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ALGORISTIC FOUNDATIONS TO COGNITIVE PSYCHOLOGY¹

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"It is a curious fact . . . that as reasoning improves, its claims to the power of proving facts grow less and less. Logic used to be thought to teach us how to draw inferences; now it teaches us rather how not to draw inferences."

—Bertrand Russell

"The specialist is one who learns more and more about less and less and thus ends up knowing everything about nothing."

—attributed to George Bernard Shaw

What is known and *how* it is known are relative questions that make no sense independent of the question of *who* knows. Indeed, our opinion is that the central question of cognitive psychology concerns the essential nature of a knowing-agent, rather than just what is known or even how what is known *is* known. Only a certain kind of sceptic can hold that all things are relative without falling into the absurdity that if *everything* were relative, there would of course be nothing for it to be relative *to*.

In the past decade or so psychology has relinquished its obsessive concern for the question of how organisms behave, or can be made to behave, in favor of a broader set of questions. It is now popular to assume that *what* people process is

¹Preparation of this paper was supported, in part, by a Career Development Award to Robert Shaw from the National Institute of Child Health and Human Development (1 KO4-HD24010) and by grants to the University of Minnesota, Center of Research in Human Learning, from the National Science Foundation (GB-35703X), the National Institute of Child Health and Human Development (HD-01136 and HD-00098), and the Graduate School of the University of Minnesota.

information, where for some theorists information is a pure mathematical measure of the uncertainty conveyed by disjunctive decision possibilities, and for others it is a semantic measure of the ecological significance of structured stimulation for a particular class of organisms. *How* information is processed is sometimes said to be by rules, procedures, algorithms, or some other functional analogue to computer programs. And, finally, the clinician reminds us that people are somehow the thinking, planning, dreaming, loving, hating, and behaving agents in whom the above processes reside, and for whom information is valuable. Unfortunately, we have so far failed to reach a consensus about the interrelationship of the *what*, *how* and *who* questions, much less how they might be answered.

This chapter discusses the concepts and principles by which these three problems might be related and attempts to specify the metatheoretical criteria by which answers to each might be recognized. Our intent is to provide a cogent rebuttal to reductionistic claims that theoretical solutions to any one of them alone will suffice for them all. We also hope to demonstrate that the inadequate interest shown thus far by cognitive psychologists in the nature of the epistemic-*who* (the knowing agent) has been a chief stumbling block in our attempts to understand the nature of what information is, and how it is processed. It is, of course, both trite and tiresome to argue merely that psychologists should study the *whole* man without explaining how this might be done. A more adequate approach must at least discuss the theoretical principles and methodological tools by which a start can be made.

The first section provides a theory-sketch of the class of dynamic systems to which knowing-agents belong, and tries to suggest why a study of man and other organisms at this level has dividends for the study of the other two questions. Section two discusses a new and admittedly radical approach to theory construction and evaluation which is especially tailored to the needs of cognitive psychology. The final section attempts to apply principles and concepts introduced in the earlier sections. Thus, to a great extent, the cogency and relevance of the arguments presented in the earlier sections must be judged in the light of the success of the final one.

I. ALGORISTIC BASES TO THE EPISTEMIC-WHO

Warren S. McCulloch, one of the founders of the cybernetic movement in the early 1940's, was an intellectual catalyst of the highest magnitude. His seminal papers seeded the minds of many students and colleagues, which he cultivated by periodic trips to the world's major life science laboratories. The titles of his papers tell us much, not only about the man, but about the loftiest aspirations psychobiologists and cognitive psychologists might entertain: "What's in the brain that ink might character?"; "Machines that think and want?"; "*Mysterium Iniquitatis*: Of sinful man aspiring into the place of God"; "Why the mind is in the head"; and many others. What we have called the question of the "epistemic-who," or knowing agent, is meant to be synonymous with the question so elegantly suggested by his title, *The Embodiments of Mind* (McCulloch, 1965).

John von Neumann, the mathematician and close friend of McCulloch, once argued that the real challenge for psychology and biology was not merely describing

the behaviors of man, nor the topography of the brain, but of discovering logical methods by which we might determine what purposive activities (what he called "effectivities") a system might support as its structural complication approached that of the human brain. He was afraid that the usual hypothetico-deductive techniques and modelling procedures might fail when applied to living systems of great structural complexity (Shaw, 1971). He speculated, moreover, that the problem might ultimately arise from the fact that new effectivities emerged in quantal jumps as the systems become more complex.

In the light of von Neumann's arguments the task of cognitive psychology is not so much to describe what behaviors man might emit, nor even what stimulus conditions might evoke them, but to determine what is in the nature of man that requires and supports the need and purpose of such activities—whether they be physical or mental. Again the marvelous balance of poet, mathematician, physiologist, psychologist, and theologian in McCulloch expressed the issue more eloquently than most.

When as a young student at a Quaker college McCulloch was asked by his advisor what he wanted out of life, he replied; "I have no idea; but there is one question I would like to answer: What is number that man may know it, and a man, that he may know number." His Quaker advisor smiled thoughtfully and replied, "Friend, thee will be busy as long as thee lives."

McCulloch and Pitts (1953), by providing an abstract neural model, did not intend primarily to give an accurate description of real neurons, for they chose to leave out of the design of their modules important properties such as refractory periods. Nor did they merely intend to simulate behaviors, nor even to demonstrate that at limit extremely complex finite neural nets approached the computational power of Turing machines (i.e., the most powerful class of computing machines), as many reviewers have claimed. Rather they were after a far deeper question: How can a complex system of neurons, a living brain, come to know the world and to perform adaptively in it?

McCulloch (1951), like James J. Gibson, believed that what best characterizes the effectivities of living brains is their ability to detect and use invariant aspects of sensory stimulation, which reliably specify environmental "universals" such as shape, distance, size, position, and time. Gibson pushes us even deeper into the question of *what* is perceived, and hence might be learned, by arguing that what is directly perceived is the value or functional utility (affordances) of objects and events for the organism. The nature of such value as afforded by the organism's pick up of the invariants of sensory information, Gibson tells us, is both "formless and timeless" — a statement that echos the tenor, if not the substance, of McCulloch's concern for how a neural brain can be a knower of universals.

How might we, however, avoid getting lost in the rarefied atmosphere of platonic ideals, if we pursue this question? Can this question be pursued empirically as well as theoretically? We believe these issues can be favorably resolved by an approach which provides an integral attack on the *what* and *how* questions by using the *who* question as a theoretical fulcrum. If we can even roughly decide on the nature of the epistemic-who we will, at the same time, *have* to take a stand on the

nature of the information processed from and about the environment, as well as on the nature of the psychological processes required to do so.

Who? What? and How?: A Closed Class of Questions

It is often the case that methodologies of sufficient precision to study one fundamental question are inadequate for the study of others. Controls required to rid experimental designs of confounding factors may at the same time rule out inferences that go beyond the single question posed by the theorist. For instance, if one is studying the effects of a drug on learning one must control any effects due to individual differences. In this way the study of the effects of *what* on *how* is only interpretable when the *who* variable has been controlled.

In a deeper theoretical sense, it may be the case that theoretical assumptions about the fundamental nature of certain psychological processes (e.g., that they are *serial* rather than *parallel*, *discrete* rather than *continuous*, *probabilistic* rather than *deterministic*, *learned* rather than *innate*, *voluntary* rather than *automatic*, *conscious* rather than *unconscious*, etc.), impose such logical restrictions as to make inferences to other views of the same processes impossible, since by definition they lead to contradiction. When this is the case, many fruitless and interminable controversies arise (witness the "nature-nurture" controversy earlier this century and reviewed in the Chomsky (1959) and Skinner (1957) debates, the "structuralist-functional" arguments between Lashley (1950) and the Gestaltists over the status of the memory trace; or the Neisser-Sternberg arguments over visual processing as parallel or serial, respectively).

A classic case took place in the 1930s between Einstein and Heisenberg over the issue of whether physical reality is fundamentally based on chance or deterministic laws. "God does not play at dice!" proclaimed Einstein, "But he does!" countered Heisenberg. Who is right?

"Both and neither," suggested Niels Bohr, for whether a significant degree of arbitrariness can be imputed to the nature of physical events depends not so much on what is true as on the way one asks the question.

Different questions may require such radically different experimental methodologies and theoretical contexts that the answers they yield are complementary. To achieve an answer to one question one necessarily sacrifices any claim to the other (a veritable "you can't have your cake and eat it too" limitation on the techniques science can use in its interrogation of nature). Thus, Bohr's principle of complementarity suggests a deeper epistemological insight than either of the stubborn positions adopted by Einstein or Heisenberg, since it provides a reason for the methodological chasm that exists between relativistic and quantum physics.

Similarly, complementary relations exist in fields other than physics. Consider, for instance, the conflict in goals that arises when one anatomist wishes to base the theory of brain functions on the dissection of "dead" brains while another insists that the proper approach is implantation of electrodes into the living brain. Clearly, the anatomical scope of "dead" anatomy may be broader than that of "living" anatomy, while the latter may be able to peer deeper into brain functions than the former. The two approaches can not even in principle be integrated since an attempt to increase the scope of surveillance by simultaneously implanting more

and more electrodes would destroy the normal functioning of the living brain as surely as any attempt to dissect it by scalpel.

Consequently, we see such a complementarity holding between current approaches that seek to study the *what* of perception, as opposed to those seeking to explain the *how* of perception. The former view is exemplified by James J. Gibson's (1966) investigations into the direct sources of information pickup by the visual system, which he terms "ecological optics," as opposed to the more Helmholtzean approaches which assume preconscious constructive processes, say as exemplified by the information-processing theories of Sperling (1960), Neisser (1967), Gregory (1966), Kolars (1968), Norman (1969), Broadbent (1958), and Haber (1969).

These constructive theories, as compared to Gibson's, do not really qualify as theoretically commensurable alternative explanations of the same phenomena, as might seem at first glance. Rather, they differ so radically in their definition of perception as to begin from contradictory assumptions which lead to quite different theoretical goals. Gibson defines perception as the direct pick-up of informational invariants by the senses operating as a perceptual system, while the other theorists (although often at odds on many secondary issues), seem to agree that perception is not a direct psychological process but involves the mediation of memory, learning, or cognitive subprocesses. Gibson's concept of information is dependent on the genetic preattunement of the species, and the education of attention for the individual organism, in the detection of certain fundamental invariants of stimulation from the terrestrial environment (e.g., gravity, sky-above and ground-below, horizon, texture gradient of the ground plane, texture-flow field properties of the optic array, etc.). Such informational invariants specify important properties of the organism's environment, to which it must adapt to survive and maintain its health and well-being; hence, the perceptual information can be said to specify "affordances" of great ecological significance.

On the other hand, the so-called "dynamic" perceptual theorists rarely discuss the ecological significance of perceptual information and often ignore or denounce the concept of invariance. Instead, they discuss the ordered stages of processing and the rules or procedures used by each stage. Their approach essentially assumes that perceptual processing can be analyzed into discrete temporal cross-sections, perpendicular to the flow of information.

In contradistinction to this approach, Gibson assumes that perceptual processing of information is a continuous interactive relation between the organism and its environment. The all-important invariant relationships exist *not* in the discrete cross sections of the information flow, but in the constant longitudinal relations picked-up over time by an "afferent-efferent-re-afferent" continuous process loop.

If, as Gibson claims, ecologically significant informational invariants exist only over time, then it is not surprising that theorists who study discrete cross sectional segments of the information processes fail to find them, and therefore deem them to be of little theoretical importance. Thus, the claim that the two approaches are complementary is not so far-fetched as might first be thought.

It is fair to conclude that the major difference between Gibson's ecological approach to perception and the dynamic approach is that Gibson is really asking

the *what* question while the other theorists are asking the *how* one. Where Gibson emphasizes the study of information as determined by the organism-environment interaction, the other theorists emphasize the study of the information-processing capabilities of organisms after they are presented with any information – whether of ecological value or not.

However, only the *what* and *how* questions are touched by the union of these two approaches. Unfortunately, this leaves virgin the question regarding the nature of the perceiver *qua* agent that processes what the environment has to offer and hence knows how to survive. Since the intersection of the *what* and *how* questions is in the nature of the epistemic-*who*, we might hope that due consideration of this third side of the issue will help integrate the two approaches discussed above and weld the three together into a coherent account of “knowing” organisms.

The question of the epistemic-*who* should not be confused with the claim that the concept of a knowing agent refers to an unanalyzable metaphysical “spook” or ghost in the biological machine. On the contrary, we will attempt to analyze the class of systems to which knowing-systems might belong. Nor does the proposed analysis degenerate into descriptive phenomenology, nor become a virtual sink-hole of introspective analysis. Rather we shall attempt to relate the class of systems that best describe knowing-agents to various classes of well-defined abstract automata (i.e., “machine” theory).

There is a major distinction between the so-called “algorithmic” approach, which attempts to characterize psychological processes in terms of input (stimulus) – output (response) functions, and what we will call the “algoristic” approach. The algorithmic approach fails to prescribe the intrinsic requirements which must be satisfied by complex systems which evolve, grow, develop, and function under the dominion of natural law.

Clearly, cognitive psychology must offer a description of agents more precise than that provided by the description of algorithms which psychological processes might follow in satisfying observed input-output functions. The class of abstract machines that may be competent to process such algorithms is obviously much larger than the class of machines that may be physically realized by construction as artifacts or by natural evolution. In fact, mathematics as such does not address this narrower question of what natural constraints must be satisfied by abstractly competent machines for them to be as effective as humans when functioning under real time and real space constraints imposed by the conservation of energy and work laws. (Exactly what these constraints might be will be discussed later.)

Moreover, there is considerable reason to suppose we have not yet defined the nature of “machine” with sufficient precision or abstraction to characterize how a neural brain might also be a “knowing” brain. Since, we believe, this is the question of the abstract *who* rather than of the *what* or *how* of knowing, a new set of assumptions about analysis may be needed to move toward the required theoretical basis for cognitive psychology. If we are lucky, or nature should be benevolent, we may be able to discover a *closed* set of variables which allow us to express the complementarity relation holding among the three primary analytic concepts of psychology (i.e., the *who*, *what* and *how* concepts). If so, then whatever properties we discover about one of these factors will imply the existence of a

compatible set of properties with regard to the other mutually dependent ones. As a concrete illustration of how these concepts may belong to a closed set consider the following: The degree of hardness of a sheet of metal tells us something about the nature of the saw we must use to cut it (i.e., something about *what* is to be done); a blueprint or pattern must be selected in the light of what can be cut from the materials with a given degree of tolerance (i.e., *how* it is to be done); while both of these factors must enter into our equations to determine the amount of work that must be done to complete the job within a reasonable amount of time. This latter information provides a job description that hopefully gets an equivalence class of existing machines rather than a class that might accomplish the feat in principle but not in practice (i.e., implies the nature of the *who* or *what* required to do the task).

