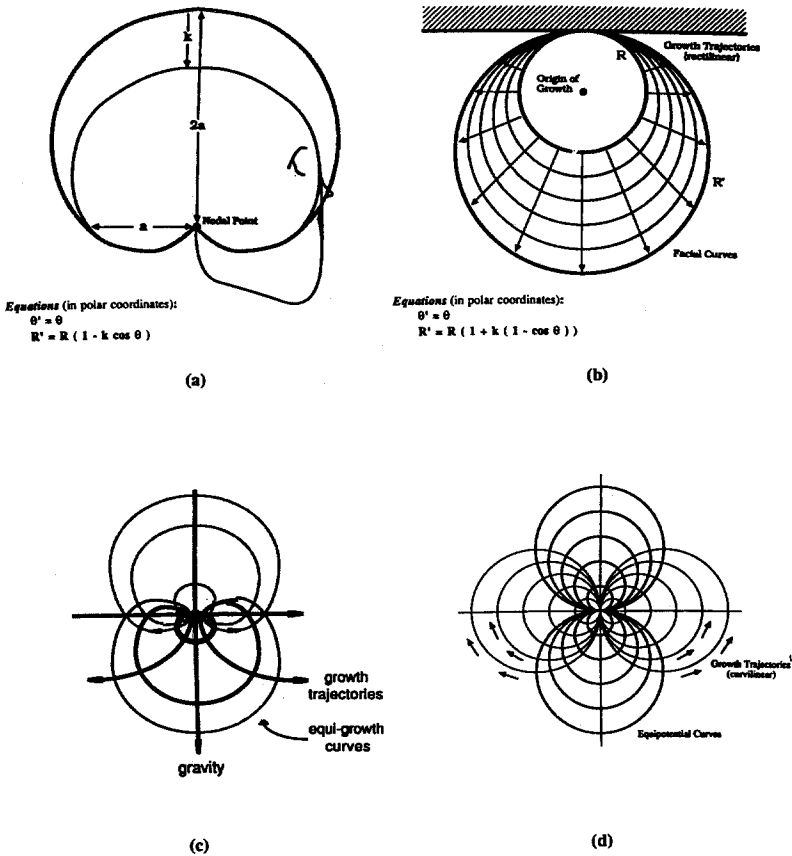


FIGURE 1 The underlying geometries for the four growth transformations: (a) The cardioidal strain model, (b) The hydrostatic model of growth, (c) The orthogonal cardioidal strain model, and (d) the hydrodynamic model of growth. See Appendix A for a detailed description of each model.

With the exception of the null transformation, each of these transformations is a nonrigid global transformation, but each differs in terms of the properties of the object it leaves invariant and the properties it varies. Except for the similarity transformation, all transformations produce remodeling (deformity) of the underlying object. In a similarity transformation, the size of the object is varied while a ratio of similarity is preserved. In an affine transformation, inner angles of the object are varied while a ratio of division is preserved. A topological transformation is the least restrictive transformation in terms of the properties preserved because even transforming a square or other polygon into a circle and vice versa is legal under a topological (i.e., homeomorphic) transformation. Nevertheless, a topological transformation preserves connectivity, endlessness of a curve, the property of being a closed curve, incidence relation, linear order, and cyclic order (see Shaw & Pittenger, 1977, for a detailed discussion).

The four topological transformations used differ by virtue of their underlying geometric invariants. For example, the ratio $1 - k \cos\theta$, in polar coordinates is invariant under a cardioidal strain transformation, and the ratio, $1 + k(1 - \cos\theta)$, is invariant under a hydrostatic transformation. On the other hand, the topological transformations are similar in that all characterize viscoelastic changes frequently observed in biological processes, including growth. In fact, the four topological transformations (hydrodynamic, hydrostatic, orthogonal cardioidal, and cardioidal strain transformations) were originally defined as growth models because they result in the perception of growth when applied to a variety of objects, including facial profiles. In contrast, the nongrowth models (*x* axis shear, *y* axis shear, and similarity) do not produce perceptions of growth. (For discussions on the motivation for developing these specific transformations, see Carello, Groszofsky, Shaw, Pittenger, & Mark, 1989; Mark, Shaw, & Pittenger, 1988; Mark et al., 1981; Pittenger & Shaw, 1975a; Shaw, Mark, Jenkins, & Mingolla, 1982; Todd, Mark, Shaw, & Pittenger, 1980).³

The stimuli used were profiles of human heads to which transformations were applied. To construct the stimuli, tracings were made of the soft tissue from X-rays of the profiles of four females, obtained from the University of Connecticut Health Center at Farmington. The tracings were digitized by a MacTablet,⁴ input into a SUN 4/260 color workstation with a TAAC-1 graphics accelerator board, and collected in a 900 × 900 pixel window to be used in developing the stimuli for the experiments. Cephalometric measurements were taken of the Articulare-Pogonion linear dimension and the facial angle. Age, facial angle,

³Although the four topological transformations were developed originally to characterize growth of biological structures; in this study, they were used solely as examples of geometric transformations that give rise to unique styles of change when applied to various structures.

⁴The number of points collected from the tracings varied for each tracing. For the four base profiles to which transformations were applied in Experiment 1, the number of points selected were 220, 253, 307, and 328, respectively.

and measures of the Articulare-Pogonion linear dimension of the four individuals used in the present study are given in Table 1.

The ranges of the transformations were equated by assigning values to the free parameter corresponding to the range of change that natural growth would produce along a profile dimension, namely, a measure of the linear distance between Articulare and Pogonion (Figure 2a). Although a measure of facial angle (Figure 2b) has been used in previous craniofacial growth research (Mark, 1979; Mark & Todd, 1985; Mark et al., 1981), the Articulare-Pogonion linear dimension was chosen because of documented difficulty in identifying the anatomical landmarks defining the facial angle (Baumrind & Frantz, 1971; Hatton & Graninger, 1958). As evidence for the reliability problem, in Table 1 we compare the relation of age and facial angle with the relation of age and the distance measure of Articulare-Pogonion based on data obtained during our own landmark identification and digitization process. Specifically, linear regressions of age against facial angle and age against the corresponding linear measure between Articulare and Pogonion resulted in an r^2 of .10 and an r^2 of .86, respectively. Similar reliability discrepancies were reported by the Center for Human Growth and Development at the University of Michigan in their longitudinal growth studies (Riolo, Moyers, McNamara, & Hunter, 1974).

TABLE 1
Age, Facial Angle, and Linear Measure Between Articulare and Pogonion for the Actual Growth Profiles Used

Individual	Profile				
	1	2	3	4	5
A					
Age ^a	4.8	7.8	10.8	19.1	
Facial angle ^b	83.2	83.3	89.9	86.7	
Art-Pog ^c	215.0	236.5	256.3	285.5	
B					
Age	3.8	4.8	11.8	13.8	18.0
Facial angle	79.3	81.8	87.5	81.8	88.9
Art-Pog	222.3	237.1	282.7	295.0	313.9
C					
Age	5.3	9.7	11.8	14.7	20.1
Facial angle	76.5	83.8	75.7	76.5	85.0
Art-Pog	212.0	256.9	262.7	288.7	295.8
D					
Age	4.8	7.8	17.1		
Facial angle	84.0	85.7	83.6		
Art-Pog	218.0	240.4	283.4		

Note. The values shown are for each of the four patients whose profiles were used.

^aAge in years.

^bFacial angle in degrees.

^cLinear distance between Articulare and Pogonion in pixels.

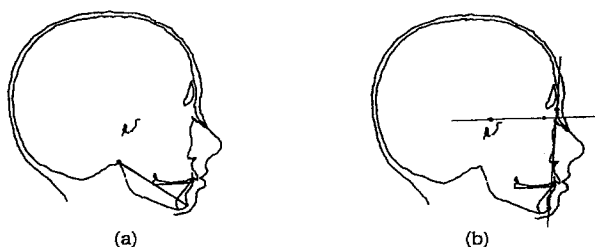


FIGURE 2 The linear dimension between Articulare and Pogonion. (b) Facial angle defined by Frankfort Plane and Nasion-Pogonion.

From each of the four profiles of women and girls, the youngest profile (corresponding to roughly 4 years of age) was selected. Those profiles were taken as the base profiles to which all transformations were applied. The total amount of transformation applied produced changes along the linear dimension that corresponded to those changes produced by normal growth occurring between the ages of 8 and 20 years. See Table 2 for the values of free parameters used in each transformation to produce changes in the linear dimension between Articulare and Pogonion⁵ corresponding to a given age.

For each trial, a target sequence was generated by randomly selecting one of the four faces and applying one of the growth or nongrowth transformations. A four-member sequence for each transformation resulted in which the amount of transformation applied to each sequence member produced changes corresponding to 8, 12, 16, and 20 years of age. A choice set of eight profiles was constructed by applying the eight transformations to a randomly selected base face. The amount of transformation applied was identical to that applied to the targeted missing member. Sequences with their inner (third) members missing constituted the interpolation task and those with outer (fourth) members missing constituted the extrapolation task.

Display. One target sequence and one eight-item choice set were displayed on a 19-in. color monitor with a pixel resolution of 1152×900 on a Sun 4/260 workstation. On the screen, a window of 1000×645 pixels was defined in which displays were drawn using Sun View graphics tools (Figure 3). This display area was further partitioned into three rows and four columns. Hence, each rectangular region was given a size of 250×215 pixels.

⁵Because this measure is of a linear dimension, measures are relevant only with respect to the original 900×900 pixel window in which they were defined. As a display window changes in size, these measures also change. Accordingly, instead of the actual linear measures, free parameter values corresponding to these measures designated the amount of transformation applied. Clearly, angular measures such as facial angle would have been preferable if such a measure could have been reliably identified in the X rays, because they remain constant regardless of size changes produced by different display windows.

TABLE 2
Amount of Transformation Applied in Terms of Free Parameter along the Dimension
of Articulare and Pogonion

Age (in years)	Articulare-Pogonion (in pixels)	Free Parameter						
		cs	hs	oc	hd	sx	sy	sc
4	222	.04	.02	.02	.03	.05	.05	.03
8	245	.18	.07	.07	.12	.24	.24	.13
12	268	.32	.14	.11	.21	.43	.42	.23
16	292	.47	.19	.16	.28	.61	.59	.33
20	314	.62	.25	.19	.35	.78	.76	.44

Note. Values have been averaged over the four patients used in the study. hd = hydrodynamic transformation, hs = hydrostatic transformation, oc = orthogonal cardioidal strain transformation, cs = cardioidal strain transformation, sx = shear x transformation, sy = shear y transformation, sc = similarity transformation, no = null transformation.

The target sequence was presented in the top row, with the missing member indicated by a question mark. Profiles were arranged from left to right in ascending order of amount of transformation applied. The eight profiles making up the choice set were distributed randomly in the bottom eight rectangles. All profiles were oriented so that the origin of the coordinate system coincided with Articulare and the Frankfort horizontal (a line connecting the top of the ear hole and the bottom of the eye socket, as shown in Figure 2b) lay parallel to the x -axis.

Design. Task was a between-subjects factor and target face, choice face, and transformation were within-subjects factors. This resulted in a 2 (Task) \times 4 (Target Face) \times 4 (Choice Face) \times 8 (Transformation) mixed design with a total of 128 trials in the experiment.

Procedure. Participants were asked to select from the choice set the profile that best completed the target sequence. Participants indicated their selection by using a mouse to position the cursor over one of the eight candidate profiles, then pressed the left mouse button to record their choice.

A short practice session was provided in which target sequences were defined by transformations of a square and the choice set by transformations of a triangle rather than by transformations of human profiles. The object to which transformations were applied was selected such that the side of the square corresponded to the base of the triangle. The range of transformation applied was the same as that applied to facial profiles. All figures were oriented so that the origin of the coordinate system coincided with the center of the figures (where the center corresponded to the midpoint on its median). The triangle was inverted with vertex down. The same task, the same transformation, and the same range of values were used in the practice session as were used in the main

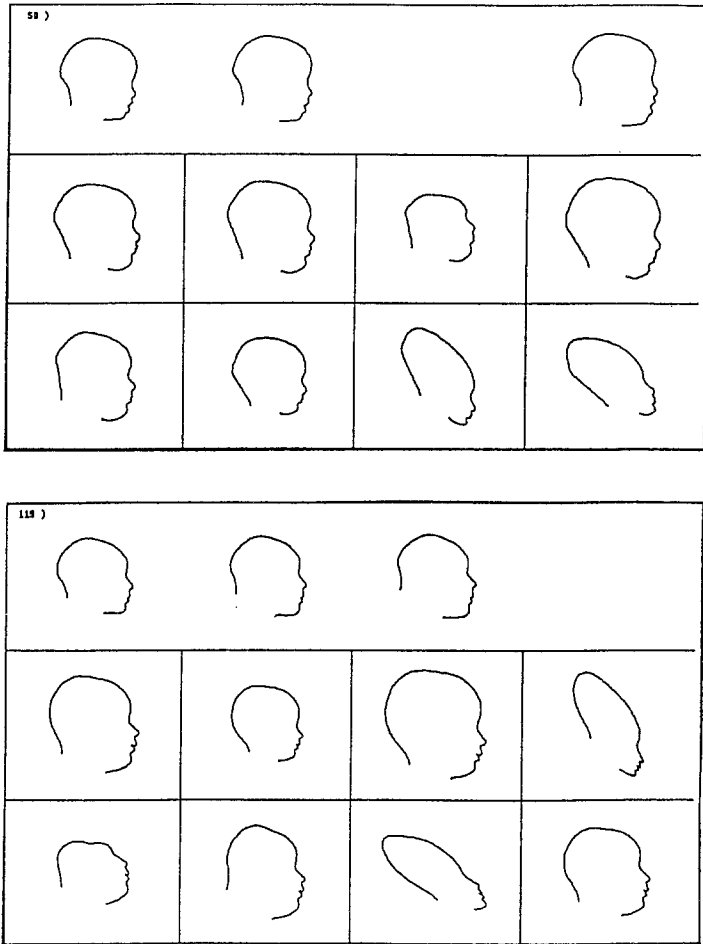


FIGURE 3 Stimuli used in the interpolation task (top) and the extrapolation task (bottom) of Experiment 1. In the interpolation task, the target sequence (top row) was composed of profiles of individual C that had undergone a hydrostatic transformation (HS). The choice set was composed of profiles of individual B that had been transformed by HS, HD, CS, SC (middle row) and by OC, NO, SY, SX (bottom row) transformations from left to right. In the extrapolation task, the target sequence was composed of profiles of individual A that had undergone an orthogonal cardioidal strain transformation (OC). The choice set was composed of profiles of individual D that had been transformed in order by HD, NO, SC, SY, CS, OC, SX, and HS transformations. To select the right profile from the choice set, participants had to either interpolate or extrapolate beyond the samples provided in the target sequence. Because transformations were nonrigid transformations, profiles in a target sequence were not identical in terms of overall shape. The choice set comprised profiles of the same individual that had been equated on the basis of the linear dimension between Articulare and Pogonion, but differed in the transformation each had undergone. The target set was composed of profiles of another individual.

session. This resulted in eight trials (1 Target Object \times 1 Choice Object \times 8 Transformations).

Participants were tested individually. The order in which trials were presented was randomized among participants. Feedback about performance was not provided in practice or in test trials. The entire experiment took about 20 min.

Results and Discussion

Results from the interpolation and extrapolation tasks of Experiment 1 are shown as a function of transformation in Table 3. Average accuracy of participants in the interpolation task was 78%, ranging from a low of 62% to a high of 88%. Average accuracy in the extrapolation task was 74%, ranging from a low of 61% to a high of 88%. Overall performance was significantly better than a chance level of 12.5% (1 out of 8 choices), $t(15) = 36.65$, $p < .0001$, in the interpolation and $t(15) = 27.66$, $p < .0001$ in the extrapolation task. A 2 (Task) \times 4 (Target Face) \times 4 (Choice Face) \times 8 (Transformation) analysis of variance (ANOVA) revealed no main effect of task, $F(1, 30) = 1.48$, $p > .05$, indicating that participants could interpolate as accurately as they could extrapolate. The

TABLE 3
Percentage of Time that a Choice Set Transformation was Selected for Each Target Transformation in the Interpolation and Extrapolation Tasks in Experiment 1

Base Transformation	Choice Transformation							
	hd	hs	cs	oc	sx	sy	sc	no
Interpolation task								
hd	47	36	0	6	0	0	8	3
hs	34	52	0	6	4	0	4	3
cs	0	1	94	3	0	0	0	2
oc	9	11	3	76	0	0	0	0
sx	0	0	0	0	92	7	0	0
sy	0	0	0	0	5	95	0	0
sc	14	9	0	0	0	0	74	3
no	1	5	0	0	0	0	1	93
Extrapolation task								
hd	45	39	0	0	0	0	11	5
hs	24	52	0	2	0	0	9	12
cs	2	3	84	8	0	0	0	3
oc	20	21	2	54	0	0	2	2
sx	0	0	0	0	96	4	0	0
sy	0	0	0	0	4	96	0	0
sc	7	5	0	1	0	0	82	5
no	3	4	1	0	0	0	4	88

Note. Sixteen participants participated in each task condition.

ANOVA showed a main effect of transformation, $F(7, 210) = 76.91, p < .0001$, as well as a significant interaction between task and transformation, $F(7, 210) = 4.36, p < .001$. The significant effect of transformation indicates that certain transformations were confused more than other transformations, whereas the significant interaction indicates that participants did not make the two kinds of judgments equally well for all transformations. Detailed analysis is reserved until later.

Target face was a significant main effect, $F(3, 90) = 2.87, p > .05$; choice face was not, $F(3, 90) = 1.48, p < .05$. A post hoc *t* test showed that participants' judgments involving individual B as the target face differed from their judgments involving individuals C and D at the .05 level. There were also significant interactions between target face and transformation, $F(21, 630) = 1.86, p < .05$, and between choice face and transformation, $F(21, 630) = 2.33, p < .001$, as well as a significant three-way interaction involving task, target face, and transformation, $F(21, 630) = 1.69, p < .05$. These effects suggest that the morphology of the profiles might have had some influence on participants' judgments; if so, that effect must be limited because there was no interaction between target face and choice face, $F < 1$. Participants' performance on each target profile is presented in Table 4 as a function of profiles constituting the choice set. In Table 4 we clearly demonstrate that participants' performance was not markedly different whether the target and choice set profiles were of the same individual or of different individuals. In other words, although the effect of the target face on performance may suggest that some morphological structures reveal a particular style of change better than others, the fact that the same effect was not found in the choice face or in a choice face–target face interaction lessens its significance.

To further examine the effects of morphological structure on participants' detection of the transformational invariant, results were collapsed over the 16

TABLE 4
Percent Accuracy as a Function of Target-Choice Profile Pairing in Experiment 1

Target Set Profile	Choice Set Profile			
	A	B	C	D
Interpolation task				
A	81	74	80	80
B	81	82	80	77
C	83	82	84	82
D	84	80	80	84
Extrapolation task				
A	82	76	74	83
B	72	75	70	73
C	80	77	76	80
D	82	79	78	81

possible combinations of target and choice faces and entered into an ANOVA. This 2 (Task) \times 16 (Face Pair) ANOVA revealed no main effect of face pair, $F(1, 15) = 2.34, p > .05$, and no interaction between task and face pair, $F < 1$. The results of this additional analysis confirmed that morphological differences actually had little effect on participants' detection of the transformational invariant.