The value of seeking closure to our set of analytic concepts lies in the hope that we will succeed in discovering a logic of co-implication on which to build the laws of cognitive psychology. Only by making a cogent case for the possibility of such a logic of closed concepts can we justify introducing additional variables into our science in defiance of Occam’s wise admonition not to multiply entities beyond necessity.

Now we must attempt to clarify the claim that the intuitive meaning of the concept of a knowing-agent is not exhausted by a description of what he does or how he does it. We will also argue that the “logic” of instantiating abstractly competent systems requires that the functions carried out by the device be supported at the physical, biological, and psychological levels; that is, the components of the system must be defined to satisfy natural constraints imposed by each of these levels of analysis. The multilevel support requirement rules out the possibility that some levels can be reduced to others. This is the only “metaphysical” postulate required and is asserted more for its pragmatic value as a working hypothesis than as a necessary truth. For instance, one immediate consequence of the multilevel support postulate is that it provides an hypothesis which accounts for the spurious evidence favoring reductionistic theories, say those that argue that psychological phenomena are epiphenomena of, or identical to, biological ones, or that biological phenomena can be totally explained by physical laws. From this postulate we can argue that the evidence presumed to support reductionistic arguments is due to the theorist mistaking co-implicative relations among the closed set of variables for an identity relation among them. Thus in order to lay the foundations for a truly general cognitive psychology we must both distinguish and relate the concept of information to the physical energy distributions which manifest it, the biological processes which preserve informational invariants, and the psychological processes which detect and use them. Our claim is that each of these levels constrains and supports the others. Thus, since none can be functionally independent of the others, reductionism is ruled out.

Toward a Rigorous Definition of Algorist

In the ninth century A.D. Al-Khuwarizimi wrote a book on procedures for arithmetic calculation. After his book became known to European scholars his name

was latinized to 'Al-Gorism.' Later any procedure for calculation came to be an "algorithm," while the man who executed the procedures was known as an "algorist." Our current term "algorithm" is a later corruption of this earlier term. But what became of the term "algorist?"

Often, when a word disappears from technical discourse, it is because the concept referred to has been discovered to be useless or fallacious and therefore abandoned (for instance, such terms as "aether" in reference to a medium which supports electromagnetic propagation, or "phlogiston" as the substance liberated when a material burns). More recently the usefulness of the terms "force" and "causation" have been seriously questioned by physicists and philosophers of science (Russell, 1945).

There are other cases, however, in which terms disappear from use that are more difficult to account for. For instance, the concept of vitalism (*elan vital*) served no useful purpose in biology although this is not due to the fact that current definitions of life or organism are sufficiently precise to disallow application of the term. Consider as evidence that this is still an open issue the fact that although we are willing to call ourselves, as well as animals, insects, paramecia, and even plants, "organisms," we do not know what to call viruses. It could be that we just do not know enough about them; or it could be that there is something fundamentally wrong with our intuitive conception of "organism" which guides our use of the term.

Such slippage in our technical vocabularies must be taken very seriously since it often indicates significant conceptual problems at the basis of a science. Many philosophers continue to agonize over the definition of such words as "life," "consciousness," "free-will," and "value," for this very reason. We submit that the passing from usage of the term "algorist" is similar evidence that no science, psychology the least, has successfully formulated a technically useful term for agent, not because the concept is itself intuitively barren (the way "force" has become in theoretical physics), but because of the dramatic advances in physics and more recently in computer science, that psychology as a science has been encouraged to press more toward *what* and *how* questions than *who* questions. Assuming that co-implication holds among the variables involved in this closed set, we should not be surprised that no major breakthroughs in cognitive psychology have yet occurred. If we fail to ask *who* is conscious or living, or *who* has free will, or for *whom* something has value, but only how he might be conscious, living, free, or served by value, or of what he is conscious, by what means he is living, free, or served by value, then we should not be surprised that our experiments uncover more phenomena than they explain, that our theories raise more questions than they answer. By concentrating too much on a few of the questions, we have given up whatever logical leverage we might have on all the questions. Whitehead once observed that if you wish to understand truly the conceptual framework of a historical period, ask not what answers they give to the questions they raise, but what questions they *fail* to raise.

In order to illustrate our underlying confusion regarding the distinction between *how* and *who* questions, consider the development of the field of computer science: Parallel to the etymological development of the term "algorithm" was the concept of calculator. In ancient time the calculator was merely a

mechanical aid to calculation by a human agent, (e.g., a set of counting beads, measuring sticks, or vessels), while an algorithm was considered a procedural aid to calculations by algorists. A clear-cut intuitive distinction existed between algorithms as procedures, calculators as mechanical aids, and algorists (or agents) who used the two.

Now notice what has become of the precise distinction among these terms due to the spectacular developments of computer science: Ask yourself, "Is the programmer of a modern multipurpose digital computer or the machine itself most appropriately consider the algorist?" If this question poses no difficulty, consider how fuzzy the distinction becomes as we progress toward the construction of truly intelligent autonomous devices capable of dynamic self-programming when but provided an initial program and presented suitable data to work on. A closer analogy between such machines and humans can be seen than might be obvious since there is good reason to believe humans are not born *tabula rasa*, and that perceptual equipment may be genetically pre-attuned to the perceptual information of our terrestrial world.

Not only is our concept of living agent semantically "squishy" but so is our concept of machine or calculator. If we assume that the technological development of machines continues unabated, there is the realistic possibility that the dividing line between nonliving machine and living system might become increasingly obscured. Consider, for instance, the malevolent and egotistical computer named Hal on the space ship in the Stanley Kubrick movie *2001* which took over the ship and began killing the crew. Here is a hypothetical case of a machine that was more than just a mechanical aid to calculation. Hal surely had autonomous agent-like properties and thus qualified as an algorist. Many philosophers of science have unsuccessfully agonized over the question of what criteria, if any, are sufficient for deciding when a complex machine with human effectivities ceases being nonliving and nonhuman (MacKay, 1952; Putnam, 1960; Scriven, 1953). At what point would it be necessary to give machines like Hal, or even machines that are developmentally his intellectual inferiors, civil rights? When would we consider pulling the plug on such machines (as was eventually Hal's fate in the movie) as homicide and therefore punishable by law? Would we want to argue that such advance machines are not really algorists since they did not naturally evolve? Or, would it be more rational to argue that there are two kinds of living agents—those that are born of woman and those that are constructed by man?

Not only does the concept of intelligent agent shade off into that of calculator when considering sophisticated machines, but so does the notion of calculator merge with that of an algorist when a man is sufficiently competent to eschew mechanical aids and executes all the algorithmic steps mentally. As Wittgenstein so well knew, this kettle of logical worms is a simulation theorist's nightmare. Very strong assumptions about the nature of cognitive processes are required to disentangle the notions of "rules involved in behavior," "rules entailed by behavior," and "behavior entailed by rules." No wonder that many of us find the phrase "rule governed behavior" difficult to use in a technically precise way. Yet however we decide this issue, the solution will contribute only indirectly to a clear intuitive account of algorists. For at best an analysis of the algorithmic, or heuristic,

procedures which approximate them tells us only *how* what is done is done, and does not address either the question of what information presented to the senses might specify about the environment, nor what significance it may have for any class of algorists. Moreover, no one should be surprised that attempts to provide a logical analysis of scientific phenomena often (or perhaps always) require *ad hoc* augmentation by extralogical principles to be adequate.

A classic example of this was Bertrand Russell's attempt to characterize physical events so that the results of quantum and relativity physics might be incorporated into our commonsense belief system about physical phenomena (Russell, 1948). One troublesome result of advances in these fields was to bring into question the causal analysis of physical events due to the recognition by most logicians of science that a *ceteris paribus* clause (i.e., other things being equal) must be included among the antecedent premisses of such analysis (Lakatos, 1970) to render the condition for causal interaction sufficient. Russell provided the following extralogical postulates which he believed sufficient (although not necessary) to render highly probable the conditions for causal interaction among physical events (an object itself being an event). They are (a) quasi-permanence, (b) separable causal lines, (c) spatio-temporal continuity, (d) a structural postulate, and (e) the postulate of analogy.

Similarly, one might ask what extralogical assumptions must be entertained regarding the epistemic-who, if both information and cognitive processes are to be adequately characterized. To arrive at the metatheoretical postulates of a truly general cognitive theory which incorporates all three questions, we must first determine where the field is at the moment.

Paradigm Shift? When Descartes wrote "On the automation of Brutes," spring-wound mechanical toys were the highest development of scientific models for men and other animals. When telephone switchboards were considered an engineering triumph (at the turn of the 20th century), Sherrington's switchboard-based reflexology provided a theoretical mechanism popularly received by behaviorists. By midcentury, feedback mechanisms and servomechanisms had been developed to provide a goal-directed, quasi-purposive, control system for anti-aircraft artillery and to steer ships and missiles. Consequently, homeostatic principles gained popularity in both biology and psychology. Currently, programmable, multipurpose computers epitomize man's technological development. Hence, we are not surprised to find concepts borrowed from computer science, and its cognate disciplines of mathematical linguistics and automata theory gaining popularity among psychological theorists. New terms like "psychological competence," "rule-governed behavior," and "simulation model," are replacing more shop worn terms such as "habit structure," "associative network," and "stimulus-response model."

It is ironic, if not surprising, that such innovative concepts derive from theorists whose training was in fields other than psychology. Theorists such as Piaget, a biologist, Simon, a chemist, Minsky, a computer scientist, and Chomsky, a linguist, have had a vitalizing and lasting influence. Where probability theory was once the dominant method for formalizing psychological phenomena, algebraic structure theory (that part of mathematics which includes the theory of formal languages and

grammars, abstract automata, switching theory, semi-groups, groups, lattices and categories) is gaining prominence (Arbib, 1969a; Chomsky, 1963; Clowes, 1967; Ernst & Newell, 1969; Norman, 1970; Pylyshyn, 1972; Quillian, 1967; Scandura, 1970; Selfridge, 1959; Suppes, 1968; Winograd, 1972).

Although it is difficult to give a general characterization fair to all these views, and probably impossible to cite a single set of premises and principles commonly assumed by such new and diverse approaches to the study of cognition, they nevertheless seem to embody a common ideal; namely, they attempt to provide a rigorous, detailed formal analysis of cognitive processes. (For the sake of generality, we are incorporating the various modes of processing involved in perception, thinking, communication, memory, etc., under the single term "cognition.")

If we follow the suggestion by metamathematicians to consider algebraic functions (or equivalently, relations) to be well-defined (in the finitistic sense) when an algorithm is constructed by which they might be computed (or generated), then all the above approaches can be said to be attempts to discover the *algorithmic* bases of cognitive processes. The intuitive notion of an *algorithm* for a computable function is that of an *effective* procedure which consists of a sequence of instructions so simple, discrete, deterministic, and finite that a noncreative agent (a machine) can execute them to find the values of the functions without guesswork or inference.² Thus, the attempt to discover the algorithmic bases of cognitive functions is the attempt to define the algorithms by which neurological mechanisms might compute them.³

We can all agree it is too soon to decide the ultimate validity of any of these algorithmic approaches to psychological theory, but it is by no means premature to ask whether theoretical efforts should be focused solely on this level of analysis and whether investigations at this level alone are even potentially capable of providing adequate explanations. This is not to argue against the importance of algorithmic analysis, since we feel it to be a necessary part of any approach. Rather, our contention is that the algorithmic bases of cognitive processes can only be defined relative to what we wish to call their *algoristic* bases.

Earlier we discussed how slippery the distinction between the concepts of algorithm and algorist has become due to their converging etymological heritage

²The fact that some of the theorists mentioned above may prefer to think of psychological processes as heuristic and probabilistic, rather than as algorithmic and deterministic, does not invalidate their inclusion under the rubric of the algorithmic approach. It can be argued that for all finite devices the probabilistic success of heuristic procedures executed by them can only be rigorously defined relative to the ideal limits prescribed by algorithms which they most closely approximate. Recognition of this fact seems to be the primary motivation of Chomsky's *competence-performance* distinction.

³A word of caution: The desire for formal precision characteristic of these contemporary algorithmic approaches should not be confused with the now *passé* attempts by earlier psychologists, such as Hull and Lewin, to axiomatize various aspects of psychology. Axiomatization is not the issue: Rather the substantive issue concerns what *ideal* goals cognitive theory should strive toward to achieve the most adequate forms of explanations for significant psychological phenomena. The issue then is primarily one of the pragmatic utility of the theoretical focus prescribed by the algorithmic presuppositions of these approaches.

and the fact that the notion of mechanical execution of computational procedures has been easier to describe rigorously than has the intentionality of agents. The reason for this difficulty is that just as the intuitive notion of function rests conceptually on the intuitive notion of an *effective* procedure, or algorithm, so the intuitive notion of algorithm rests ultimately on what is meant by an epistemic agent, or algorist.

The history of contemporary American psychology can be viewed as falling into three major periods which embody the distinctions we wish to make: The first period, running from the early 1920's until the mid 1950's, was characterized by attempts to discover the behavioral bases of psychological phenomena. The second period, from the 1950's until now, has been characterized by attempts to discover the algorithmic bases of behavior.

Currently, however, we seem to be on the threshold of a third period, in which theorists attempt to discover the algoristic bases of the "algorithms" of mind. Although many earlier theorists have anticipated the goals of this third period, no clear formulation of what constitutes an algoristic basis has yet been given. Disenchantment with the pure syntactic approach to psycholinguistic theory and the rebirth of interest in its semantic foundations, the growing concern over the bases of human intentional behaviors and the desire for reintroducing "value" into perception by assuming an ecological approach, the bankruptcy of computer models for psychoneural processes, the limited success of simulation models for cognitive and perceptual processes, as well as the renewed interest by contemporary philosophers in the mind-body problem in the guise of the "man-machine" analogy, all seem to argue strongly that the field is once again in transition.

Consequently, a first and necessary step to reformulating the goals of cognitive theory is to clarify the distinction between the algorithmic and algoristic bases of psychological phenomena. We turn now to this problem.