Inspection of Table 3 clearly demonstrates the source of the main effect of transformation in the first ANOVA. Participants confused the hydrodynamic, hydrostatic, and orthogonal cardioidal strain models most frequently, which together accounted for 57% of all errors in the interpolation task and 52% of all errors in the extrapolation task.

These three models, originally defined as growth models along with the cardioidal strain transformation, are unlike the nongrowth models because they involve topological strain transformations. Hence, all growth models belong to topological spaces, whereas the x -axis shear and y -axis shear belong to affine spaces and the scale transformation belongs to a similarity space. The particular way the strain transformation is applied in each model further differentiates the cardioidal strain from the other three growth models. In the cardioidal strain model, strain is applied maximally along the vertical dimension, diminishing to zero strain along the horizontal dimension. The other three growth models apply differential amounts of strain along the vertical and horizontal dimensions. This difference results in a negative change in facial profile size under the cardioidal strain model as the measure of Articulare and Pogonion increases, but positive size changes under the other three growth models for the same direction of change in the Articulare-Pogonion dimension. Since growth, according to Enlow (1968), is a process involving changes in the size of an object and its shape, unless a profile is renormalized to compensate for the negative size change, the cardioidal strain transformation cannot result in a perception of "growth," hence, it cannot be categorized as one of the "growth" transformations. Specifically, the cardioidal strain transformation flattens the object maximally along the vertical axis, diminishing, in effect, to a limit along its horizontal axis. The other three growth models, on the other hand, strain the object vertically while expanding it horizontally (see Figure 1 for the underlying geometries).

The most frequent confusion among the three growth models (hydrodynamic, hydrostatic, and orthogonal cardioidal strain) was between the hydrodynamic and hydrostatic models, which accounted for 39% and 31% of the errors in the interpolation and extrapolation tasks, respectively. This may be because the hydrodynamic and hydrostatic models are embedded in circular spaces, whereas the orthogonal cardioidal model is embedded in a cardioidal space (see Figure 1). In fact, the hydrostatic model and hydrodynamic models are, in a sense, identical because the latter can be derived from the former by continuous application of the former. When these two models are treated as

equivalent, that is, when we count the selection of either of these two models by participants as correct when given one of these models in the target sequence, accuracy involving these two models improves to 84% in the interpolation task (from 47% in the hydrodynamic model and 52% in the hydrostatic model) and 80% in the extrapolation task (from 45% in the hydrodynamic model and 52% in the hydrostatic model). Thus, the similarities of the underlying geometries of the hydrostatic and hydrodynamic models may be causing greater confusion between these two models than among other models, while at the same time differentiating the two from the other topological strain transformation—the orthogonal cardioidal model (see also Mark et al., 1981, for a similar result with different transformations).

The confusability between the hydrodynamic and hydrostatic models was further confirmed by other types of errors. Specifically, confusing the similarity transformation with this pair of transformations accounted for 20% of the errors in the interpolation task and for 16% of the errors in the extrapolation task. In contrast, the similarity transformation was seldom confused with the orthogonal cardioidal transformation (accounting for less than 1% of errors). As noted earlier in this article, all three growth models undergo scaling changes with a preferred direction, that is, a strain tending toward gravity. The effect of strain, however, differs between the hydrodynamic and hydrostatic models, on the one hand, and the orthogonal cardioidal model on the other because in the former, global remodeling occurs in a circular-based space and in the latter it occurs in a cardioidal-based space. Specifically, the circular-based transformations transform a circle or any other closed form into a series of oblate forms that enlarge disproportionately in the direction of gravity, whereas the cardioidal-based transformations eventually transform a circle into a cardioid. This may account for the confusion of the similarity transformation with the hydrodynamic and hydrostatic models and not with the orthogonal cardioidal strain model. That is, although (in both cases) objects undergo expansion, in the similarity transformation, the center of expansion remains fixed, whereas in the hydrostatic and hydrodynamic transformations, the center of expansion shifts in the direction of gravity. This results in a degree of strain not seen in a scaling transformation.

Because participants responded differently to growth and nongrowth transformations (63% and 89% accuracy, respectively), the transformations were divided into two types—growth and nongrowth—and were entered into a 2 (Task) \times 2 (Transformation Type) ANOVA in an attempt to understand the significance of the task-transformation interaction. For this analysis, growth models were defined as the hydrodynamic, hydrostatic, orthogonal cardioidal, and cardioidal strain transformations, and nongrowth models were defined as the shear x , shear y , similarity, and null transformations. Although we noted earlier that the cardioidal strain transformation without size renormalization does not actually qualify as a growth transformation, it was categorized as a

growth model simply to allow for the statistical comparison of an equal number of "growth" and "nongrowth" transformations. This ANOVA revealed a significant effect of transformation type, $F(1, 30) = 272.54$, $p < .0001$, and a significant interaction between task and transformation type, $F(1, 30) = 10.63$, $p < .01$.

An additional ANOVA comparing only the four nongrowth transformations revealed no interaction between transformation and task, $F(3, 90) = 1.89$, $p > .05$, whereas the same ANOVA on the four growth transformations revealed an interaction between transformation and task, $F(3, 90) = 4.32$, $p < .01$, as well as a main effect of task, $F(1, 30) = 5.53$, $p < .05$. Task had little effect on nongrowth models (participants showed 89% accuracy in the interpolation task and 90% accuracy in the extrapolation task). However, task did affect participants' decision on growth models (in which participants showed 67% accuracy in the interpolation and 59% accuracy in the extrapolation task). Specifically, participants' performance on the two cardioidal-based (cardioidal and orthogonal cardioidal) transformations was less accurate in the extrapolation task than in the interpolation task—84% and 94%, respectively, in the cardioidal transformation condition and 54% and 76%, respectively, in the orthogonal cardioidal transformation condition. An analysis of the simple effects showed that the main source of the interaction was the orthogonal cardioidal transformation, $F(1, 101) = 15.89$, $p < .001$. An analysis of subject errors showed that participants confused the orthogonal cardioidal transformation with either the hydrodynamic or hydrostatic transformation more often in the extrapolation task (which accounted for 20% of all errors) than in the interpolation task (12% of errors).

In short, it appears that the extrapolation task is more difficult than the interpolation task, primarily in the orthogonal cardioidal transformation condition. This may be because although the control dimension chosen for the transformations (the distance between *Articulare* and *Pogonion*) is a linear measure, the overall effect of the transformations is a global remodeling of the profiles—a nonlinear effect. Under small amounts of transformation, this nonlinear effect may be minimal. However, as the amount of transformation increases, the nonlinear effect of global deformation may become more pronounced. This nonlinear effect may vary from transformation to transformation. The transformations with the most significant effects of nonlinear shape change may be the two cardioidal-based transformations. This would explain why when the final member of the sequence was present as in the interpolation task, identifying the missing third member was relatively easy. When the final member had to be extrapolated from a subsequence, however, the problem was more difficult.

Overall, participants were very successful in identifying the correct profile to fit the sequence. Finding this high level of accuracy in participants with no specialized training with the transformations under study can best be explained

by assuming they were detecting the transformational invariant specifying a style of change, thus confirming Gibson's earlier conclusion that participants can detect differences in nonrigid styles of change (Gibson & Gibson, 1957; von Fieandt & Gibson, 1959). That participants are often confused by similar styles of changes, especially in the growth transformations, is a further indication that it is the transformational invariant, not the structural properties of the object, to which participants were attending. The minimal effect of specific facial morphology on participants' detection of the transformational invariant also supports the hypothesis that this information is highly abstract and relatively indifferent to the structural properties by which it is specified.

The results also showed that participants can identify the transformational invariant as reliably from discrete samples in a static sub-sequence, demonstrating that this information is also generative. That is, the informational value of a discrete sample of a dynamic event is not restricted to the specific moment at which the sample is taken, but goes beyond that moment to specify prospectively the future and retrospectively the past.

As discussed earlier in this article, under certain transformations the extrapolation task posed more difficulty for participants than did the interpolation task. Interestingly, a similar outcome was obtained in a series of representational momentum studies (Finke & Freyd, 1985; Freyd, 1987; Freyd & Finke, 1984). In a typical experiment, participants were presented with a sequence of static displays depicting an object undergoing a coherent motion such as rotation and asked to recall the exact configuration of the object in the last display. Results showed that participants tended to recall the object farther along the implied path than had actually been visually presented. These results have been interpreted as evidence for dynamic mental representation, and the observed memory distortion has been called representational momentum in an analogy to physical momentum. The results, however, could have been interpreted as evidence that participants detected the transformational invariant underlying the coherent motion. Despite being presented with static displays and being asked to remember the exact configuration of the last display, participants were not storing in memory a dynamic representation of a series of static images, but were instead, perceiving an event—an object undergoing a unique style of change. As demonstrated in this experiment, the transformational invariant is sufficiently generative to specify the sequence in its entirety despite its being presented in discrete samples. The fact that this phenomenon was not obtained when the order of displays was reversed suggests further that participants in the representational momentum experiments were indeed apprehending events rather than memorizing individual pictures (see Jenkins et al., 1986, for similar findings). Thus, the phenomenon observed by Finke and Freyd may not be a case of memory distortion, but simply may be additional evidence for the generative nature of the transformational invariant and the ability of human perceivers to detect this information.