Algoristic bases to algorithms. The intuitive notion of an algorithm, or computation procedure, can only be made formally rigorous when the class of mathematical functions that are mechanically computable can be distinguished from those that are not. To understand what this means consider a simple case: There is no known algorithm for trisecting an angle using just a straight-edge and compass. Since early Greek geometers first began searching for such algorithms, many hours have been squandered on this problem by holiday mathematicians. It can now be shown that this particular function—the "trisection-of-angle-by-compass-and-straight-edge" function—is not computable. The question of the existence of an algorithm for computing this particular "trisection" function could be settled because its intuitive content was precisely understood by everyone.

It is generally agreed that the meaning of the assertion, "X is a function computable by an algorithm" or less redundantly "X is a computable function" was exhausted when definition of the total class of such functions was provided by Church (1936) and Kleene (1936) in 1935 (Kleene, 1967). Church proposed that the intuitive class of all computable functions was identical to a well-defined class of functions he and Kleene independently showed to be computable by a system of simple logical rules termed the "lambda calculus."

A little later A. M. Turing independently demonstrated that a very simple

hypothetical computing machine consisting only of symbols on a linear tape, a reading and writing head, and an alphabet, could be "programmed" to compute any function computable in the manner of the lambda-calculus. This hypothetical device became known as the universal Turing machine since Turing made the same claim for it as Church did for his calculus; namely, that what is meant intuitively by a function being "computable by an algorithm" was identical to it being computable by this abstract machine. This conjecture, often called the Church-Turing thesis, is not capable of mathematical proof in the strict sense because it asserts the equivalence between the concept of a formally precise object, a Turing machine, and one that is merely intuitive, an algorithm.

However, it should be pointed out that the thesis does seem to be true on grounds of inductive generalization (although it has its detractors, e.g., Kalamar, 1957) since each time a universal device, say an abstract neural-net, Post or Wang-machine has been designed that appears *prima facie* different from a Turing machine, it turns out to be formally equivalent in that it can execute no new algorithms.

Does this mean then that the universal Turing machine and the class of abstract automata equivalent to it provide a rigorous instantiation of what we intuitively mean by an algorist? The answer is "No," because such abstract automata do not satisfy the natural constraints that must be satisfied by any real agent. For instance, Turing machines are assumed to possess infinite memory capacity, to be perfectly reliable, and to compute as fast as you please—all ideal properties not representative of any organism or actual machine. Furthermore, if we take into account the constraints placed on the working of organisms by the laws of physics and biology, many *cost* variables other than *amount of storage space* and *reliability* become relevant to the concept of an actual agent of computation. For example, due to the very general design required to insure its universality, the Turing machine would be a very inefficient computing machine if actually constructed. A special-purpose machine constructed to compute just a small class of functions would be far superior in efficiency to the multipurpose universal Turing machine, as regard the cost of the computation.

As algorists, such special purpose devices are quite different from Turing machines since the *ideal* limits on the "cost" of computation must be determined by variables that play no essential role in our intuitive concept of algorithm—variables such as the *material composition* of components (e.g., How much speed or heat can they withstand?), their *geometric configuration* (e.g., Are they large or small? Numerous? Do they maximize adjacency relationships?) as well as the resultant effects of the environmental conditions surrounding the machine (e.g., temperature, gravity, atmosphere, humidity, etc.). There can be no doubt that determination of appropriate cost variables involve extralogical principles that go beyond the logical variables required to describe the algorithms the machine may be capable of executing. It goes without saying that humans and other organisms are subject to a wide range of such cost variables including psychological cost variables, the capricious functioning of faulty components, and even design errors due to the logical slack in evolutionary principles (Wistar Institute, 1967).

A final reason why a completely rigorous understanding of the concept of

algorithm gets us no closer to our goal of an explicit theory of algorithms is the observation that *the notion of algorithm is in fact presupposed in the very concept of algorithm*, as the ancients well knew. A little discussed condition that must be satisfied by any procedure that can be deemed "effectively" rigorous is that it be stated in a "natural" way—that is, it must be described in terms of "self-evident" primitive concepts and stated in terms of an elementary notation system that is truly easy to understand by whatever algorithm is selected to execute the algorithm. By an algorithm we mean a procedure whose first step as well as each subsequent step can be unequivocally understood by the user. A procedure that is not self-evident in this sense must either be potentially analyzable into steps that are, or it is not an algorithm. Similarly, if its instructions are couched in a language too vague or too complicated to be clearly understood by the potential user, then it fails to qualify as an algorithm since the rationalistic criterion for it being a natural or clear intuition is also violated.

But clearly this is a *cognitive* assumption about the ability of the human algorithmist to understand some things better than others. In this sense, the concept of algorithm can only be unambiguously understood when that of the algorithmist is. It is quite perplexing to those who wish to keep the foundations of mathematics logically pure to admit that underneath it all lies a truly cognitive assumption; but here is the concept of the algorithmist, like the legendary Atlas, bearing the world of mathematics on its shoulders. Moreover, to Bertrand Russell's famous witticism regarding the nature of mathematics that "mathematics may be defined as the subject in which we never know what we are talking about, nor whether what we are saying is true. . ." we must add, "nor to whom we speak."

Therefore, if the primary evidence for an algorithmic level of analysis is the *a priori* need for an agent which naturally "compiles" certain algorithms more efficiently than others, then the intuitive concept needs to be rigorized in the following way: First, the cost parameters must be shown to follow intrinsically from the logic by which the system is designed, and secondly, the cost parameters must be shown to be compatible with the *intentionality* requirements of the system (i.e., the cost of computing a behavioral or mental function must not be so dear as to preclude successful adaptation to the exigencies of its environment, nor so lenient as to allow unrealistic achievements).

Thus, the cost of appropriate functioning for a *natural* system derives from both an *intrinsic* and an *extrinsic* source, while that of an *abstract* system derives merely from an intrinsic one. Regarding intrinsic cost constraints, neither type of system can achieve a level of computational efficiency which surpasses that permitted by the most economical algorithms that can be shown to be logically possible. However, the natural system if further restricted to levels of functional efficiency dictated by the mechanical efficiency of its material components, which must labor under the space-time restrictions imposed by natural law. This leads us to expect that the class of algorithms that can be defined which are *in principle* capable of computing well defined psychological functions will be spuriously larger than the class that can actually be executed on the neurological machinery with its *real time/real space* processing limitations.

Moreover, the logically possible algorithmic bases to cognitive processes must be

shrunk still further to accommodate the particular properties of the energy distributions which carry the information required for an organism's adaptive responding. For all these reasons, as argued earlier, the algorithmist or epistemic—who can be considered to be the logical intersection of the algorithmic basis (the *how*) of cognitive processes and the informational sources (the *what*) of the environment.

Of course, Chomsky, Simon, and other theorists characterized earlier as proponents of the algorithmic approach to theory construction in psychology recognize the need for an approach that incorporates cost parameters into models of cognitive processes. In fact both Chomsky (1965) and Simon (1969) have discussed the problem. Moreover, both express optimism regarding the possibility of introducing cost parameters as adjunct adjustments to algorithmic (competence) models. At times this optimism seems tantamount to the reductionistic belief that the algorithmic basis of cognitive processes can be 'mapped' onto the algorithmic ones. So far such direct augmentation of algorithmic models by cost parameters has not proven systematically possible. However, Chomsky's admonition to theorists that a formal grammar is not to be taken as a "process" model, but to consider it neutral with respect to the processes of both speaking and comprehending, seems to be evidence that he recognizes the incommensurability that exists between cost and competence.

Two current areas of research in algebraic structure theory seem especially cognizant of the problem of determining the cost parameters of algorithms and machines which compute functions of the considerable complexity required of living systems. The first is the exciting work being pursued in the new branch of recursive function theory known as the theory of computation, especially that part called the theory of computational complexity (Blum, 1967; Hartmanis & Stearns, 1965; Minsky & Papert, 1968). A second source of relevant techniques may issue from attempts to model systems that perform biologically significant functions, such as self-reproduction and regeneration or reparation of parts (Arbib, 1969). This work is ultimately aimed at the explanation of evolution of species and the growth of individuals. The most dramatic result in this area is von Neumann's proof of the existence of a universal constructor machine which can reproduce any other machine, analogous to the ability of the universal Turing machine for computing any function. This proof is of special interest because for the first time it suggests that mathematicians, and not just physicists or biologists, must interest themselves in the idealization of structural rather than just algorithmic properties of machines. Such work can be thought of as pointing toward a geometry of machines. In order to have a machine that will reproduce itself, or another machine, there must, in addition to the program providing the 'blueprint' which guides construction, also be formally described an available stockpile of compatible parts and a suitable space in which these parts can be retrieved and assembled. Consequently, if the parts are to be assembled into a machine, they must be structurally compatible with the manipulative organs of the parent machine. The material composition as well as intrinsic geometry of both parts and manipulative organs must be "naturally" designed for one another. For instance, it would be impossible for a parent machine with mechanical manipulators to reproduce itself if the stockpile was liquid. Clearly, the parts must be discretizable, suitably rigid, etc., if they are to be

naturally suitable for the desired construction. Discretizability and rigidity are intuitive mathematical concepts whose idealization have played a prominent role in set theory and geometry respectively.

To interpret the intuitive notion of a "natural" medium for the instantiation of a given algorithm, we need to gather up into a coherent theory all such idealized properties of media which are required as support for the computation of various kinds of functions. Consider, for instance, the abstract properties of materials in which analogs to digital conversion can and cannot take place, or in which serial simulation of parallel processes would be impossible, etc.

The discovery of the appropriate idealizations of the structural properties of materials that form a suitable stockpile of parts and assembler organs, as well as the discovery of proofs regarding which of these properties define a *universal* stockpile, are problems whose solution would shed light on what we mean by agent-like machines. A universal stockpile of parts would require the abstract specification of all properties which all types of materials must satisfy if they are to be mutually compatible as components of a system which has a *natural* algorithmic basis, i.e., which computes functions satisfying both the natural cost parameters and intentionality of physically, biologically, and psychologically realizable systems.

Unfortunately, such proofs may not be possible so long as we assume the laws governing such materials to be abstractly equivalent to the laws of physics. The properties of algorists as psychological agents may transcend even those prescribed by current biological theory. If so, what might we expect to be the nature of such algorithic laws? In the next section we explore a possible answer to this question.

Mind: A Higher Phase of Matter

Since Thales of Miletus in the fifth century B.C. asked, "What is matter?", many diverse and remarkable answers have been given. It has been suggested that the science of the ordinary phases of matter, e.g., gas, liquid and solid, was adequately accounted for by the classical physics of the 19th century. Recently, however, a new and most predominant phase of matter was discovered—the so-called "plasma" phase—whose properties were found to transcend the laws of classical physics and to require explanation in terms of the new physics of quantum mechanics.

Since Driesch and other vitalists debated the biological mechanists, there have been perennial claims that living systems consist of a phase of matter not adequately covered by the laws of physics, classical or otherwise. Several contemporary scientists have argued that in living systems we encounter matter in the 'protoplasmic' phase that is governed by 'biotonic' rather than physical laws (Elsasser, 1958; Wigner, 1970). In support of this claim, Wigner, a Nobel laureate in physics, has provided a proof that quantum mechanical laws do not provide a sufficient account of explaining the logic of biological reproduction—a property many theorists believe criterial for distinguishing living from nonliving matter (Ashby, 1962; von Neumann, 1966). Claims by some mathematicians (Arbib, 1969; Block, 1967) to the contrary do not refute this, since as yet no one has been able to show that their models are able to satisfy the stringent "cost" requirements of reliability and real-time and real-space development set by natural evolution.

If it is indeed true that the laws for lower phases of matter discovered by classical physics can no more explain the higher phase of matter dealt with by quantum mechanics than the latter laws can explain the biotonic phase of matter, then still higher phases of matter might exist that are beyond the province of even biotonic law. Such speculation that intemperately proliferates levels of analysis understandably is apt to send reductionists and nonreductionists alike away to sharpen Occam's razor.

There does seem to exist dramatic, if not precise, evidence for the existence of a still higher phase of matter than even the protoplasmic. We refer, of course, to the "psychological" phase and to "consciousness" as its crucial property. Many plants and lower organisms presumably reproduce and, thus, are governed by "biotonic" law, but do so with no wit of consciousness, just as many physical systems exist which possess plasmic properties but no biotonic ones.

Whatever might be meant by the intuitive concept of alorist, we suggest that an essential part of its theory must account for that aspect of man, organism, or machine that both expends the cost of executing algorithms as well as constrains the selection of processing goals. Furthermore, the concept of the alorist seems to play as fundamental a role in physics as we have argued it does in mathematics. Many of the greatest theorists in physics agree that it does.

In 1912 a group of scientists met in Berlin to co-author a manifesto to "oppose all metaphysical undertakings" and to champion the view that a better philosophy "should grow in a natural manner out of the facts and problems of natural science" (Clark, 1971). The manifesto continued:

In the theory of relativity [physics] touches the most searching question thus far of epistemology: is absolute or is only relative knowledge attainable? Indeed: Is absolute knowledge conceivable? It comes here directly upon the question of man's place in the world, the question of the connection of thought with the brain. What is thought? What are concepts? What are laws? In psychological problems, physics and biology come together. [p. 197].

Among the three dozen signatures endorsing this document are those of Mach, Einstein, and Sigmund Freud.

Just a few years ago Wigner (1970) cautiously reiterated this view that psychology has a key role to play in the unification of science:

One is less inclined to optimism if one considers the question of whether the physical sciences will remain separate and distinct from the biological sciences and, in particular, the sciences of the mind. There are many signs which portend that a more profound understanding of the phenomena of observation and cognition, together with an appreciation of the limits of our ability to understand, is not too distant a future step. At any rate, it should be the next decisive step toward a more integrated understanding of the world... That a higher integration of science is needed is perhaps best demonstrated by the observation that the basic entities of intuitionistic mathematics are physical objects (e.g., models), that the basic concept in the epistemological structure of physics is the concept of observation, and that psychology is not yet ready for providing concepts and idealizations of such precision as are expected in mathematics or even physics. Thus this passing of responsibility from mathematics to physics, and hence to the science of cognition ends nowhere [pp. 36-37].

We believe that cognitive psychology buttressed by the concept of the alorist,

can at last begin assuming its full responsibility in the efforts to find a unitary basis for all science.

Unfortunately, Wigner's pessimism seems warranted at the present since no precise theory of algorists yet exists. In the next section we want to discuss the metalogical principles required of all sciences which desire to rigorize the algoristic foundations we are admonished to seek by the poet Dylan Thomas when he says, "Man be my metaphor!"

II. TOWARD INVARIANCE LAWS IN PSYCHOLOGY

We have argued that the concept of algorithm as an *effective* procedure depends upon the notion of an agent, or algorist, who computes the procedure. What is meant by the intuitive term "effective" is always relative to the context of constraint within which the algorist must function. Hence some knowledge of the capabilities of the agent, or class of agents for whom the algorithm is intended, is a necessary prerequisite for providing a truly effective definition of algorithm. For this reason it follows that the true meaning of 'effective' entails the extralogical requirement that the algorithmic basis be compatible with the algoristic one.

Often the dependency of the effectivity of algorithms on algoristic factors is evidenced by the fact that the algorithm, as stated, leaves unspecified some formally inessential choices which must be left to the volition of the algorist, (e.g., the mathematician executing the algorithm or the programmer who programs the machine to do so). For instance, if several numbers are to be multiplied together, it may be left to the algorist to choose the order in which they are combined. Of course, in all cases determinate choices must be made or the algorithm will not execute properly. Exactly how such choices are made is logically arbitrary, since the numbers can be selected in various ways and combined in different orders. That many equivalent alternative choices exist follows from the fact that multiplication is both associative and commutative.

As the functions to be computed increase in complexity, the number of arbitrary choices that must be made to execute the algorithms effectively increase proportionally. Under such circumstances, the algoristic bases for some machines may prove to be more compatible with certain strategies for choosing than others. That is, the extralogical requirements for executing the algorithm may be more costly for some machines than others.

Consequently, there seems to be no universal form for algorithms which will guarantee *a priori* that they can be compiled or executed with equivalent ease by all algorists. Mathematical knowledge, like all knowledge, has no absolute form, but seems to be relative to the processing capabilities of the agents which use it.

To summarize, there are at least two ways the realization of algorithms tacitly depends on the concept of the algorist: first, in their *definition*, since whether a procedure is deemed "effective" depends on the existence of some class of agents competent to follow the procedure without creative intervention. This extralogical constraint defines a lower limit on the precise application of even the simplest algorithms. Secondly, the actual implementation of algorithms requires that the agent be provided with a strategy by which to interpolate the formally arbitrary

choices needed to render the procedure practically effective. The fact that cost requirements for doing so may exceed the performance capabilities of the algorist, when the algorithm becomes complex, suggests an upper limit on implementation exists. Although this upper limit may appear to be only practical, it is, in fact, a theoretical limit, for there exists no algorithm for predicting *a priori* exactly where in the execution of complex procedures a given algorist may have to interpolate logically arbitrary decisions. Moreover, whenever a complex algorithm is implemented by a new algorist, it must be checked *post hoc* to see if it actually executes. But who or what does the checking?

Any new algorist competent to check whether the previous algorist is able to execute the algorithm effectively, must possess knowledge of that algorist's cost factors, structural design, etc., to determine its compatibility with the bases of the algorithm. But the checking procedure must also be an algorithm. Therefore, who checks the checker to determine if it can actually execute the checking algorithm? Since cost factors, intentionality, and other properties of actual machines which limit and direct its computational abilities can not be algorithmatized, no *a priori* solution can be found to circumvent the regress.

Granting the above argument, then the nature of the algoristic bases for machines is a topic worthy of study. The concept of a knowing agent seems to be what Whitehead called "a recalcitrant fact." It simply will not dissolve under algorithmic analysis. Instead, there seem to exist algoristic limits on the rigorous definition and effective application of algorithms. In this way cognitive variables not only enter into the foundations of pure mathematics, but into their application as well. Consequently, it is a mistake to identify the concept of algorist with what machines can do, or how they can do it.

On the other hand, there must be some lawful relationship among the three bases of support for natural phenomena that renders them naturally compatible, or they could not coexist. This is as true for man-made machines as it is for organisms. The designer of computers must satisfy mutual compatibility relations among the informational, algorithmic and algoristic bases or else, (a) the information may not be compilable by the machine, (b) the programs may not execute, or (c) the machine will not satisfy the intentions of the user.

How nature achieves similar ends through evolution, with a minimum of arbitrary contrivance, is one of the most perplexing problems of science. This process can be seen as either marvelous or inevitable. For Leibniz the process was inevitable.

Compatibility and existence. To Leibniz should go the credit for recognizing the importance of developing a logic of synergistic relations among natural systems. Although his metaphysical doctrines have been ignored, we would not be wise to dismiss lightly the work of the co-inventor of the calculus and a thinker who has been acclaimed as "one of the supreme intellects of all time [Russell, 1945, p. 581]."

Leibniz held that substances can not interact. Moreover, he believed there were an infinite number of them, which he called "monads." No causal relation could hold between them. What seems to be an action of one body on another in physics is not a true causal interaction. Rather he assumed there existed a "pre-established

harmony" between the changes of state in one monad and those in another that produced the appearance of interaction. Thus, what passes for causal interaction among substances is but a parallel coordination of state changes.

Leibniz also explained perception in terms of a harmonious change in the state of the observer on the occasion of a change in state of the event. Coordination of state changes was not itself considered miraculous, but rather due to the inexorable unfolding of natural laws according to a primitive harmony or symmetry among substances. Consequently, this is the best of all possible worlds because it alone is inevitable.

Leibniz, as a consummate logician and rationalist, refused to leave anything to chance. He endorsed a "principle of sufficient reason" according to which nothing happens without a reason. What then is the reason for the preestablished harmony of natural events? To this question he gave the following argument:

Only those things may coexist which satisfy certain fundamental compatibility relations. Thus, it may be possible that some Structure A should exist, and also possible that some Structure B should exist, but not possible that both A and B should exist; that is, they may be logically incompatible. For instance, ice can exist and fire can exist but ice may not be compatible with a universal conflagration, nor fire with a frigid, energy-dead universe. Two or more things are only "compossible" when it is possible for all of them to coexist.

Leibniz made compatibility relations among logically possible structures the defining criterion for existence. He argued: "The existent may be defined as that which is compatible with more things than is anything incompatible with itself." That is to say, if A is incompatible with B, while A is compatible with C and D and E, but B is only compatible with F and G, then A, but not B, exists *by definition*. Hence, to exist is to be mutually compatible with the most things.

Notice how Leibniz's argument inverts the usual evolutionary argument for the "struggle of things to exist." The mechanism of natural selection proposed by Darwin provides a means for weeding out those species of organisms that are not sufficiently compatible with their environments to continue to exist. Leibniz, on the other hand, claims an even more central role for compatibility relations; namely, that only those things can coexist to compete which are sufficiently compatible to do so.

Darwin's theory addresses the question of the algorithmic mechanism by which nature selects from among existing species those that might continue to evolve. By contrast, Leibniz's theory addresses the deeper question of how the existence of such systems might be explained. This question is aimed at clarifying the algoristic basis of life as well as all natural phenomena. Perhaps this is the reason it has proven considerably more difficult to investigate.

As we shall see, Leibniz's insights, as curious and radical as they may seem, have been partially vindicated by contemporary science. Causal interaction among substances has been brought into question in physics by Heisenberg's principle of indeterminacy and Boltzmann's law; it has been questioned in biology by Weiss (1969), Elsasser (1958) and others; and has never really gained a strong foothold in psychology due to the intractability of the mind-body problem.

The compatibility relation that is assumed to hold among the different phases

of matter hints at a notion of preestablished harmony not unlike that proposed by Leibniz. This may seem quite far-fetched and an unwarranted metaphysical assumption; yet it is neither, as we will argue in the next section.

Laws of Nature and Invariance Laws

By a *natural law* we mean a law which explains *intraphasic* interactions, that is, the interactions among phenomena instantiated in the same phase of matter. There can then be physical laws of at least two varieties as well as biological laws and psychological laws, corresponding to the mechanistic, plasmic, biotonic, and psychological phases of matter, respectively. The nature of interactions between different phases, however, is not causal in the usual sense at all. We will explore this claim in a moment.

The coimplicative relationship among the questions concerning the bases of natural phenomena, as well as the transitive closure of the phases of matter, suggests that sufficient compatibility holds among the various phases of phenomena to permit some kind of *interphasic* interaction as well. Whereas in quantum physics one primarily studies *monophasic* phenomena (i.e., one phase of matter), psychology is by its very nature more complex (This, of course, is not a novel claim). In psychology we must ask what effect physics has on biology such that psychological experiences of a certain sort result. This means that psychology essentially deals with "*polyphasic*" phenomena. Thus, it is the laws of interphasic interactions that must be captured. In physics such laws are called *invariance laws*, since they refer to the structure of the interaction of natural laws.

The conclusion we will argue for in this section can now be stated: *The laws of psychology must be invariance laws; the explicit form of such laws is mnemonic rather than causal; the polyphasic interactions are macro-deterministic rather than micro-deterministic; the nature of scientific theorizing is assertive (adjunctive) rather than hypothetico-deductive; and, finally, symmetry group theory provides a meaningful analysis of invariance laws.*

We assert that all the above conclusions can be supported by scientific argument, rather than by metaphysical speculation. Indeed, it is a matter of record that theorists in other fields have already reached a consensus on most, if not all, of the above points. We fully expect that as the field of psychology matures and greater precision is achieved, similar conclusions will be pressed upon us.

A final point, that is a methodological corollary of those above, is the claim that the concept of *algorist* plays a central role in all sciences (as it apparently does in mathematics). Thus, as Wigner, Einstein, Mach, and Freud all seem to agree, cognitive psychology may prove to be the study of variables intrinsic to all sciences. We will now discuss each of these points in turn.

Knowledge of the Physical World

Creatio ex nihilo is the cosmological principle, entertained by some scholastic philosophers, that the world was created from nothing. The astronomer Hoyle, when asked to defend his use of this principle in astronomy, replied that it is often necessary in science to tolerate bizarre and exceptional assumptions to preserve more fundamental ones, such as the conservation laws and the consistency of

mathematics. Contemporary theoretical physicists have had to be extremely tolerant in this regard with the advent of such concepts as antimatter, time reversals, ephemeral particles, curved space, and complementarity conditions (e.g., wave and particle aspects of light).

Recently, some theorists in quantum mechanics have reached the exceptional conclusion that cognitive variables enter directly into the wave equations describing physical events. If this turns out to be a "recalcitrant fact," major repercussions will be felt throughout all sciences.

Most people have little trouble accepting the premise that physical events can somehow act upon the "mind" to produce changes in psychological states. But they balk, and feel common sense is violated, by the inverse claim that mental states may somehow directly affect physical states, (psychosomatic effects, perhaps, being a "gray" case).

As bizarre as the inverse assumption may seem, there now seems to be evidence for it.

Postulates of scientific knowledge: What are the general conditions that make knowledge of physical events possible? First, if the world consisted of an unstructured chaos then no knowledge would be possible. Even if the world consisted of a plurality of uncorrelated events, the exigencies of life (assuming it were possible) would be totally arbitrary, thus precluding adaptation by any living creature. For knowledge to be possible, then, we must assume uniformity of structure at some level of analysis across all classes of events, and assume that such structure is preserved to some significant extent by the knowledge-gathering processes of organisms.

In addition to structural uniformity across events, there must exist natural laws which can be learned so that similar responses are adaptive across families of event classes. However, the assumption that natural laws exist is not alone sufficient to guarantee continued adaptation by the species. We must also assume that the natural laws governing each phase of matter remain globally invariant, i.e., do not change arbitrarily with the passage of time or change of locale.

Consider what would happen if the laws of physics were variant from moment to moment, or from place to place. Objects would have no relatively permanent shape, hence food and mates would be unrecognizable and dangerous situations unavoidable. If the laws of biology were not invariant, then mating would not guarantee the survival of the species, vital functions might cease to support life, and information-processing capabilities of the sensory systems might not permit interpretable perceptions. The lack of invariance of psychological laws might preclude learning, memory would be unreliable, and problem solving impossible. Moreover, adaptive responses emitted by an organism would be accidental.

It is not even enough to assume events exhibit higher order structural uniformities, or that monophasic natural laws are relatively unchangeable with respect to each other. A final assumption is required, if knowledge of the world is to be possible, and this assumption brings us to the highest abstraction in all of science. We must somehow guarantee the compatibility of the various sets of natural laws; they must not only be invariant within their particular phase, but must exhibit conjoint invariance across phase boundaries. This highest order of

structure among phases constitutes an invariance law or symmetry postulate.

Thus, for knowledge of the physical world to be a possible achievement of knowing-agents, there must be four levels of structure to experience: (a) events must be structured; (b) families of events must be lawful; (c) natural laws must be globally invariant; and (d) invariance laws must hold over phases of matter.

Wigner (1970) calls the progression from events to natural laws, and from natural laws to invariance laws, "the hierarchy of our knowledge of the world around us." Although the progression seems to provide an accurate account of the foundations of scientific epistemology, it does not actually seem to be hierarchical.

In a hierarchy, categories at the same nodal level do not interact. However, there is ample evidence that the various phases of matter, all of which have the same level in the classificatory schema, do in fact interact—although not in a causal manner. If this is the case, then either the assumption that the phases of matter are at the same level in the hierarchy is false, or else scientific knowledge of the world is not strictly ordered in a hierarchical fashion.

*Types of invariance laws.*⁴ Physics distinguishes two types of invariance laws: the classical laws, which had their most precise formulation in the Special Theory of Relativity, and the newer type (not so well understood) that the General Theory of Relativity provides. Theoretical physicists have now shown that laws of nature must be derived from invariance laws rather than vice-versa.

In fact, the structure among the natural laws, which is what we mean by invariance laws, has proven so symmetrical that in some cases new laws of nature have been inferred from the presumption that they were needed to complete the symmetry of that structure. The existence of antimatter, the symmetry of chemical properties as captured by Mendeleev in the periodic table, chemical steroids, the law of parity, and other discoveries have been progeny of this principle. What more dramatic testimony could Leibniz have wanted in support of his postulate of preestablished harmony than this evidence of a higher order symmetry in nature?

As Wigner points out, and Einstein heartily emphasized, neither type of invariance law is an *a priori* category of pure reason, nor are they speculative results of metaphysical theory. Rather, both are products of careful observation in science and a conservative evaluation of experimental data. There are cases where invariance laws have been found wanting in these regards and were subsequently abandoned.

Fourier's principle of similitude is an example. It was abandoned, not because it lacked theoretical plausibility, but because it was inconsistent with empirical results. The principle claimed that the absolute magnitude of objects was irrelevant with respect to their behavior on the proper scale. The discovery that atoms possessed elementary charges, and that light was the limiting velocity in the universe, were serious anomalies that led to the principle being discarded.