Also related to the present study is a study conducted by Kolers and Pomerantz (1971) that used an apparent-motion paradigm. By using disparate pairs of objects such as a square and a circle, an arrow and a triangle, or a trapezoid and its transformed image in an apparent motion context, the researchers were able to separate the underlying change from objects undergoing the change and thereby investigate the change dimension alone. Motions tested included simple translations (or beta motions), scalings, rotations, reflections, inversions, or a combination of scaling and rotation. Even with these disparate objects, participants still saw apparent motion. Kolers and Pomerantz postulated constructive synthetic processes to explain their results. The results, however, could as legitimately be interpreted as evidence that participants simply detected the transformational invariant defined over the experimental stimuli. As we just demonstrated, this information is highly abstract and relatively indifferent to the structure over which it is specified. This may explain why participants in the Kolers and Pomerantz study reported perceiving coherent motions even when the motion was applied to disparate objects.

Moreover, rather than assuming that perception and motion must occur prior to figural analysis as did Kolers and Pomerantz, we would contend that perceiving an event simply evokes two kinds of awareness—one of change and one of permanence (Gibson, 1986). It seems likely that these two event components could be isolated and studied separately in the current paradigm through an experimental manipulation similar to that of Kolers and Pomerantz (see also Gibson, 1957, for a similar manipulation). It does not follow, however, that one component would necessarily prove more fundamental than the other as Kolers and Pomerantz have claimed. Instead, as will be shown later, by reversing the control, the controlled dimension (the structural invariant) will come to the forefront, whereas the uncontrolled dimension (the transformational invariant) will recede into the background. Hence, what looks primary in one condition will look secondary in the other condition. Once again, under the current view, the two components of an event cannot be distinguished in importance. Rather, both are equally fundamental and real because without one, the other cannot exist. The opportunity to show that structural properties are primary when the style of change is controlled will be reserved until the last experiment. In the next experiment, the abstract nature of the transformational invariant will be investigated further.

EXPERIMENT 2: PERCEIVING STYLES OF CHANGE OVER DIFFERENT STRUCTURES (COMPARING MORPHOLOGICAL AND GEOMETRICAL OBJECTS)

In Experiment 2, the abstract nature of the transformational invariant was explored further in two different ways. First, the method used in Experiment 1

to construct the stimulus sequence was extended to the whole sequence such that a sequence was made of the profiles of different individuals but were transformed using the same style of change. Second, transformations were applied not only to human faces, but also to geometric objects (circle, square, triangle, and cardioid). Experiment 1 revealed some influence of structure on participants' detection of the transformational invariant. By applying the transformations to geometric objects, we sought to determine whether styles of change are, in fact, sensitive to the structure to which changes are applied. In other words, if the transformational invariant is indeed abstract, a particular style of change should be recognized with equal facility even when that change occurs over different structures. Any result other than this would suggest that abstractness is relative and that the two event components are not independent, but are to some extent mutual and reciprocal.

Method

Participants. Twenty-four undergraduates at the University of Connecticut participated in the experiment in partial fulfillment of a course requirement. All had normal or corrected-to-normal vision. None had participated in the previous experiment. Twelve participants served in the morphological condition and 12 served in the geometric condition.

Stimulus materials. In the morphological condition, the same profiles produced for Experiment 1 were used with a different arrangement (see Figure 4) defined by the following scheme: A stimulus sequence was produced by applying the same transformation to profiles of four different faces, designated as A, B, C, and D. Next, a four-place sequence was constructed by random selection of the four profiles without replacement, B, C, D, A; D, C, A, B; . . . etc. If the row headings of a 4×4 stimulus array designate the ordinal positions in the four-place stimulus sequence, then the four age levels—8, 12, 16, and 20 years—represent the column headings of this stimulus array. The four randomized sequences of profiles were assigned to the columnwise cells of this array under the restriction that each row and column must have only unique cell values. In this way, a balance of profiles and age levels for each ordinal position was guaranteed in each of the four stimulus sequences.

In the geometric condition, four geometric figures (square, triangle, circle, and cardioid) were used as base objects (Figure 5). The geometric figures were prepared as in Experiment 1. For the circle and cardioid, the object to which transformations were applied was selected such that the side of the square corresponded to the diameter of the circle and to the altitude of the cardioid (i.e., the height $2a$, where a is the maximum length of the x component of the generating radius vector).

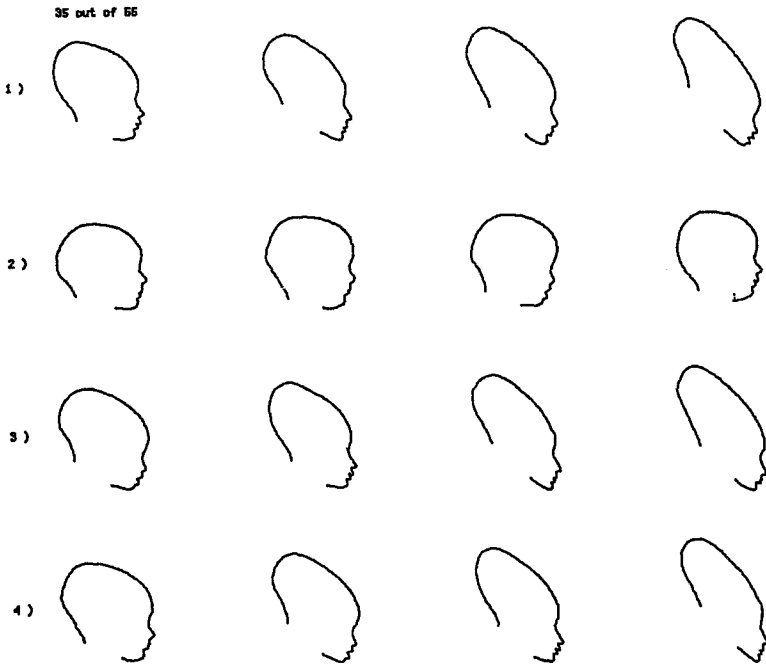


FIGURE 4 Stimuli used in the morphological condition in Experiment 2. All the profiles are produced by a shear y transformation, except for Row 2, which is produced by a null transformation. With determinant of 1, a shear y transformation belongs to the group of *equiaffine* transformations, which preserve area size. Because the null transformation clearly has no effect on area size of the object to which it is applied, size can be eliminated as a source of information in both these conditions, making it more likely that the transformational invariant corresponding to each transformation is the relevant source of information in this task.

Display. The stimuli were presented in a 1000×840 pixel window using Sun View graphics tools. This window was divided into 4×4 grids without explicit demarcations. Each of these subdivisions was occupied by a facial profile or a geometric form, depending on condition.

Procedure. A four-item, forced-choice paradigm was used. From the eight transformations available (including the null transformation), two were selected randomly for each trial. One was designated the base transformation, the other was designated the target transformation. Hence, three sequences were produced by one transformation, and a fourth sequence was produced by a different transformation. Eight base transformations combined with seven target transformations resulted in a total of 56 trials in the experiment.

The participants' task was to select from among the four rows of objects (faces

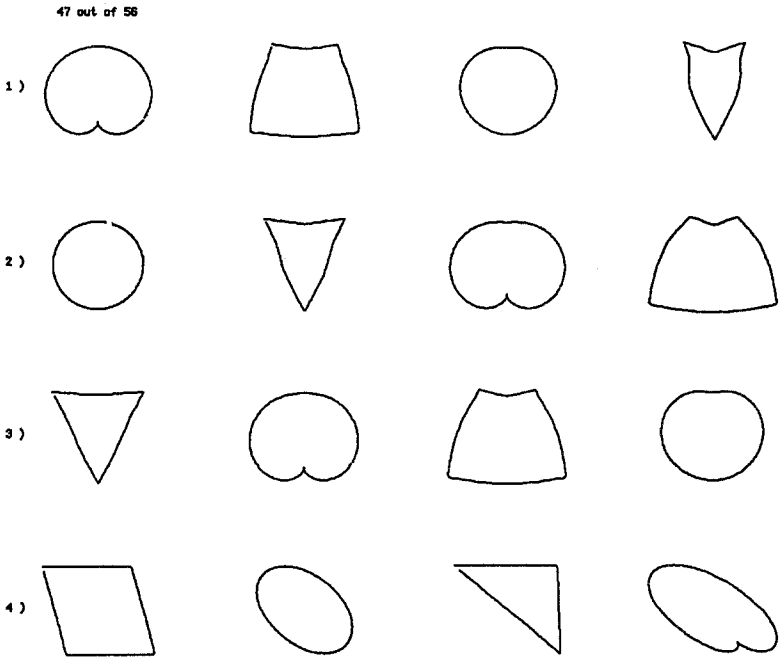


FIGURE 5 Stimuli used in the geometric condition in Experiment 2. Objects used in this experiment are made of geometric forms such as a circle, a triangle, a square, and a cardioid. Rows 1, 2, and 3 are produced by a cardioidal strain transformation, whereas Row 4 is produced by a shear x transformation.

for the morphological condition or geometric forms for the geometric condition) on the computer screen the one row that was most different from the other three. Participants indicated their selection by using the mouse to select the button corresponding to their choice.

A short practice session was provided for each participant. In the morphological condition, the practice stimuli were those used in the geometric condition. In the geometric condition, the practice stimuli sequences were constructed using profiles of human heads. Only three transformations (hydrostatic, cardioidal strain, and y -axis shear transformations) were used in the practice session, yielding six practice trials (3 Transformations \times 2 Target Transformations). No feedback about performance was given during either practice or experimental trials. The entire experiment took about 20 min.