The first type of invariance laws were based on geometric symmetries that hold over the space-time continuum. These are analogous to Klein's symmetry group

⁴Wigner (1970) is the main source for much in this section. We refer the reader to this collection, especially pp. 3-51, 153-200, for an authoritative discussion of many of the topics overviewed here.

representations for various types of geometric spaces (more will be made of this analogy in Section III). These geometric invariance laws are formulated in terms of structural symmetries among events themselves, such as objects remaining rigid when displaced (i.e., preserving their shape), or the so-called "constant radius of curvature" which holds throughout space.

The second type, the newer invariance laws, are dynamic in the sense that they apply to laws of nature, rather than events. These provide a formulation of types of interactions which can exist among events. In doing so dynamical laws, however, do not apply to events or correlations among events, but to types of interactions. Mathematically, the laws of geometric symmetry can be characterized by a single group while the dynamical laws of interaction symmetries can only be characterized by a different group for each type of interaction. Currently, we have no knowledge of the exact relationship that holds among the interaction groups or how they relate to the geometric symmetry group.

It is exactly these differences between phenomena governed by geometric laws of invariance and dynamical laws of invariance which specify the differences between phenomena instantiated in the first "mechanistic" phase of matter and the second "plasmic" phase of matter, respectively. Thus, the necessity of distinguishing the first two phases of matter follows naturally from the fact that distinct types of laws exist which are currently irreducible to a common law. The argument, however, for the necessity of postulating the third or "biotonic" phase of matter is more difficult to make than that for the first two phases of matter.

What must be shown is that the laws governing physical phenomena, instantiated in either of the first two phases of matter, are insufficient for explaining biological phenomena. In other words, it must be shown that biological laws can not even in principle be reduced to physical laws. Three major arguments have been offered:

1. Evidence is provided to demonstrate that the principles of biological evolution and reproduction cannot be adequately explained by physical law. Theorists of this persuasion point out that life is characterized by a change from a more homogeneous form of matter to a more heterogeneous form. Thus, this increase in structured complexities, or "negentropic" tendency in nature, violate the conservation laws. This means that neither the mechanistic laws of classical physics nor the quantum law are able to predict biological events (e.g. life) because, being negentropic, their probability is essentially *nil* (Elsasser, 1958; Quastler, 1953).

2. A second viewpoint respects the claim that physical laws (especially those of quantum mechanics) apply everywhere in nature and are sufficient to predict all significant events. Their failure, however, to explain the origin of life, biological evolution, or reproduction is due to the practical inability of any agent (e.g., his mortality) to execute the astronomically complex computations required of such complicated events (Shaw, 1971; von Neumann, 1966; Wigner, 1970).

3. Still another argument, and one that seems most cogent to us, argues from the circularity of the logical interaction of the three bases of reality support for any phenomenon—physical, biological or psychological. The argument goes as follows:

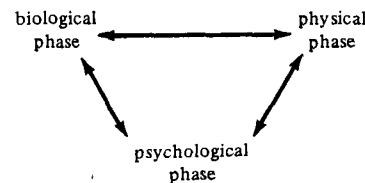
It is generally recognized that the quantum mechanical theory of energy propagation (i.e., energetic events) provides a necessary, if not sufficient,

description of the informational bases of all knowable phenomena. Furthermore, one can argue that, in principle, the laws of biology provide a necessary description of the 'machinery' for modulating the environmental information. In this way, biological laws provide the logical interface between physical laws and psychological laws.

As argued earlier, most theorists agree that psychosomatic interactions occur, so that mental states (e.g., fear, worry, excitement, etc.) may produce changes in physiological states (e.g., perspiration, blood pressure, pilo-erection, ulceration, heart palpitation, etc.). It is even more obvious that biological manipulation of physical states is possible, (e.g., the displacement of objects by hand, reduction of oxygen to carbon dioxide, vocalic sound productions, energy storage, etc.)

These mutual interactions among the phases of matter can be illustrated diagrammatically: physical phase ↔ biological phase ↔ psychological phase. But can there be a mutual interaction between the psychological and physical phases of matter, i.e., can mind affect matter? Parapsychology calls this phenomenal possibility "telekinesis." If such an interaction were in any sense possible, then the hierarchical model of scientific knowledge breaks down, and we must consider some other structure to be a more appropriate model for our epistemology than a hierarchy.

In other words, if we could show that the following schema holds for the polyphasic interactions, then the type of invariance law we need would have a different characterization than might be otherwise sought.



If this state of affairs actually is the case, then it would follow that:

- (a) reduction of biological law to physical law is logically impossible;
- (b) reduction of psychological law to biological law is also impossible;
- (c) there would exist a *transitive closure* to the phases of matter; and
- (d) the co-implicative relationship imputed to hold among the informational, algorithmic, and algorithic bases of phenomena, would thereby possess existential reality (i.e., be instanced by the transitive closure of the phases of matter).

Not only would the verification of these four hypotheses rule out the hierarchical ordering of natural phenomena (as claimed incidentally by Wigner and more emphatically by Simon, 1969) but it would require a new conception of the relationship of mutual interdependence of these phenomena. A term suggested by von Foerster (1962) and Shaw (1971) for the mutual interdependence of the complex interactive structures is "coalition." A *coalition* consists of polyphasic laws, a symmetry of acausal interactions among all components, and super-additive-

ty of effects (i.e., emergent properties) to which natural laws apply nonlinearly, since they must be applied across phase boundaries.

Still further important implications would follow from the validity of the above hypotheses. Claims by epiphenomenalists that life 'emerges' from 'dead' matter, or that sentient and conscious states emerge from torporic matter would be invalidated. Rather, a Leibnizean view of mutually compatible phases of matter would be more reasonable; where physical, biotonic, and psychological phenomena would be in all ways compossible actualities of equal existential rank.

The symmetry of the relations among the different phases of existence would be an invariance law of the highest degree. Such a law would constitute a unitary law common to all sciences. Einstein once explained that he had discovered Relativity theory because he had been "so firmly convinced of the harmony of the universe" (Clark, 1971, p. 343). This is but one example of the implicit faith in symmetry, as the highest law of nature, that guides most scientists as it did Leibniz.

The role of the epistemic-who in physics. Heisenberg (1958), in characterizing the current state of physics, observed: "The laws of nature which we formulate mathematically in quantum theory deal no longer with particles themselves but with our knowledge of the elementary particles." The laws of quantum mechanics can not be formulated in a fully consistent way without reference to what is consciously experienced by the observer. The indispensable role of conscious experience in formulating our ultimate conception of physical events can be illustrated in the following way:

It is assumed that given any event, all possible knowledge about the event can be given in its wave function. The exact form of such equations need not concern us. The mathematical language of quantum mechanics provides a means by which the probabilities that an event will be perceived to be in various possible states can be precisely determined within certain practical limitations (Bohm, 1951). For instance, if the event is a pulsing radiation field, its wave function will provide an estimate of the likelihood it will be experienced by an observer who looks in a certain direction, or that it will leave an impression on a photographic plate.

Although complete knowledge of the wave function does not always permit exact predictions of what will be experienced by an observer interacting with the system, in most cases it does permit one to predict later experiences with increased certainty. Thus, one may be sure that, if a flash is not experienced from one direction at a certain time, then one will surely experience the flash from another direction at some later time.

The important point is that the wave function is a way of predicting what will be experienced given that something else has been experienced. What is meant by the future behavior of a natural system or event is ultimately based on what has been directly experienced by a conscious observer in the immediate past. In this way the information based on the environment, described in terms of physical laws, consists of probability connections between subsequent perceptual impressions that it makes on the observer, if the observer *interacts* with it repeatedly. Thus, the result of an observation mathematically modifies the wave function of the perceived system. Wigner summarizes the argument this way:

The modified wave function is, furthermore, in general unpredictable before the impression gained at the interaction has entered our consciousness: it is the entering of an impression into our consciousness which alters the wave function because it modifies our appraisal of the probabilities for different impressions which we expect to receive in the future. It is at this point that consciousness enters the theory unavoidably and unalterable. If one speaks in terms of the wave function, its changes are coupled with the entering of impressions into our consciousness. If one formulates the laws of quantum mechanics in terms of probabilities of impressions, these are *ipso facto* the primary concepts with which one deals [pp. 175-176].

Hence, at this point the theoretical physicist is operating *de facto* as a cognitive theorist.

The natural question to raise is whether or not the wave function describing an event is the same when a conscious observer interacts with the event as when an inanimate measuring device does. It is extremely surprising to find that, according to Wigner, the wave function describing the event is actually quite different in the two cases. One consequence of this fact is that a new postulate must be introduced into physics which says that the laws of motion of quantum mechanics become grossly nonlinear if conscious beings enter the picture.

Thus it seems proper to argue that mind affects body (in the above sense). This of course is not the dramatic interpretation usually given to the term "telekinesis" by science fiction writers, but it is a legitimate one nevertheless. This is really not so surprising a conclusion when one considers that we know of no phenomena in nature in which one component of an interaction affects another component without the second also affecting the first—although the strength of the reciprocal counter effect may be very, very small.

Recall that the mechanical effects on light are easily measured, while the measurement of the effects of light on the mechanical motions of bodies are much more difficult to make. Indeed, as Wigner points out, it is unlikely that the latter effects would have been detected at all, if theory had not first shown the necessity for their existence.

Invariably, one of the main functions of theory is to alert us to the great significance of small effects which might otherwise go unnoticed. This is amply illustrated in the fact that although Newtonian mechanics and Relativistic mechanics make essentially the same predictions with respect to gravitational attraction in weak or moderately strong fields, they differ by small but measurable amount in strong gravitational fields.

The most precise test of the difference predicted by the two theories had to do with whether or not a shift of the wavelength of light occurred as a function of gravitational attraction. The direct test of this hypothesis did not take place until nearly half a century after Einstein's original paper. Robert Oppenheimer (in Clark, 1971) wrote of the test of the proposed Einstein shift:

The most precise and, I think, by far the most beautiful example of this is a recent experiment conducted at Harvard in which light was simply allowed to fall down from the third floor to the basement of the Physics Building. One could see how much bluer it had become; one part in 10^{14} ; not very much [p. 261].

It is ironic that in science so much so often rests on such little differences between competing theories. Consequently, it is likely that definitive psychological theory will require a comparable level of precision. Thus, there is little wonder that current theoretical descriptions of psychological processes are insufficiently precise to significantly constrain the selection of neural functions that might support those processes.

There is evidence then that the phases of matter constitute a closed set, since the psychological phase may indeed interact directly with the physical phase *vis à vis* the wave function. The conclusion that the phases of matter represent a closed set of concepts is important for two reasons: First, it precludes a regress to higher and higher phases of matter, since the various phases have transitive closure, i.e., A implicates B, B implicates C, and C implicates A. Secondly, transitive closure to the phases provides additional evidence for the claim that the co-implicative relation holds among the *what*, *how* and *who* questions, and that this reflects the compatibility condition which must hold among the informational, algorithmic, and algoristic bases.

The Logic of Invariance Laws

The preceding section attempted to show that although the different phases of matter interact, they do so in a nonlinear fashion, i.e., in some higher order way than mere linear, causal interaction. By demonstrating their transitive closure, we at the same time invalidated certain logical argument forms that might otherwise be applied to model them.

Normally, one can argue that if A implies B, B implies C, then by transitivity, A implies C. But given that cognitive variables, instantiated at the psychological phase, must be assumed to be primitives for the physical phase, such transitive arguments no longer hold. von Neumann (1966) and McCulloch (1965) both suspected that such logical problems might emerge whenever biotonic considerations were introduced into mathematical formulations of natural law.

To see this, consider the following case: Let $(x > y)$ represent the relation "founded upon" in the sense of furnishing necessary conditions (Read: x is founded upon y). As argued earlier, it is reasonable to assume that physical laws provide the necessary bases for the information processed by organisms, and that biological laws provide the necessary bases for the algorithms computed by organisms in processing the information. This can be expressed as:

algoristic bases $>$ algorithmic bases $>$ informational bases,
and, thus, by transitivity:

algoristic bases $>$ information bases.

But by the argument for the co-implication of these bases we also have:

informational bases $>$ algoristic bases.

Transitive closure introduces a symmetry of relationships among the bases of phenomena which destroys the asymmetry required for a valid interpretation of the dominance relation. Moreover, we cannot assume that the above propositions have the logical form of the hypothetical conditional, since the co-implication relation among the bases would have to be represented as a set of biconditionals, i.e., "if x

then y" and "if y then x." The biconditional form is inappropriate since it implies that the bases are formally equivalent. This is obviously an invalid interpretation of the compatibility relation among the bases of phenomena, since their natural laws are monophasic and nonlinear across phase boundaries. Laws for each phase are quite different and not generalizable given our current tenets of science. Thus, the hypothetical propositional schema is an invalid model for the invariance laws which are needed to bridge the gap between phases.

Is there another propositional form that might provide a more adequate model? Before attacking this question, let us explore in more detail the properties that such a propositional interpretation of invariance laws must possess.

One of the chief properties of invariance laws is that they must be *polyphasic*. They must capture the compatibility relations holding among the different phases which explain the nature of the symmetry of natural laws across phase boundaries. In this sense invariance laws can be said to describe all possible *interphasic* interactions. It follows that a causal interpretation is not applicable to invariance laws since all natural laws, (including causal ones if they exist), become nonlinear when so applied.

Intuitively, we recognize this when we question the simplistic causal argument that there exists an unbroken causal sequence from events in the environment to events in the central nervous system to changes in psychological states. Cornsweet (1970) cogently illustrates why such functions, what he calls "modular transfer functions," often become nonlinear. The nonlinearity of such modular transfer functions has long been recognized in psychophysics in the concepts of thresholds and adaptation levels. We deal next with the polyphasic property of psychological laws.

Invariance Laws as Mnemonic Relations

Bertrand Russell in *The Analysis of Mind* (1921) suggested that there is an important difference between the traditional mechanistic interpretation of causal laws and those required for psychology. Although he did not explicitly recognize the polyphasic property of psychological laws (i.e., that they must be invariance laws), he provided a logical analysis which fits nicely with this interpretation.

Traditionally, a causal law in physics is one which expresses the results of micro-deterministic (i.e., point-by-point) interactions among physically contiguous events. Two events are said to be physically contiguous, if they have neighboring values on the space-time continuum; that is, they must be spatially and temporally adjacent. Indirect causal interactions may take place between noncontiguous events if and only if they are connected by an interpolated sequence of events which interact in a direct causal manner. Such a causal sequence can be said to form a *reduction chain* that causally links the non-contiguous events by a well-defined series of linear micro-deterministic relations.

Of course, everyone is aware of Hume's criticism that contiguity alone is not sufficient to guarantee that a causal interaction takes place between events. Similarly, we have been repeatedly admonished that a mere correlation between the changes in state of the contiguous events, even when perfect, does not logically

imply that a causal relation holds. Some extralogical assumptions are needed to supplement the contiguity assumption.

Intuitively, causal relations between events must satisfy the same abstract requirements for simple and immediate conceptual dependence as do adjacent steps in an algorithm. As argued in Section I, a procedure is effective (algorithmic) when the agent, by a process of immediate rational assessment, is able to move mechanically from each antecedent step to each subsequent step without recourse to *ad hoc* inferences or guesswork.

In the physical situation, causal interactions must satisfy similar criteria to be equally effective. A sequence of interactions among a chain of contiguous events is effectively linear and mechanistically causal when, (a) the relation between each event is microdeterministic in the sense that no simpler relations can be interpolated between them, and (b) there is a direct transference of energy from one event to the next exhibited by a measurable complementary change in the state vectors of the events involved.

Unfortunately, the above situation does not provide an adequate model for explaining what transactions take place between *heterophasic* events that are not only separated in space-time, but which also involve several phases of matter. Under such circumstances single natural laws become grossly nonlinear when applied across phase boundaries. The difficulty of providing a causal interpretation for interphasic interactions is compounded by the tendency of unspecified cognitive variables to creep into the interaction. Russell termed the relation between such noncontiguous, psychologically dependent events, *mnemic*.

Mnemic relations can be shown to play a necessary role in the description of all psychological phenomena: Let A be an event which reliably evokes an experience B in some conscious organism O when presented. Consequently, we say O perceives A at time t_k whenever experience B occurs. Now let a be a distinctive element of A , such that under appropriate conditions if a is presented to O , then either experience B , or some close correlate experience B' is evoked, (i.e., where B' is in the equivalence class of B). Under such circumstances we will say that A *specifies* the event A by reintegrating the original experience of A .

The above formulation has the virtue of not favoring any particular theoretical interpretation of psychological phenomena, while, at the same time, providing a minimal description for what counts as a psychological phenomenon. Thus, it provides the following logical schema for psychological phenomena:

If $A \rightarrow B$ at t_1 and $a \rightarrow A$, then $(a \rightarrow A) \Rightarrow B$ at t_k
(or B' , as the case may be).

We can now define a *mnemic relation* as that transitive relation which holds between a and B (or B'), i.e., $a \Rightarrow B$ (or B').

Traditionally, the relations $A \rightarrow B$ has been given a mechanistic causal interpretation on the grounds that A and B are events linked by a reduction chain of biological events, while the relation $a \rightarrow A$ was said to be an associative relation and was treated in the usual behavioristic way. However, given the possibility that cognitive variables may ultimately be needed to provide precise physical description of events (e.g., A or a), the $A \rightarrow B$ relation can hardly be said to be mechanistically causal.

The claim that $A \rightarrow B$ is causal in the classical mechanical sense also ignores the fact that such relations defined over physical, biological, and psychological events are interphasic. This means that the function mapping A onto B is nonlinear and that any law explaining how this mapping occurs must be polyphasic. It follows, of course, that such explanatory laws for psychological phenomena must be invariance laws rather than natural laws (in the sense defined earlier).

Similarly, the relation $a \rightarrow A$ can not be adequately explained in terms of associative laws, since cognitive variables also enter irreducibly into the precise physical description of the events to be associated. This follows directly from the fact that a and A are contiguous physical events whose wave functions, when observed by a conscious agent, necessarily interact to produce a nonlinear mixture (Wigner, 1970). Hence relations $A \rightarrow B$ and $a \rightarrow A$ like $a \Rightarrow B$ are mnemic rather than mechanistically causal (i.e., linearly micro-deterministic). We will call such polyphasic interactions *macro-deterministic* instead of micro-deterministic for these reasons.

Hence, since psychological phenomena always involve mnemic relations, they cannot be explained by mechanistic causal law. Rather they must be explained by invariance laws, since such laws are needed to coordinate macro-deterministic interactions that take place across phase boundaries. Luckily, there already exists laws for macro-deterministic interactions in science. By studying the form of such laws we might discover exemplars for psychological laws.

Boltzmann's law. Strict micro-deterministic laws demand that the degree to which each element of a complex structure contributes to the interaction with another complex structure be precisely specifiable (in principle). We have already seen how this is not possible in quantum physics when cognitive states are involved, nor when attempts are made to apply quantum laws to biological phenomena (e.g., biological reproduction), nor to psychological phenomena which are mnemic. One begins to wonder whether there are any nontrivial interactions to which micro-deterministic laws apply.

Weiss (1969) has argued that nearly all significant biological interactions fail to yield to micro-deterministic analysis. He suggests that this is due to the fact that Boltzmann's law applies equally well to biology as to thermodynamics, celestial mechanics, statistical mechanics, information theory, and quantum mechanics (and now we might add psychology to the list).

The significance of Boltzmann's law is best explained by means of a simile: In an economy not everyone can be the most wealthy. Only a few people can be most wealthy. Either everyone has the same amount of money, and in that sense nobody is wealthy, or some people must have more money than others. Now assume that money is continuously changing hands. To apply the simile to Boltzmann's law simply interchange the words "energy" for "wealth," "particle" for "person," and "population" for "economy": In a population not every particle can have the most energy. Only a few particles can be most energetic. Either every particle has the same amount of energy, and in that sense there is entropy, or some particles must have more energy than others. Now assume that the energy states of the particles are continuously changing.

This bears on the possibility of micro-deterministic analysis as follows: When two complex populations come into proximity, any interaction that transacts energy is going to be primarily due to very few particles at any given moment. This is due to the fact that only a small percentage have sufficient energy to break away from the others. At some later moment, after the initial interactions, the previous small subset of particles will have dissipated their energy, and a new subset must take over if the interaction is to continue.

Therefore, at any given moment the probability that a particular sample of particles, small enough to be analyzed, contains all those particles responsible for the interaction is very, very small. Since it is not practically possible to track the individual particles over time and change of position (to determine their individual contribution to the overall interaction, micro-deterministic analysis is not scientifically feasible.

Weiss argues that for similar reasons, although you may have a transaction going on between large cell populations in biology (what he calls a "macro-deterministic interaction"), micro-deterministic analysis is practically impossible. This is due to the fact that the cell by cell communication lines seem to be coaxial, with considerable time-sharing of the same neural fibers and chemical messengers by a large number of noncontiguous cell assemblies. In this respect, biotonic law proves to be nonlinear also. Thus, biotonic law, like psychological law, is not reducible to physical law nor is it modelled by strictly linear causal law.

For all the above reasons, there is ample support for the claim that phenomena of different phases may interact at a macro-deterministic level but not at micro-deterministic one. The laws governing interactions among various phases of matter seem invariably to assume a mnemonic form which is manifested in their nonlinearity when applied across phase boundaries.

The question remains, however, whether there actually exists an invariance structure to the natural laws applying to the different phases of matter, so that macro-deterministic, interphasic interactions might be explained. If we could determine the propositional form of these laws, we would have taken an important step toward that goal. This is our last question before considering the applicability of invariance laws to psychology in Section III.

The adjunctive logic of invariance laws. Whatever the laws of psychology, they must explain mnemonic relations, since causal relations cannot be said to hold. Such laws must also explain the macro-determinism that apparently holds across the different components of polyphasic phenomena.

If the distinct natural laws peculiar to each phase of matter were incompatible and totally asymmetric, psychology as a science would not be possible, since it deals essentially with polyphasic phenomena. Consequently, the laws of physics would remain unelucidated because cognitive variables would not be well-defined in the wave function equations.

For such reasons, study of the logical structure of scientific theories which invoke invariance laws, or symmetry postulates, is worthwhile. For this study to be successful, however, the relationship between natural and invariance laws must be clarified. We must also consider how invariance laws can be given empirical support.

Finally, if causal interactions do not hold, then we must show how macro-deterministic ones can explain mnemonic relations.

The Stoic philosophers distinguished several kinds of logical propositions, among them the *hypothetical*, *causal*, and *adjunctive*. The hypothetical or conditional proposition take the form, "If x , then y ." This can be contrasted with the causal proposition, "Because x , then y ," and the adjunctive proposition, "Since x , then y ."

The hypothetical proposition is invalid whenever the premise is true and the conclusion is false, and valid otherwise. It can be said to be conditionally true or *correct*, if the opposite of its conclusion contradicts its premise (i.e., by *modus tollens*). However, the opposite of the conclusion (e.g., not y) is not necessarily inconsistent with the premise (e.g., x). For instance, "If this is Monday, I go to work." However, I may not go to work because it is Labor Day. This does not, of course, imply that it is not Monday. To argue so is to commit the fallacy of affirming the consequent.

Hypothetical propositions make a poor model for natural laws, since there are too many ways in which they can be invalid. They also fail to provide a necessary relationship between premises and conclusions. The reason why these limitations make the hypothetical proposition an inappropriate model for either natural laws or invariance laws becomes apparent as soon as one attempts to fit these laws to this propositional schema.

Let the initial and auxiliary conditions that define the domain of application of the natural law correspond to the premise of the proposition. The valid outcome predicted by the natural law will then correspond to the consequent of the proposition. A principle is considered to be a natural law (*a*) if the denial of its prediction or consequent is necessarily inconsistent with the premises and (*b*) when its premise can be shown to be true (i.e., when its initial and auxiliary conditions can be shown to be satisfied). This is to say, a natural law is a principle that predicts true outcomes whenever it can be shown to validly apply.

It clearly violates what we mean by a natural law to say the law validly applies but does not predict the outcome. In such a case, either we would not accept the principle in its stated form as being a valid law, or else we would deny that the conditions for its application had actually been satisfied.

This is not the case for hypothetical propositions: Where a law cannot be validly applied to any situation where its premises are not true, a hypothetical proposition is valid by definition even when its premises are false. Attempts to interpret laws as hypothetical propositions have also led to paradoxes in what we all accept to be the function of scientific theory but which cannot be formally shown to be the case.

Popper (1959) and others have shown that neither natural laws nor theories are logically verified, when stated in hypothetico-deductive form, simply because their predictions are confirmed. Affirming the consequent of a hypothetical proposition does not affirm the premise: Hence, if law x predicts outcome y , given that y is true does not imply that x is true.

Unfortunately, Popper's attempt to find an alternative way to evaluate scientific laws and theories does not work either. Popper argues that even though

we cannot verify laws or theories directly, we can evaluate them by attempting to falsify them, that is, by showing that their predictions do not hold. The falsifiability procedure is based on the valid argument schema known since antiquity as the *modus tollens*: If law x , then outcome y , but not y , therefore, not x .

It has been argued (Lakatos, 1970), however, that falsifiability is never achieved in practice, since the premises for a law or theory are such a complex of variables that it is not possible to determine which one has been falsified. This allows the theorist to choose at his discretion whether a major or a minor assumption of his theory is at fault. Given this choice, we would not reject the core of the theory or law that had been so arduously developed.

When Einstein was told that Eddington's measurement of light bending around the eclipsed sun agreed with the predictions of his theory, he replied: "But I knew that the theory is correct." When asked what if there had been no confirmation of his prediction, he candidly countered: "Then I would have been sorry for the dear Lord—the theory *is* correct [Clark, 1971, p. 369]." Theories, like laws, seem logically inaccessible. It is not, however, the case that logic makes no difference in theory evaluation (and a law is, of course, just an accepted consequence of a theory), but that logic makes *so little* difference. The fruitfulness of a theory in explaining anomalies and bringing general consistency into science are more important than either logical verification or falsification. In other words, the degree to which a new law applies symmetrically across a wide domain of natural phenomena and relates other laws is the highest criterion of its worth. It is also a realistic measure of the resistance scientists will show in abdicating it.

What other logical forms than the hypothetical proposition might be better for representing these facts about theory evaluation? Let us consider the Stoics' conception of causal propositions.

A causal proposition begins with a true premise and ends with a necessary consequence, e.g., "Because it is day, it is light." This way is more appropriate to expressing the form of laws, since the hypothetical form is still valid if its premises are assumed false. For instance consider the following hypothetical: "If it is night, it is light"; given "It is night," then it follows that "It is light"—a valid logical argument but scientifically false.

It is not possible, however, to perform such a trick on our scientific intuition with causal propositions. A causal proposition is incorrect (by definition) if it begins with a false premise or ends with a conclusion which does not follow from it. Thus, unlike hypothetical forms, the causal interpretation demands that the premise and conclusion correspond.

Although the schema for natural laws seems to be satisfied by causal propositions, the intuitive notion of causal relation cannot be effectively captured in formal statements. Moreover, as argued earlier, invariance laws which characterize the symmetry relations existing among natural laws do not seem to fit the schema for causal propositions.

The mutual compatibility of the various phases of matter, which permits some kind of macro-determinism to hold among their distinct phenomena, does not permit (nor does it require) the micro-deterministic relations necessary to the

concept of causal interaction. The adjunctive propositional form offered by the Stoics, although essentially ignored by history, seems more promising.

An adjunctive proposition begins with a true premise and ends with a necessary consequence; e.g., "Since it is day, then the sun is shining." This proposition is incorrect when it either begins with a false premise or ends with a consequence which need not follow. The adjunctive proposition professes both that the second member follows from the first and that the first member is true. It is this propositional schema which, we believe, best fits the sense of both mnemonic and macro-deterministic relations expressed by natural laws, as well as by invariance laws.

If we analyze the adjunctive proposition, "Since x then y ," in terms of truth tables, it is the case that in order for the adjunctive relation to hold both x and y must be true. The adjunctive relation is false otherwise. Now this looks suspiciously like the truth functional definition of a conjunctive relation (e.g., x and y). It differs, however, in one important way: Where conjunctive relations are commutative (i.e., " x and y " is equivalent to " y and x "), adjunctive relations are not; hence "Since x , then y " does not imply "Since y , then x ."

The adjunctive formulation also seems to capture the sense that laws of nature apply in an inexorable manner to grind out reality. This is expressed simply as the adjunctive proposition that "Since the law applies, the observed outcome *must* follow (necessarily)." If the outcome does not follow invariantly upon correct application of the law, we have grounds for falsifying the whole proposition, since in an adjunctive proposition the truth of the conclusion follows necessarily from the truth of the premises. Thus, in this special sense, the falsifiability criterion is preserved.