Results and Discussion

Results are shown in Table 5 for both morphological and geometric conditions as a function of transformation. Average accuracy for participants in the

TABLE 5
Percent Accuracy as a Function of Base-Target Transformation Pairing for the
Morphological and Geometric Conditions in Experiment 2

Base transformation	Target Transformation							
	hd	hs	cs	oc	sx	sy	sc	no
Morphological condition								
hd	–	25	100	100	100	100	92	92
hs	42	–	100	100	100	100	100	92
cs	100	100	–	100	100	100	100	100
oc	58	75	100	–	100	100	100	100
sx	100	100	100	100	–	100	92	100
sy	100	100	100	100	92	–	100	100
sc	100	100	100	100	100	100	–	92
no	100	100	100	100	100	92	100	–
Geometric condition								
hd	–	42	100	58	100	100	100	83
hs	42	–	92	92	100	92	92	75
cs	33	50	–	33	100	92	92	83
oc	33	33	75	–	100	100	92	92
sx	92	92	100	92	–	83	100	100
sy	100	100	100	100	92	–	100	83
sc	100	83	100	100	100	92	–	58
no	100	92	100	100	100	100	100	–

Note. Twelve participants participated in each condition.

morphological condition was 96%, ranging from a low of 91% to a high of 100%. Average accuracy in the geometric condition was 86%, ranging from a low of 82% to a high of 91%. Performance was well above a chance level of 25% (one out of four), $t(11) = 126.01, p < .0001$, in the morphological condition and $t(11) = 63.44, p < .0001$ in the geometric condition. A 2 (Morphology) \times 8 (Transformation) mixed-design ANOVA showed significant effects of morphology, $F(1, 22) = 66.12, p < .0001$, and transformation $F(7, 154) = 11.18, p < .0001$, as well as an interaction between condition and transformation $F(7, 154) = 6.33, p < .0001$. Although the accuracy results appear to indicate that the transformational invariant is specific enough to distinguish its underlying style of change, the interaction between condition and transformation suggests that its specificity is relative with respect to the structure over which its underlying order unfolds.

Inspection of Table 5 suggests, as in Experiment 1, that participants responded differently to growth and nongrowth transformations. Following the same rationale as that used in Experiment 1, responses were divided into two groups on the basis of transformation type (growth vs. nongrowth) and entered into a 2 (Morphology) \times 2 (Transformation Type) ANOVA. The difference between the growth (85%) and nongrowth (97%) transformation types was significant, $F(1, 22) = 48.28, p < .0001$, and the interaction of transformation

type with morphology was significant, $F(1, 22) = 9.01, p < .01$. Further inspection of Table 5 reveals that participants' performance in the two different structural conditions involving growth models was significantly different, with 92% accuracy in the morphological condition and 76% in the geometric condition. This was confirmed by a *t* test, $t(11) = 6.78, p < .0001$. Participants' accurate judgment of growth transformations involving morphological structures and comparatively poor judgment of growth transformations involving geometric forms suggests that certain styles of change may be more compatible with certain object structures (Mark, Shapiro, & Shaw, 1986; Pittenger & Todd, 1983). Mark et al. (1986) reported that applying one of the growth transformations, specifically, cardioidal strain⁶ to a straight-line robotlike structure did not result in the same perceived style of change as applying the transformation to a more curved and less angular biomorphic structure such as a human profile. This explains why the growth transformation is perceived as growth over not only human heads, but also over dog, bird, or other animal heads as well as over some softly curving, inanimate objects such as Volkswagens (Pittenger et al., 1979), but not over armchairs, shoes, or robotlike objects (Shaw & Carello, 1979; Mark et al., 1986). In short, it appears that structure and change are not totally independent but, instead, are interdependent to the degree that certain object structures can better support a given style of change. How the specificity of the structural invariant relates to the structural properties of an object and, reciprocally, how the relativity of the structural invariant relates to the styles of change under which it is revealed, will be explored further in Experiment 3.

Overall, the results of Experiment 2 confirmed once again the abstractness of the transformational invariant and its expected salience for human observers. The results also showed that there is a range of structural properties over which a given style of change can be better preserved. That is, structure and change may not be totally independent, but may be interdependent.

EXPERIMENT 3: EXAMINING THE STRUCTURAL INVARIANT

In Experiment 3 we focused on the other component of event information, the structural invariant. This invariant has been hypothesized to provide information specific to the structural properties of an object in an event. Like its reciprocal partner, the transformational invariant, the structural invariant is assumed to be both generative and abstract.

To examine the abstract nature of the structural invariant, the four profiles

⁶Cardioidal strain was not treated as a growth transformation in this study because size normalization was not used.

used in the previous experiments were transformed by one of the eight transformations (the choice set) and presented along with one of the original nontransformed profiles (the base profile). Participants were asked to select the profile from the choice set which was (individually) identical to the base profile. If an individual's identity is specified by a structural invariant⁷ as hypothesized, then despite having undergone different transformations, the individual's identity should be easily recognized.

To explore the generative nature of the structural invariant, we controlled the amount of transformation applied. If the structural invariant is indeed generative, participants should be able to recognize the same individual despite the amount of transformation applied (or, analogously, the timespan between samples). For example, we can commonly recognize the same individual in photographs taken at 4 years and 8 years of age. It may be the case, however, that with increased (temporal) distance between the samples shown, for example, photographs of an individual at 4 years and at 44 years of age, generativity breaks down somewhat (e.g., Bruck, Cavanagh, & Ceci, 1991; Seamon, 1982). To examine the effects of the amount of temporal separation between samples on participants' perceptions of structure, the experiment was divided into two parts. In one part, the base profile was paired with a profile to which an intermediate amount of transformation (i.e., corresponding to 12 years of age) had been applied. In the other, the base profile was paired with a profile to which the maximum amount of transformation (i.e., corresponding to 20 years of age) had been applied.

Method

Participants. Twelve undergraduates at the University of Connecticut participated in the experiment in partial fulfillment of a course requirement. All participants had normal or corrected-to-normal vision. None had participated in the previous experiments.

Design. Amount of transformation, target face, and transformation were controlled. The same base face was used, corresponding to an age of approximately 4 years. For the intermediate transformation, the amount of transformation applied corresponded to an age of 12—the second profile of the sequence used in Experiments 1 and 2. For the maximum transformation, the amount of

⁷Our usage of structural invariant is quite abstract because we use the term to describe any structural aspects that are preserved under a given set of transformations. Thus, the structural invariant may describe not only individual identity but also race, sex, identity, species, or family resemblance (Pittenger & Shaw, 1975a). It should be pointed out that within the context of specific problems such as face recognition, structural invariants have been described more concretely as independent features or arrangements of features (for a more extensive review, see Davies, Ellis, & Shepherd, 1981, and Bruce & Young, 1986).

transformation applied corresponded to the age of 20—the last profile of the sequence used in Experiments 1 and 2.

To determine whether the amount of transformation seen in an initial recognition task has any effect on a subsequent recognition task, the presentation order for amount of transformation applied was controlled as a between-subjects factor. Group A experienced first the base profile paired with a choice set composed of faces to which a medium amount of transformation had been applied, followed by the base profile paired with a choice set of faces to which a maximum amount of transformation had been applied. The presentation order was reversed for Group B. This resulted in a 2 (Presentation Order) \times 2 (Amount of Transformation) \times 4 (Target Face) \times 8 (Transformation) mixed design with a total of 64 trials in the experiment. Participants were tested individually.

Display. The four transformed profiles were presented together with the base face in a window of 900 \times 600 pixels in a Sun View graphics environment (Figure 6). This window was divided into three columns 300 pixels wide. The outside columns were further divided into two rows 300 pixels high. One of the four base faces was selected randomly and placed in the middle column. Each of

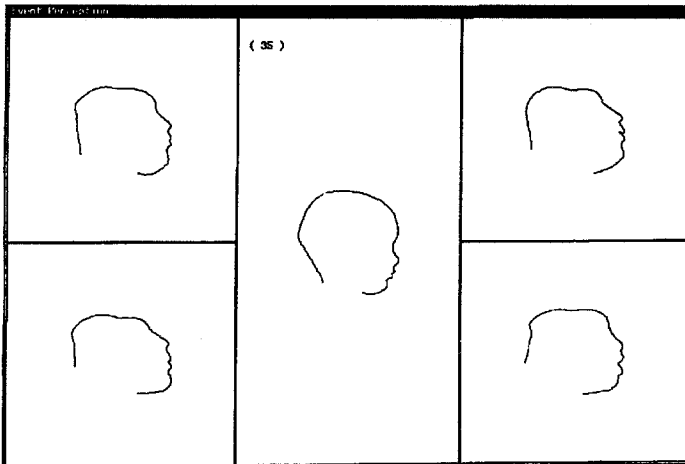


FIGURE 6 Stimuli used in Experiment 3. All the profiles occupying the four surrounding subsectors were transformed maximally (equivalent to an age of 20) by a cardioidal strain transformation. Without size normalization, application of a cardioidal transformation actually results in a negative change in profile size. Consequently, under these conditions, it must be considered a nongrowth transformation. The results of Experiment 3 in which participants' performance was significantly worse in the maximum transformation condition than in the medium transformation condition further supports this. Of the profiles occupying the four surrounding subsectors, the upper left profile is identical with the base profile.

the four transformed profiles was assigned randomly to one of the four outside sectors.

Procedure. Participants were asked to select from the four profiles shown in the outside sectors the profile that belonged to the same individual as the profile shown in the center of the screen. Participants indicated their selection by positioning the cursor over the sector occupied by one of the four target profiles and pressing the left mouse button.

Each participant was given a practice session in which target and choice sets were constructed by applying a similarity transformation to each of the four geometric figures used in Experiment 2. The amount of transformation applied in the practice trials was fixed at the maximum amount (equivalent to 20 years). This yielded four trials in the practice session (4 Figures \times 1 Transformation). No feedback about performance was given during either practice or experimental trials. The entire experiment took about 10 min.

Results and Discussion

The results of Experiment 3 are shown in Table 6 as a function of magnitude of transformation, face type, and order of presentation for the eight transformations. The average correct judgment for the 6 participants making up Group A was 91% in the medium condition (with a low of 81% and a high of 100%) and 85% in the maximum condition (with a low of 72% and a high of 94%). For participants in Group B, the average correct judgment was 88% in the medium condition (with a low of 69% and a high of 100%) and 79% in the maximum condition (with a low of 72% and a high of 88%). Overall performance was significantly better than a chance level of 25% (1 out of 4 choices), $t(11) = 27.80$, $p < .0001$, for the first group and $t(11) = 19.26$, $p < .0001$ for the second group.

A 2 (Presentation Order) \times 2 (Magnitude of Transformation) \times 4 (Target Face) \times 8 (Transformation) mixed-design ANOVA was conducted to analyze the accuracy difference just shown. The ANOVA revealed a main effect of magnitude, $F(1, 10) = 7.79$, $p < .05$, a main effect of transformation, $F(7, 70) = 3.67$, $p < .01$, an interaction between magnitude and transformation, $F(7, 70) = 2.26$, $p < .05$, and an interaction between face and transformation, $F(21, 210) = 1.62$, $p < .05$.

The absence of a significant effect of target face ($F < 1$) indicates that participants were able to detect the structural invariant specifying identity equally well for the four different individuals. On the other hand, the main effect of magnitude of transformation indicates that gap size between two samples does affect the generative specificity of the structural invariant. The bigger the gap between samples, the more difficult it is to recognize an individual's identity by detecting the structural invariant. In addition, the main effect of transformation suggests that not all styles of change are equivalent in their

TABLE 6
 Percent Accuracy as a Function of Transformation, Magnitude of Transformation, and
 Order of Presentation in Experiment 3

Transformation	Base Faces							
	A		B		C		D	
	Med	Max	Med	Max	Med	Max	Med	Max
Group A								
hd	83	100	67	67	100	83	100	83
hs	100	83	100	83	100	83	100	100
cs	100	67	100	67	83	83	100	67
oc	83	83	100	83	100	100	100	83
sx	100	83	83	83	83	100	83	67
sy	67	83	100	83	67	67	83	67
sc	100	100	83	100	100	100	67	67
no	83	100	100	100	100	100	83	100
Group B								
hd	67	83	50	83	100	100	100	67
hs	67	83	100	67	83	83	100	100
cs	50	50	83	67	100	50	83	17
oc	67	83	100	67	67	100	100	83
sx	83	67	50	67	67	67	67	67
sy	33	50	83	83	83	67	67	33
sc	83	83	67	67	83	67	83	17
no	83	83	83	67	100	83	50	83

Note. Each group was made up of 6 participants.

ability to support the specificity of the structural invariant. That is, the degree to which the structural invariant specifies identity appears to be dependent on the style of change applied to the object. The significant magnitude-transformation interaction suggests further that the effect of magnitude depends on the type of transformation. An analysis of simple effects revealed that the cardioidal transformation was the transformation most affected by the different amount of transformation, $F(1, 10) = 14.41, p < .01$. A similar analysis of simple effects in the face-transformation interaction showed that individual morphology influenced participants' decisions significantly only in the hydrodynamic transformation, $F(3, 30) = 3.13, p < .05$, and in the similarity transformation, $F(3, 30) = 2.97, p < .001$.

Inspection of Table 6 reveals that overall judgment of identity was relatively poor when the two shearing transformations were applied compared with performance in other transformation conditions. Moreover, there was a dramatic drop in participants' performance in the maximum transformation condition when cardioidal strain transformations were applied, with participants achieving only 60% accuracy—a significant drop from the 94% accuracy achieved in the medium transformation condition.

As indicated earlier, the cardioidal strain model differs from the other three growth models in that it not only deforms, but also shrinks the underlying object. Unless object size is renormalized as magnitude of transformation increases, the effect of the transformation on the profile to which it is applied is to make the profile appear somewhat nonhuman. The two shearing transformations induce similar effects; that is, given a sufficient magnitude of transformation, the resultant profile is less human in appearance. In other words, biomorphic structures such as facial profiles support perceptions of growth or aging under growth transformations (the hydrodynamic, hydrostatic, and orthogonal cardioidal transformations), which suggests that the structural invariant specifying identity must be quite salient under these transformations (Mark et al., 1986). Presumably, because the effect of applying growth transformations to facial profiles kept those profiles within the range of "humanness," the structural invariant specifying individual identity was preserved. In contrast, when the effect of applying nongrowth transformations (the two shear transformations and the cardioidal transformation without size renormalization) to facial profiles exceeded the range of humanness, the structural invariant was obscured. As the amount of transformation was increased, the degree to which the structural invariant was specified was increasingly reduced.

However, it is not always the case that structural invariants are better specified under growth transformations and underspecified under geometric (nongrowth) transformations. Recall the geometric condition of Experiment 2 (Figure 5). Under growth transformations, squares can be transformed into marshmallowlike figures, or triangles can be transformed into tuliplike figures. In fact, all the geometric objects used in Experiment 2 can be transformed into nondistinguishable forms under growth transformations because topological transformations preserve few properties. In these situations, identifying the underlying structural invariant for a geometric object may become very difficult. In contrast, under nongrowth transformations (such as affine transformations), identifying the underlying structural invariant may be relatively easy because more properties are preserved under these transformations. For example, under shearing transformations a square can be transformed into a rhombus, but never into a circle. Furthermore, under a similarity transformation, the size of an object is changed but shape is preserved, facilitating object recognition.⁸ In short, geometric objects may be better specified under affine or similarity

⁸Klein's Erlanger Program categorized various geometries such as Euclidean, similarity, affine, projective, etc., in terms of the properties that remain unchanged under the transformations of one group or another. Two geometric figures are said to be equivalent with respect to a group of transformations if and only if the group contains a transformation that maps one figure onto the other. Moreover, two figures equivalent under a group of rigid motions (translation or rotation) are called "congruent"; two figures equivalent under a group of similarity transformation are called "similar"; and two figures equivalent under a group of affine transformation are called "affine equivalent" (Gans, 1969).

transformations because these transformations preserve certain aspects of the metric properties inherent to geometric objects. On the other hand, geometric objects are underspecified under growth (topological) transformations, which do not preserve metric properties.

In summary, the results of Experiment 3 were consistent with the results of the preceding experiments. Structural invariants specifying a particular individual's identity were detected easily for each of the profiles used despite those profiles having been transformed in various ways. The results also suggest that persistence and change are relative; that is, structural properties such as individual identity are well-specified under growth transformations and are underspecified under nongrowth transformations such as shear. This parallels the results of Experiment 2 in which growth transformations were well-specified over facial profiles. In addition, the results of this experiment indicated that the structural invariant is as generative as its transformational invariant counterpart was shown to be in previous experiments. However, the degree to which the structural properties of an object were preserved depended on the kind and amount of transformation applied. Under growth transformations, an individual's identity was easily recognized despite the various amounts of transformation applied, thus showing the generative nature of the structural invariant. In contrast, in the nongrowth transformations, the generativity of the information decreased as the amount of transformation increased.

GENERAL DISCUSSION

These studies explored the hypothesis that event perception normally involves detecting an invariant specific to the style of change and an invariant specific to the object undergoing the style of change—the transformational and structural invariants, respectively. The key assumption underlying the hypothesis is that information about an event, in the form of transformational and structural invariants, is abstract and generative. Information is abstract because it contains the underlying order of the event, not the event itself, and it is generative because it is not about the present moment or about a current form (e.g., the retinal image), but it is about the event. Hence, detection of the structural invariant, information specific to the permanent properties of the event, would allow a perceiver to go beyond the event's current spatial extension, thus eliminating the need for exhaustive perspectival sampling of an event to perceive a whole event. By the same token, detection of the transformational invariant, information specific to the underlying style of change in an event, would allow a perceiver to go beyond the event's current temporal duration, thereby allowing the need for memory or inferential processes to be usurped by generative specification that projects retrospectively and prospectively beyond a given sample. In addition, because it specifies the underlying successive and adjacent

orders of the event, information (in the form of the transformational and structural invariants) preserves the identity of the style of change and the object, respectively, thus enabling the observer to identify the two informative aspects abstracted from the event itself.

Various geometric transformations were applied to a variety of objects to produce various styles of change. Transformations ranged from topological transformations to a similarity transformation, all defined as nonrigid transformations that deform an object globally in more or less complex ways. Each transformation differed from the other transformations in terms of the underlying geometric invariant by which, it was hypothesized, an observer perceives the unique style of change an event undergoes. This technique allowed us to examine the salience of each event component—that is, change and structure—independently of one another. The events produced ranged from those that are more familiar such as growth transformations applied to human facial profiles or affine transformations applied to geometric objects, to those that are less familiar such as growth transformations applied to geometric figures or affine transformations applied to facial profiles. Structure and change were shown to be perceived reliably through detection of the corresponding information in the form of the structural and transformational invariants. Moreover, the two kinds of event information were shown to be highly abstract and generative. The transformational invariant specifying a unique style of change was reliably detected even when applied to a variety of objects. Reciprocally, the structural invariant specifying an individual's identity was reliably detected even when undergoing a variety of changes. The results also showed that a discrete sample of an event component could be generative enough to predict and postdict the entire event.