The verification criterion, however, does not hold for adjunctive propositions at the level of natural law. The observation (y) that some event (x) occurs as predicted by physical laws x does not verify that x is a law.

Thus, given y , it is fallacious to affirm x . Although natural laws apply to predict outcomes, no number of observed outcomes can be used logically to verify the law. This fact is expressed in the noncommutativity of the adjunctive propositional form of the natural law [i.e., $(x \supset y) \neq (y \cdot (y \rightarrow x))$].

One might even question the utility of the verification procedure, since natural laws are postulated on more general grounds than observations. The major grounds for accepting or rejecting principles as natural laws is whether or not they fit into the invariance structure of a science and clear up anomalies and relate other principles.

The accuracy with which natural laws predict effects is not so important as the degree to which they contribute to the coherence of explanations for natural phenomena. Since one might predict what one does not understand, prediction alone is an insufficient criterion of the explanatory worth of either theories, hypotheses, or laws. Scientific theories or natural laws which help simplify a field will never be abdicating solely on the grounds that they are not predictive. Indeed, they should not be, since the conditional logic of verification does not apply.

At the higher level of invariance law, however, the postulated symmetry of interaction among the phases of matter lends a special validity to the verification-

ist's argument that cannot be found at the level of events and natural laws. In a sense the successful application of laws to phenomena in one phase implies the existence of similar laws that apply to correlated phenomena in other phases.

Due to the symmetry relation that must hold among phenomena in various phases (if they are to be compatible), it follows that there must exist some degree of reciprocity between the laws of observation which hold in the psychological phase and the laws of physics. Hence, perception of an event is due to psychological laws which must have some invariant relationship with the physical laws that determine the event. This invariance law can be expressed adjunctively as $X \diamond Y$ where X stands for the laws of physics and Y for those of psychology. The invariance relation " \diamond " can be defined as: $(X \diamond Y) \equiv [(X > Y) \cdot (Y > X)]$. In words, the laws of physics adjunctively imply the laws of psychology and vice-versa. Of course, the other phases of matter can be included in a similar fashion. But let us see if adjunctive logic is sufficiently powerful to express some of the conclusions argued for earlier regarding the interpenetration of psychological variables into the description of physical events.

By the definition of natural law we have $X > x$, where x is the physical event or set of events predicted by X , as well as $Y > y$, where y is the experience of observing x . We can now prove the following: Given $X \diamond Y$, $X > x$, and $Y > y$, we derive $X \cdot (X \rightarrow Y)$ from the definition of the invariance law and $Y \cdot (Y \rightarrow y)$ from the definition of natural law. By simplification we have $X > Y$ and $Y > y$. Then by transitivity we derive $x \rightarrow y$. Since X is given, by conjunction we have $X \cdot (X \rightarrow y)$ and by the definition of the adjunctive we finally derive $X > y$. By a similar line of proof we can also derive $Y > x$.

In words, we can prove that since the laws of physics are what they are, the perceptual experience y is what it is. This is in agreement with the view that the laws of physics determine the informational bases for perception. Indeed, we would be disturbed if adjunctive logic could not express this assumption. More importantly, however, we have also demonstrated that the laws of psychology enter into the determination of physical events, that is, $Y > x$. This is a nontrivial conclusion which generalizes the earlier claim by Wigner that psychological laws determine the nature of the algoristic bases of physical events. This is exactly the conclusion one expects to be the case if the adjunctive analysis of the invariance law of physics and psychology is valid.

The final conclusion which needs to be expressed adjunctively is $x \diamond y$. This expression asserts that a mnemonic relation holds between a physical event and the perceptual experience of it. As argued earlier, a mnemonic relation is symmetrical and captures what is meant by a macro-deterministic interaction existing between a physical event and its psychological correlate. Consequently, it follows directly from the definition of the invariance relation presumed to hold whenever there is a veridical perception of a physical event, that $x > y$ and $y > x$.

The adequacy of the adjunctive analysis of macro-deterministic interactions is further supported by the fact that, given $X > x$ and $Y > y$, as well as $x > y$ and $y > x$, we can again derive transitively the earlier conclusions, $X > y$ and $Y > x$. This completes the adjunctive analysis of natural and invariance laws. Moreover, we have shown what we set out to do in this section (as outlined on p.325).

In Section III, we provide some concrete illustrations of how symmetry postulates allow us to derive specific invariance laws in psychology.

III. INVARIANCE LAWS FOR PSYCHOLOGY

Our primary goal in Section III is to provide a tentative theory of cognition based on invariance laws which relate the three bases of support. Such a theory must explain polyphasic phenomena that arise primarily from the interaction of psychological laws with physical and biological laws.

More specifically, a theory of cognition should explain the nature of information by which events are known, what is contributed individually by physical, biological and psychological factors (i.e., how the algorist interacts with the information made available by the environment), and, finally, it should describe the mechanism by which cognitive capacities are acquired. Unfortunately, such a large order is beyond our scope. We will, however, sketch the framework we feel such a theory must ultimately fill.

Cognitive Symmetry: The Fundamental Invariance Law for Psychology

A theory of cognitive functions should explain the invariant relationship which must exist between what an agent truly knows and what can be known. Ultimately, this would be nothing less than a theory of the exact algorithmic relationship that must exist between the informational bases of events and the algoristic bases of the knowing agent. It must explain how organisms select and process the information made available by environmental events to satisfy intentions necessary to the achievement of adaptive goals.

A precise theory will provide mathematical characterization of those cognitive functions which map informational structures onto decision states of the agent. Before the proper mathematical formulations of the algorithmic bases can be discovered, the cognitive principles governing the knowledge transaction must be made intuitively clear.

Eventually, due to the polyphasic nature of psychological phenomena, even the biological processes which support these cognitive functions must also be rigorously defined. We concur, however, in the popular belief that a solution to the physical and psychological problem will greatly enhance the possibility of solving the biological one, since a precise "job" description is likely to provide an important source of constraint for selecting among competing biological models.

The fundamental problem of cognitive theory. The basic problem of cognitive psychology can be expressed schematically in terms of the adjunctive analysis of invariance laws proposed earlier: Let ϕ refer to the physical states of sources of information in the environment and ψ to those cognitive states resulting from the processing of that information. The expression $(\phi \diamond \psi) = [(\phi > \psi) \cdot (\psi > \phi)]$ denotes the adjunctive relationships that must be preserved by the biopsychological processes if the organism's knowledge of its world is to be sufficiently true to be adaptive. Recall that since these are mnemonic relations, no micro-deterministic interaction can hold between the variables designated.

A theoretical explanation of the left side of the adjunctive expression, $(\phi > \psi)$,

would answer the question of how the information determined by the environment macro-deterministically interacts with the psychological states of the organism. Conversely, a theoretical explanation of the right side ($\Psi \gg \Phi$), would answer the question of how psychological processes macro-deterministically interact with informational states (e.g., events) of the environment. The expression $\Phi \diamond \Psi$ signifies that this reciprocal interaction is symmetrical, such that some invariant relations (properties) in Φ are preserved in Ψ and *vice-versa*.

Consequently, the expression $\phi \gg \psi$, (i.e., since the structure of the environment is what it is, then psychological experiences are what they are), is an elegant statement of the problem of how organisms perceive their worlds. With respect to this issue, we side with J. J. Gibson (1966), who should be given credit for emphasizing the importance of the invariance concept for perceptual theory.

Gibson argues that perception is logically a *direct* process by which invariants of energy distributions (i.e., physical information) are detected by organisms. The hypothesis that perception is direct means that no other psychological processes mediate the detection of information. Neither memory, inference, nor images play a necessary role in the pick-up of information which invariantly specifies an event, although these secondary processes may all be accompaniments or byproducts of the perceptual process. Since this is admittedly a radical hypothesis, we will return later to defend it.

The expression $\Psi \gg \Phi$, (i.e., since psychological processes are what they are, then the nature of information is what it is), is essentially an elegant statement of the problem of how psychological states interact with physical events to determine new states of information beyond those determined by physical factors alone. It addresses the issue of what informational aspects of energy distributions exist because organisms perceptually interact with their world.

Although we also view this interactive process as being direct and in no sense psychologically mediated, we feel it has received less emphasis than it deserves by Gibson and his students. This view is not, however, inconsistent with Gibson's fundamental theory. Indeed, it is implicit in his theory of the ecological foundations of perception.

To recapitulate, two main concepts must be explained by any cognitive theory: first, the nature of invariant physical information that is intrinsic to energy distributions that need only be directly modulated by the organism; and, secondly, the nature of invariant psychological information that is a direct product of the biological processes which modulate physical information.

This formulation of the problem of cognitive theory allows two different interpretations of the concept of psychological information. The most obvious interpretation is that perceptual systems differentiate physical information in the course of modulating it. For instance, it is known that the lens and macula region of the human eye are slightly yellow in color. This has the effect of filtering out those wavelengths toward the blue end of the spectrum that contribute most to chromatic aberration. Similarly, the shift from rods to ones in bright light displaces vision toward red wavelengths that cause less trouble (Wald, 1950).

Still more dramatic is the ability of the perceptual systems to track, focus, and attend to subtle aspects of physical information while ignoring others. In spite of

the ambient flux of multiply reflected light in a well-lighted environment even a single eye is able to detect the optical invariants for edges of objects, and texture, size, shape, and brightness gradients which yield the precise layout of complex environments. Different voices and other distinct sounds can be readily identified in spite of their being nested simultaneously within complex ambient noise, say as experienced in the hub-bub of traffic or a cocktail party.

The psychological enhancement of specific dimensions of physical information is a "negentropic" process, since attentional processes objectify partitions of complex energy fields that often exceed mere intensity level differences (e.g., by pick-up of phase differences). The main function of the perceptual systems seems to be not only the discovery of objective "seams" in energy distribution, but the insertion of them when it is ecologically adaptive to do so.

Thus, a second interpretation of the expression $\Psi \gg \Phi$ is this apparent ability of the cognitive system to exceed mere differentiation of given structural contours of ambient energy, as specified by what we have called physical information. Although this ability of the cognitive systems to "broadcast" structure is admittedly problematic and surely requires more precise empirical evidence than yet exists, it is nevertheless predicted by the symmetrical form of the invariance law. If such a claim can be experimentally validated, then Wigner's (1970) observation that natural laws are often conjectured on the grounds that they are needed to fulfill the symmetrical structure of invariance laws would again be supported.

The above adjunctive statement of the invariance law holding between psychology, biology, and physics, specifies the complete content of the field of cognitive psychology and demarcates it from other sciences.

Adaptation as the dynamic expression of symmetry. How might invariants of energy distributions be detected by organisms? How might adaptive states of organisms be conditioned by informational invariants? There is considerable evidence that energetic systems have a natural tendency to reorganize their states to symmetrically balance those forces tending to disturb their equilibrium. Mach (1902) observed, "In every symmetrical system every deformation that tends to destroy the symmetry is complemented by an equal and opposite deformation that tends to restore it.

One might generalize this symmetry postulate from single systems to interacting systems in the following way: What is meant by *equilibration* of one system to another is that a symmetry exists between the energy states of the two systems such that a change in the state configuration of one system invariantly induces a corresponding change in the other. We would like to generalize this symmetry postulate still further to incorporate the symmetry of states that might be induced across phase boundaries (i.e., by the invariance laws) to account for macro-deterministic interactions.

The basic postulate of cognitive theory envisioned here must accord with the adjunctive analysis of the proposed invariance law, $\phi \diamond \psi$, given above. Here is a tentative statement of the required symmetry postulate: *An organism achieves the highest degree of equilibration with its environment (i.e., has ecologically relevant knowledge of it), when there exists a persistent symmetry between its psychological states and the informational states of the environment.* This postulate is called the

principle of cognitive symmetry (for a detailed discussion of this principle see Shaw, McIntyre, & Mace, 1973).

Whenever, in the ensuing discussion, we invoke this principle, it should serve as a reminder of all conditions governing the applicability of invariance laws to psychology argued for earlier (e.g. macro-determinism, mnemonic relations, and adjunctive analysis). There are many reasons why little progress has been made toward the study of the invariance laws of psychology.

Problems of the ecological approach to psychology. There are two essential aspects to an ecological approach: First, the effects of the environment on the organism must be determined, and secondly, the effects of the organism on the environment must be determined. An ecological approach to psychology emphasizes the effects of physical information, made available by the environment, on the cognitive states of the organism, and the reciprocal effect of these states, *vis à vis* the modulatory activities, on that information. Hence an ecological approach to cognitive psychology could naturally be founded upon the principle of cognitive symmetry.

The further assumption that the perceptual mode is a direct pick-up of invariant information poses several problems. Invariant information consists of structural relations in energy distribution which remain the same although other structural relations may undergo change. That organisms can come to know their worlds by means of perceived invariants, and thereby interact adaptively with it, requires the validity of two assumptions: First, the ecologically significant properties of the environment must determine invariant physical information. This is properly a problem of physics. However, since there is at present no well demarcated macrolevel, ecological physics, the solution to this problem falls by default to the cognitive psychologist. Lack of training in fundamental techniques of physics perhaps explains to some extent the lack of interest shown by most psychologists in pursuing an ecological approach.

A second assumption that must be justified is that given the first assumption, the invariants of perceived information must be shown to logically specify the ecologically relevant environmental sources of that information. This is essentially a problem for ecologically applied mathematics, a field not yet developed. Some examples, however, of what such applied mathematical techniques may look like can be found in Gibson, Olum, and Rosenblatt (1948), where optical expansion patterns and motion perspectives are analyzed; in Purdy (1958), where a mathematical theory of planar texture gradients is given; and in Hay (1966), where rotations and translations of rigid objects are studied.

Thus what the complete application of the ecological approach to psychology requires is considerable sophistication in mathematics, physics, and psychology, a combination of interests not often found in those qualified to undertake the approach, and a combination of qualifications not often found in those interested in doing so.

There is, of course, much ground work on the problem to be laid by experimental psychologists. The study of the first half of the invariance law, $\Phi > \Psi$, must be accomplished before a serious study can begin on the second half, $\Psi > \Phi$,

which is more central to the task of cognitive psychology. In the next section we sketch some of the problems that need to be investigated.

The Discovery of Physical and Psychological Invariants

Let us assume that a quantum mechanical description can in principle be provided for all significant aspects of the environment. As argued earlier, although such a description may provide a necessary basis to information, it will not provide one sufficient to account for the psychological experience of such information. That is, it might establish the basis for $\Phi > \Psi$ but it would not adequately characterize what is meant by $\Psi > \Phi$.