The results also indicated that structure and change may not be completely independent of one another, but may be mutual and reciprocal. The degree to which one component persists was shown to be affected by the degree to which its reciprocal persists. That is, growth transformations were better specified over biomorphic structures such as human facial profiles, whereas nongrowth transformations such as affine transformations were better specified over geometric structures. Reciprocally, biomorphic structures were better recognized under growth transformations. In short, transformation is shape specific, and shape is transformation specific (Mark et al., 1986). Within the range of shape-specific transformations applied to reciprocal transformation-specific shapes, the perceptual outcome is a salient event in which both structural and transformational invariants are well specified. If either of these constraints is violated, the information (i.e., the transformational and structural invariants) is underspecified, making the corresponding event component (structure or change) less salient. The reciprocal nature of structure and change indicates that "structure" and "change" are perspectival terms; structure is not permanent—and change does not vary at each instant. Despite changes in structure and

persistence in changes, human observers can detect structure and change by detecting information in the form of their corresponding invariants.

Hence, structure and change are relative. That is, structure and change, although conceptually distinct, are not independent of one another, but are interdependent (Jenkins et al., 1986; Mark et al., 1986; Mark et al., 1988; Pittenger & Todd, 1983). Neither occurs separately nor independently, but mutually and reciprocally. To put it more simply, there are no things that do not change and no changes that take place independently of things (Bunge, 1977; Turvey, 1992).

For this reason, it may be better to speak of structure as (relatively) persistent regions under change, and change as (relatively) persistent regions over structure. This is because an event is neither located in empty space nor frozen in time (as was classically believed), but is both spatially extended and temporally enduring. Furthermore, structure and change are reciprocals; that is, persistence along one dimension is further affected by persistence along the other. Nevertheless, this conjunction of structure and change defines a unique kind of event in space time because a particular style of change coupled with a particular type of structure determines dynamically a unique trajectory of the space-time continuum that identifies a given kind of event. One might say that becoming is extensive and being is enduring (Prigogine, 1980).

By the same token, persistence also may be relative. That is, spatiotemporal components of an event can be described in terms of their degree of persistence. Consider the following: Substances and surfaces that occur in nature differ both in rigidity and in the degree to which they resist deformation. At one extreme, one finds solid rocks and mountains; at the other extreme, flames and clouds. In between, one finds liquids, gels, plastic substances, and viscoelastic objects. Solid objects are rigid; hence, in many cases, they may endure longer but, like all objects, they will eventually disintegrate. Objects at the other end of the spectrum such as flames and clouds do not have any shape that persists over time but still are sufficiently substantial to be recognized for what they are. More abstractly, geometric transformations vary widely in terms of which properties they preserve and which they destroy. For example, some transformations such as translation and rotation preserve Euclidean metric properties while altering the properties of location and orientation, respectively. Conversely, transformations such as bending and stretching preserve only topological properties while destroying the other properties. Thus changes can affect the properties of the objects to which they are applied to different degrees (Warren & Shaw, 1985).

In conclusion, by using computer-generated displays satisfying mathematical conditions of invariance and transformation, we have shown observers' concurrent perceptual sensitivity to that which is persistent (the structural invariant) and to that which is changing (the transformational invariant). The transformational invariant is specific to the style of change, and the structural invariant

is specific to the object undergoing a style of change. Moreover, this procedure has allowed us to demonstrate that the information that specifies an event is both abstract and generative. Information has been shown to be abstract because it does not replicate, but specifies, an event. That is, a transformational invariant specific to a given style of change persists over various objects to which it is applied. Similarly, a structural invariant specific to a given object persists under various changes to which it is subjected. Persistence of these invariants, however, has been shown to be relative. That is, there exists a range of transformations under which the structural properties of an object will be preserved. There also exists a range of structures over which a given style of change can be preserved. Hence, certain types of change are better preserved over some objects than over others, and certain types of object structures are better preserved under some changes than under others. Information has been shown to be generative because, although sampling of the information is necessarily discrete and partial, discrete and partial samples are sufficiently generative of both change and structure to specify an entire event. Specifically, the structural invariant has been shown to specify the consistency of an object despite temporally separated views of the object, and the transformational invariant has been shown to specify the continuity of change despite discrete samples of that change. Moreover, generativity of event information was demonstrated by the relative ease with which participants detected information retrospectively or prospectively. Taken together, the experiments provide support for the hypothesis that perceiving an event is essentially equivalent to detecting the two reciprocal invariants of transformation and structure that together specify an event.

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REFERENCES

- Baumrind, S., & Frantz, R. C. (1971). The reliability of head film measurements. *American Journal of Orthodontics*, 505-517.
- Bruce, V., & Young, A. (1986). Understanding face recognition. *British Journal of Psychology*, 77, 305-327.

- Bruck, M., Cavanagh, P., & Ceci, S. J. (1991). Fortysomething: Recognizing faces at one's 25th reunion. *Memory & Cognition*, *19*, 221-228.
- Bunge, M. (1977). *Treatise on basic philosophy* (Vol. 3). Boston: D. Reidel.
- Carello, C., Grososky, A., Shaw, R. E., Pittenger, J. B., & Mark, L. S. (1989). Attractiveness of facial profiles is a function of distance from archetype. *Ecological Psychology*, *1*, 227-251.
- Cutting, J. E. (1983). Four assumptions about invariance in perception. *Journal of Experimental Psychology: Human Perception and Performance*, *9*, 310-317.
- Davies, G., Ellis, H., & Shepherd, J. (1981). *Perceiving and remembering faces*. London: Academic.
- Enlow, D. H. (1968). Wolff's law and the factor of architectonic circumstance. *American Journal of Orthodontics*, *54*, 803-822.
- Finke, R. A., & Freyd, J. J. (1985). Transformations of visual memory induced by implied motions of pattern elements. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *11*, 780-794.
- Freyd, J. J. (1987). Dynamic mental representations. *Psychological Review*, *94*, 427-438.
- Freyd, J. J., & Finke, R. A. (1984). Representational momentum. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *10*, 126-132.
- Gans, D. (1969). *Transformations and geometries*. New York: Appleton-Century-Crofts.
- Gibson, J. J. (1957). Optical motions and transformations as stimuli for visual perception. *Psychological Review*, *64*, 288-295.
- Gibson, J. J. (1966). *The senses considered as perceptual systems*. Boston: Houghton Mifflin.
- Gibson, J. J. (1975). Events are perceivable but time is not. In J. T. Fraser & N. Lawrence (Eds.), *The study of time, II* (pp. 295-301). New York: Springer-Verlag.
- Gibson, J. J. (1982). Ecological optics. In E. Reed & R. Jones (Eds.), *Reasons for realism: Selected essays of James J. Gibson* (pp. 61-75). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc. (Original work published 1961)
- Gibson, J. J. (1986). *The ecological approach to visual perception*. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc. (Original work published 1979)
- Gibson, J. J., & Gibson, E. J. (1957). Continuous perspective transformations and the perception of rigid motion. *Journal of Experimental Psychology*, *54*, 129-138.
- Haber, R. N. (1983). The impending demise of the icon: A critique of the concept of iconic storage in visual information processing. *The Behavioral and Brain Sciences*, *6*, 1-54.
- Hatton, M. E., & Graninger, R. M. (1958). Reliability of measurements from cephalograms at the Burlington Orthodontic Research Center. *Journal of Dental Research*, *37*, 853-859.
- Jenkins, J. J., Wald, J., & Pittenger, J. B. (1986). Apprehending pictorial events. In V. McCabe & G. J. Balzano (Eds.), *Event cognition: An ecological perspective* (pp. 117-133). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
- Kolers, P. A., & Pomerantz, J. R. (1971). Figural change in apparent motion. *Journal of Experimental Psychology*, *87*, 99-108.
- Lee, D. N. (1976). A theory of visual control of braking based on information about time-to-collision. *Perception*, *5*, 437-459.
- Mark, L. S. (1979). *A transformational approach toward understanding the perception of growing faces*. Unpublished doctoral dissertation, University of Connecticut, Storrs.
- Mark, L. S., Shapiro, B. A., & Shaw, R. E. (1986). Structural support for the perception of growth. *Journal of Experimental Psychology: Human Perception and Performance*, *12*, 149-159.
- Mark, L. S., Shaw, R. E., & Pittenger, J. B. (1988). Natural constraints, scales of analysis, and information for the perception of growing faces. In T. R. Alley (Ed.), *Social & applied aspects of perceiving faces* (pp. 11-49). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
- Mark, L. S., & Todd, J. T. (1985). Describing perceptual information about human growth in terms of geometric invariants. *Perception and Psychophysics*, *37*, 249-256.
- Mark, L. S., Todd, J. T., & Shaw, R. E. (1981). The perception of growth: How different styles of change are distinguished. *Journal of Experimental Psychology: Human Perception & Performance*, *7*, 355-368.

- Moon, P., & Spencer, D. E. (1988). *Field theory handbook* (corrected 3rd printing). New York: Springer-Verlag.
- Peterson, R. G. (1974). *The relationship between contour deformation and perceived separation of forms in depth*. Unpublished doctoral dissertation, University of Minnesota, Minneapolis.
- Pittenger, J. B., & Shaw, R. E. (1975a). Aging faces as viscal-elastic events: Implications for a theory of non-rigid shape perception. *Journal of Experimental Psychology: Human Perception and Performance*, 1, 374-382.
- Pittenger, J. B., & Shaw, R. E. (1975b). Perception of relative and absolute age in facial photographs. *Perception & Psychophysics*, 18, 137-143.
- Pittenger, J. B., Shaw, R. E., & Mark, L. S. (1979). Perceptual information for the age level of faces as a higher order invariant of growth. *Journal of Experimental Psychology: Human Perception and Performance*, 5, 478-493.
- Pittenger, J. B., & Todd, J. T. (1983). Perception of growth from changes in body proportions. *Journal of Experimental Psychology: Human Perception and Performance*, 9, 945-954.
- Prigogine, I. (1980). *From being to becoming*. San Francisco: Freeman.
- Riolo, M. L., Moyers, R. E., McNamara, J. A., & Hunter, W. S. (1974). *An atlas of craniofacial growth: Cephalometric standards from the university school growth study*. Ann Arbor: Center for Human Growth and Development, University of Michigan.
- Seamon, J. G. (1982). Dynamic facial recognition: Examination of a natural phenomenon. *American Journal of Psychology*, 95, 363-381.
- Shaw, R. E., & Carello, C. (1979). *Are faces special?* Paper presented at the 20th Meeting of the Psychonomics Society, Phoenix, AZ.
- Shaw, R. E., Flascher, O. M., & Mace, W. (in press). Dimensions of event perception. In W. Prinz & B. Bridgeman (Eds.), *Handbook of perception and action*. Tullamore, Ireland: Academic.
- Shaw, R. E., Mark, L. S., Jenkins, D. H., & Mingolla, E. (1982). A dynamic geometry for predicting growth of gross craniofacial morphology. In A. Dixon & B. Sarnat (Eds.), *Factors and mechanisms influencing bone growth* (pp. 423-431). New York: Liss.
- Shaw, R. E., & Pittenger, J. B. (1977). Perceiving the face of change in changing faces: Implications for a theory of object perception. In R. Shaw & J. Bransford (Eds.), *Perceiving, acting, and knowing* (pp. 103-132). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
- Shaw, R. E., & Pittenger, J. B. (1979). Perceiving change. In H. Pick & E. Saltzman (Eds.), *Modes of perceiving and processing information* (pp. 187-204). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.
- Todd, J. T., Mark, L. S., Shaw, R., & Pittenger, J. B. (1980). The perception of human growth. *Scientific American*, 242, 132-144.
- Turvey, M. T. (1992). Affordances and prospective control: An outline of the ontology. *Ecological Psychology*, 4, 173-187.
- von Fieandt, K., & Gibson, J. J. (1959). The sensitivity of the eye to two kinds of continuous transformation of a shadow-pattern. *Journal of Experimental Psychology*, 57, 344-347.
- Warren, W., Jr., & Shaw, R. E. (1985). Events and encounters as units of analysis for ecological psychology. In W. Warren, Jr. & R. E. Shaw (Eds.), *Persistence and change* (pp. 1-23). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

APPENDIX A

Mathematical Formula for the Growth Transformations

THE CARDIOIDAL STRAIN TRANSFORMATION

Motivated by the coordinate transformation approach developed by D'Arcy Thompson, this model geometrically changes one biological form into another.

Growth under this approach is assumed to be initiated at a point, such as a nodal point on a stem from which an apple or a leaf might grow. Because this model turns a circle into a cardioid, it is also known as cardioidal strain (Pittenger & Shaw, 1975a; Pittenger et al., 1979; Mark et al., 1981) and is expressed in polar coordinates as

$$\theta' = \theta$$

$$R' = R(1 - k \cos\theta).$$

THE HYDROSTATIC TRANSFORMATION

Growth of all natural objects takes place in a gravitational field. Incorporating the effect of gravity with the well-known hydrostatic principle resulted in the *hydrostatic model* of growth. In this model, growth is assumed to emanate from the center of the fluid-filled sphere to the direction of gravity. Although growth of all types is reduced to one process—the infusion of fluid into the sphere and to one site, the center of the sphere—this model has proven to be biologically—as well as perceptually—valid (Mark & Todd, 1985; Mark et al., 1981).

From elementary hydrostatics, the hydrostatic transformation can be expressed in polar coordinates as

$$\theta' = \theta$$

$$R' = R[1 + k(1 - \cos\theta)].$$

THE ORTHOGONAL CARDIOIDAL STRAIN TRANSFORMATION

By applying an orthogonality condition to the cardioidal strain transformation and then reflecting the facial curves in the upper half, another model is obtained in which a set of growth trajectories cut the facial (or equipotential) curves orthogonally. The resultant geometry is mathematically equivalent to an orthogonal cardioidal field (Moon & Spencer, 1988), which is defined in the complex plane as

$$\bar{z} = \frac{1}{2}w^{-2} \quad (1)$$

where $\bar{z} = x - iy$, $w = u + iv$. Note that because z and w are complex variables, the values z and w can be assumed to be located in the planes. Moreover, the z plane, in which objects are defined, is specified in terms of a rectangular coordinate system, whereas the w plane, in which a strain transformation is

defined, is specified in terms of an orthogonal coordinate system. To apply a strain transformation to an object in the orthogonal cardioidal field, a coordinate transformation has to be carried out between a rectangular coordinate system and an orthogonal coordinate system. That is, (a) determine a 1-1 transformation from the z plane to the w plane, or $(x, y) \rightarrow (u, v)$; (b) apply a strain transformation in the w plane by the amount of free parameter, or $(u, v) \rightarrow (u', v')$; and (c) determine the inverse transformation from the w plane to the z plane so that the transformed image can be displayed on a computer, or $(u', v') \rightarrow (x', y')$.

To carry out the first step, we solve equation 1 in terms of w to obtain

$$w = \frac{1}{\sqrt{2}\sqrt{z}}, \quad (2)$$

Equation 2 may also be expressed as

$$u + iv = \frac{1}{\sqrt{2}\sqrt{x - iy}}. \quad (3)$$

From Equation 3, u and v can be computed in terms of x and y as follows:

$$u = \frac{\cos\left(\frac{\theta}{2}\right)}{\sqrt{2}\sqrt[4]{x^2 + y^2}}$$

$$v = \frac{\sin\left(\frac{\theta}{2}\right)}{\sqrt{2}\sqrt[4]{x^2 + y^2}} \quad (4)$$

where $\theta = \tan^{-1}\left(\frac{y}{x}\right)$. Then, a strain transformation is applied by the amount of free parameter

$$\begin{cases} u' = u \\ v' = v + k \end{cases} \quad (5)$$

where k is the free parameter. To determine the inverse transformation from the w plane to the z plane, Equation 1 is expressed as

$$x - iy = \frac{1}{2}(u + iv)^2. \quad (6)$$

By solving Equation 6, we obtain

$$x = \frac{(u^2 - v^2)}{2(u^2 + v^2)^2} \quad (7)$$

$$y = \frac{uv}{(u^2 + v^2)^2}$$

which are used to display the image on a computer screen.

THE HYDRODYNAMIC TRANSFORMATION

Wolff's law (cf. Enlow, 1968) asserts that stress (or force) is a direct stimulus to growth (a type of strain). Thus, gravity, a directed global force, would direct growth along lines of least work. As a rule, paths of least work will be curved, rather than rectilinear (straight). For example, a balloon filled with water suspended in space like a soap bubble will tend to assume a form that minimizes the work required to maintain its structural integrity. This means that for any closed, volume-increasing material situated in a gravitation field, its surface tends to be curved and the paths of points on the expanding surface also tend toward constant curvature (i.e., spheres), which includes heads growing under the influence of gravity. However, gravity cannot act on a water-filled sphere above a line perpendicular to the direction of gravity where there is no more fluid (i.e., the top of the head). Thus, the field is restricted to the lower half-plane. This restricted field is mathematically equivalent to a hydrodynamic field; objects in this field undergo changes in shape by what is actually a hydrodynamic transformation (Carello et al., 1989; Shaw et al., 1982).

A hydrodynamic field is specified in terms of a tangent circle coordinate system (Moon & Spencer, 1988), whose coordinate transformation is defined in the complex plane as

$$\bar{z} = \frac{1}{w} \quad (8)$$

where $\bar{z} = x - iy$, $w = u + iv$. Following the same steps identified in the orthogonal cardioidal strain transformation, Equation (8) is solved in terms of w to give

$$w = \frac{1}{\bar{z}} \quad (9)$$

Equation 9 is expressed as

$$\begin{aligned} u + iv &= \frac{1}{x - iy} \\ &= \frac{x}{x^2 + y^2} + i \frac{y}{x^2 + y^2} \end{aligned} \quad (10)$$

to produce

$$\begin{aligned} u &= \frac{x}{x^2 + y^2} \\ v &= \frac{y}{x^2 + y^2} \end{aligned} \quad (11)$$

A hydrodynamic transformation is applied in the direction of gravity by the amount of free parameter, k ,

$$\begin{cases} u' = u \\ v' = v + k. \end{cases} \quad (12)$$

To carry out the final step, we express Equation (8) as

$$\begin{aligned} x - iy &= \frac{1}{u + iv} \\ &= \frac{u}{u^2 + v^2} - i \frac{v}{u^2 + v^2} \end{aligned} \quad (13)$$

From this, we obtain

$$\begin{aligned} x &= \frac{u}{u^2 + v^2} \\ y &= \frac{v}{u^2 + v^2} \end{aligned} \quad (14)$$

APPENDIX B

Mathematical Formula for the Nongrowth Transformations

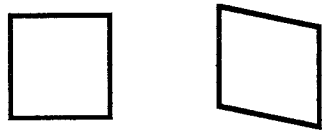
- (1) The X-Axis Shear with respect to Y-Axis (in rectangular coordinates)

$$\begin{aligned} Y' &= Y \\ X' &= X + k Y \end{aligned}$$



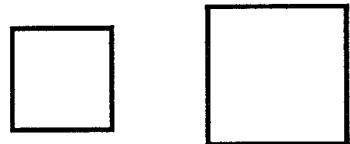
- (2) The Y-Axis Shear with respect to X-Axis (in rectangular coordinates)

$$\begin{aligned} Y' &= Y + k X \\ X' &= X \end{aligned}$$



- (3) The Similitude Transformation (in rectangular coordinates)

$$\begin{aligned} Y' &= Y (1 + k) \\ X' &= X (1 + k) \end{aligned}$$



- (4) The Null Transformation (in rectangular coordinates):

$$\begin{aligned} Y' &= Y \\ X' &= X \end{aligned}$$



k — free parameter