It is very important, however, that as much be explained in terms of physical law as possible, to lessen the task of psychological explanation. Thus we need to assay the extent of applicability of physical law to delimit the proper domain of psychological law. A systematic survey of the various types of information invariants will aid the attainment of this goal.

Types of invariants. Four types of invariant information seem evident: *global* and *local physical* invariants, as well as *global* and *local psychological* invariants. Although these invariants should be rigorously defined, for the purposes of this chapter they can be comprehended best by illustration.

The physical invariants are objective in that they do not depend on algorithmic bases and exist in spite of the sentient properties of organisms. Global physical invariants specify properties of the environment that are *coordinate-free*, in that they are conveyed by information which is available everywhere, at all times, independently of organisms. Examples of ecologically significant environmental properties of this type are the terrestrial horizon, the direction and strength of gravitational attraction, the range of terrestrial temperatures, the texture gradient of the ground plane, the refractive index of air, the perspective information from objects distributed over the ground plane, the course of the sun that determines the day-night schedule, as well as the changing patterns of shading and shadows. Since these properties of the environment are ubiquitous and perpetual, they do not depend on the point of view of the organism.

By contrast, local physical invariants are only conveyed by information that is specified for a particular point of view of an organism. Where the above properties are globally available throughout the terrestrial environment, and constitute permanent relations between organisms and their world, local physical invariants are determined by temporary relationships an organism may have with his environment. Examples of these are fixed-point properties and kinetic invariants of flow fields determined by an organism's locomotion through his world, or by objects in the world moving relative to the organism.

Whereas global invariants specify properties by which organisms orient to their world, local invariants specify properties by which they might orient to objects in the world. While global properties are coordinate-free, and structurally defined properties, local ones are coordinate-dependent and functionally defined. This difference can best be appreciated by considering in detail ecologically significant examples of each type of physical invariant for humans.

Globally Invariant Physical Information

The orientation systems of higher animals depend upon information which specifies the terrestrial horizon, the direction of gravity, and the slant of the ground plane. Due to the perpetual and ubiquitous nature of the perceptual universals and their significance for orientation, it is likely that species evolved a selective attunement to them. The genetic pre-attunement of organisms to global invariants would render unnecessary learning to perceive them. Presumably, pre-attunement can be due to the evolution of specialized sensory organs or to the natural propensity to be functionally adept at detecting their existence.

The horizon as a global invariant. In humans, as well as most animals, the vestibular sense is operative at birth and responds efficiently to changes in body posture from the upright position. Thus, direct perception of the direction and strength of gravity provides one of the three axes required for a perceptual reference system, say the y axis. Direct perception of the horizon and the extent and slant of the ground plane would provide the other two orthogonal axes needed to complete the system, say the x axis and z axis, respectively.

Although it is not known whether there is any anatomical specialization in the visual system of humans or most animals which might account for the direct pick-up of the optical invariants specifying the horizon or the texture gradient of the ground plane, there is abundant evidence that the ability to do so emerges in great strength at very early ages (Gibson & Walk, 1960). The functional development of the spatial reference system in children has been studied extensively by Piaget and Inhelder (1956) and Piaget, Inhelder, and Szeminska (1960). The chief evidence for the functional specialization of the human perceptual system with respect to each axis of spatial orientation comes from experiments demonstrating the child's differential ability to discriminate and use them at various stages of development (Pufall & Shaw, 1973).

It is interesting to note, however, the existence of the so-called "visual stripe" in the pigeon: A set of cells on its retinae appear to be especially sensitive to horizontal lines. It is quite conceivable that this specialized structure evolved to facilitate the bird's orientation to the horizon while in flight. That this is a reasonably effective way to achieve level flight patterns is indicated by the fact that few airplanes today are built without an instrument which shows the orientation of the wings to the horizon.

The horizon also provides a globally invariant reference axis by which to judge the relative height of objects distributed over the ground plane at various distances from the stationary observer. The angular distance from the top of objects of equal height to the horizon divides the angular length of the object into an invariant proportion. By simple trigonometric calculations it is possible to show that this invariant relationship provides a precise perceptual yardstick by which to judge directly the relative sizes of objects, even those with which the observer has no previous experience. In other words, neither familiarity, memory, nor intellectual inference is required to mediate the direct perception of relative size. This is a fact well-known to artists (see for instance the discussion of perspective by Paul Klee, 1953).

Although the mathematical formulation of other global invariants may not be so obvious, there is every reason to believe equal rigor is obtainable. Consider another less obvious global invariant of physical information: We see the shapes of stationary objects (e.g., trees and rocks) and protuberants and concavities of the ground plane (e.g., hills and dales) by means of light contrasts. If the light reflected from them were homogeneous (i.e., a Ganzfeld), then the layout of the environment would be optically unspecified.

Light contrasts originate because sunlight is differentially reflected by surfaces as a function of their orientation, as governed by the law of reflection and the nature of their material composition (i.e., their matte or specular quality). In spite of the variability of sunlight due to change in weather conditions, the same light contrast relations are defined over the relatively permanent surfaces of the environment.

One global optical invariant which exists is the relationship between highlights (regions of greatest reflectance) and accents (regions of deepest shading). For instance, if a coin is illuminated, the crescent shaped highlights on the side nearest the sun will always be balanced by an inverted crescent shaped accent. Thus, it will always be the case that the shape of regular objects will be optically specified by an invariant relationship between highlights and accents — what mathematicians call an enantiomorphic symmetry (i.e., a bilateral symmetry between structures of opposite contrast value).

If this enantiomorphic symmetry is truly a global physical invariant (as argued earlier with respect to the horizon), the principle of cognitive symmetry predicts that the visual system might be genetically pre-attuned to directly perceive it. There is evidence that this is so.

A picture of an object drawn by using only accents usually appears to be more than a mere jumble of disconnected black lines. Instead, the observer often sees the complete object. More importantly, he will report seeing highlights symmetrically opposed to the accents, that complete the contour, even though they are not actually in the drawing. Highly embossed lettering on a white matte surface, strongly illuminated from one side in such a way that only accents are visible, will nevertheless be easily readable. Many introductory psychological textbooks use illustrations of this type to demonstrate how familiar figures might be intellectually completed. However, if the above hypothesis is correct, direct perception of the contrast symmetry due to genetic pre-attunement of the visual system is a more viable explanation than the mediation hypothesis.

Our summary hypothesis can be stated adjunctively as follows: *Since global physical invariants are invariant over the terrestrial portion of the space-time continuum, the orientation system of organisms must be genetically pre-attuned to directly perceive them.* We now turn to a corollary of the principle of cognitive symmetry to explain the role of experience.

Locally Invariant Physical Information

If direct perception of globally invariant physical information is due to the evolutionary attunement of the biological systems supporting cognitive processes, then, by contrast, direct perception of local invariants is due to attunement of the

modulatory states of those biological systems by the experience of the organism with its world. According to the principle of cognitive symmetry this attunement arises from the symmetrical rearrangement of states of the biological system with respect to the invariant structure of the events perceived.

Presumably, the attunement is accomplished through what Gibson has called the "education of attention"; that is, organisms learn to differentiate physical information such that the invariants specifying various classes of events are preserved. Such a process can be thought of as an efferent analogy to the afferent synthesis of neuro-motor synergisms, say as required to account for the smooth intergration of complex motor acts involved in learning to speak, to play the piano, to dance, or drive a car (Lenneberg, 1967). (For a detailed development of this view applied to cognition see Jenkins, Jiménez-Pabón, Shaw, and Sefer, 1974.)

Local invariants for motions and movements. Examples of local physical invariants are plentiful. A simple one explains how organisms coordinate their actions with respect to objects in the environment. The principle of cognitive symmetry requires that an organism's attunement to these less permanent aspects of its environment be achieved through modulatory activities. The recognition of an event (up to determination of class equivalence) is achieved by the induction of a unique modulatory state configuration which is invariant with the informational invariants determined by the event and which specify the event. To be explained, of course, is the process by which the symmetrical modulatory state configuration is induced by the informational invariants of the event. However, let us first consider some detailed examples of local invariants of physical information.

A textured object that looms toward a stationary observer determines optical information specifying both its shape and its approach velocity (i.e., speed and direction). If the object is on a collision course with the observer, a symmetrically expanding radial flow field will be kinetically defined over its texture. The shape of the object will be specified projectively by the invariant termini of the lines of flow (e.g., its contour). The speed of approach will be specified by the rate of texture flow along these radial lines, as well as by the apparent increase in projected perimeter size. The direction of travel of the object will be specified by the relatively invariant center of the flow pattern defined by the intersection of the radial flow lines. This center of the flow field is mathematically termed a *fixed-point property*. So long as this property exists in the flow field, all velocities of texture units along the radial lines of flow will be equal, and the shape of the perimeter contour will only alter ever so slightly (i.e., since on-line projections of three-dimensional objects at different distances are not strictly speaking simply similarity transformations or magnifications).

On the other hand, if the center of the expanding radial flow pattern is not a fixed-point property, then the velocities of individual lines of texture flow will not be equal. The asymmetrical pattern of flow defines the direction of travel of the object relative to the point of observation. Thus, if the center of the flow translates laterally, say to the observer's right, then the object will pass by the observer on his right. On the other hand, if it has an upward translation, this specifies that the trajectory of the object will pass overhead.

The important point to note is that fixed-point properties of flow fields are, mathematically speaking, the simplest possible invariants. But they only exist for observers situated at the proper relative point of observation (i.e., on a line defined by the flight path of the object). For this reason they are called local perspectival invariants.

Notice that the intentions of the observer determine whether he wants to maximize or minimize the lateral velocity component of the center of the object's flow field. If the object is a missile to be dodged, he will want to move in such a way as to maximize the lateral velocity vector. On the other hand, if he is a baseball player and wishes to catch the missile, he will want to minimize the lateral flow vector with respect to the position of his glove. In such cases consideration of the intentions of the epistemic-who are logically prior to the informational or algorithmic analyses.

Note also that for a locomoting organism wishing to steer toward or away from objects in the environment, the same analysis holds with but minor modifications. Similarly, it can be shown that the perspective invariants determined in the light from a rotating object, sufficient to specify its shape for a static observer, are also available in the light picked up by an observer who orbits around a static object. Thus, there is a symmetry of information for the shape of an object between what can be picked up by a static observer and what can be picked up by a moving observer.

All relative displacements between observers and objects or locales in the environment can be composed from a combination of rotations and translations (i.e., the set of displacements is closed). The symmetry relationship between observers and parts of the environment means that, in principle, for every displacement of objects in the environment, a complementary displacement can be carried out by the observer (i.e., all displacements have inverses).

This means that the possible locomotions of an observer relative to the environment are a mathematical group, just as the possible displacements of objects are a group. These groups are symmetrical or isomorphic groups. Each is a *dual* representation of what might be called the *modulatory group*. Moreover, the possible complementary motions of objects and observers means that each group contains abstractly equivalent inverses of the operations performable in the companion group. In other words, the group of motions, like the group of movements, are concrete instances of the abstract modulatory group. What are the perceptual consequences of the abstract symmetry of these two groups?

The most subtle aspect of the group theory analysis of cognitive activities is that symmetry groups, such as these dual representations of the modulatory group, are but special applications of the principle of cognitive symmetry to a restricted domain of phenomena. In this case, the domain is the perception of local physical invariants. Furthermore, if the principle of cognitive symmetry is a necessary postulate, violation of it should have empirical consequences. Thus if the group symmetry relation between the algoristic bases and the informational basis is destroyed, successful cognition of the world should be disrupted. Evidence that this is the case exists.

When the perspective information specifying the shape of an object is modified

to violate the necessary symmetry relationship between an observer and the object, the object no longer can be recognized for what it is. One way of demonstrating this is to stroboscopically illuminate a rotating wire cube (Shaw et al., 1974). If the rotating object is strobed so that the observer glimpses nonadjacent discrete perspective projections in rapid succession, the cube is no longer recognizable, but is seen as a complex disconnected figure with more than six sides. Moreover the faces not only appear to rotate in contrary directions but appear to be elastic rather than rigid. Thus, when the structure of the modulatory group is destroyed, the cognition of shape is disrupted.

The modulatory group also includes the finer adjustments of the perceptual systems as well. In addition to gross locomotions and postural changes, modulatory activities of the visual system include vergence and accommodative adjustments of the conjugate binocular system. A transformation of optical information so that local invariants are either destroyed or greatly modified distorts veridical perception. For instance, by means of a "pseudo-scope" (an arrangement of mirrors) the eyes can be made to converge on distant objects as if they were much closer. In such cases the size of the objects appears greatly diminished and their distance much closer.

A reciprocal effect can be achieved by presenting the visual system with erroneous perspective information, as in the Ames' distorted room demonstration. Here a room was constructed whose floor, walls, and windows do not have perpendicular corners but which appear so when viewed appropriately. An object placed in one corner of the room appears much smaller than when placed in another corner.

Additional evidence in support of the necessity of satisfying the symmetry between the attuned modulatory states of the perceptual system and the local invariants of the environment is provided by stabilized retinal image demonstrations (Pritchard, 1961). When, by the use of a corneal attached lens device, an image is made to remain stationary relative to the movements of the eye, it fractionates and finally disappears altogether. This demonstrates that the destruction of the inverse symmetry relationship between the motions of objects in the world relative to the movements of the observer destroys the structure of the modulatory group, hence preventing cognition.

Another source of evidence for this claim derives from experiments in which observers wear spectacles that optically transform the world (Kohler, 1964). Spectacles can be worn which color the world, turn it upside down, reverse it, or shift the location of objects systematically to the right or left. In all these cases, the visual system recalibrates itself with the orientation system so that after several days the visual world no longer appears transformed. Dramatic evidence that the state configuration of the modulatory system has been systematically transformed to compensate for the transformation of optical information is provided when at last the spectacles are removed. Incredibly, the world now viewed without the spectacles looks transformed opposite to what was seen while wearing the spectacles.

What better evidence could one ask for in support of the claim that the modulatory activities must form a group? The inverse relationships induced on the

visual system by a systematic transformation of optical information is also consistent with the claim that the modulatory group is instantiated in the two symmetrical dual groups defined over the informational bases of the world and the algoistic bases of the organism. Mathematically, it is well-known that a systematic transformation of a group does not necessarily destroy its structure.

Still stronger evidence of the validity of the symmetry group analysis of cognition, as required by the principle of cognitive symmetry, would be to show that certain arbitrary systematic transformation of information can not be adapted to. Again such evidence exists.

If prismatic spectacles are worn where each eye looks through a wedge prism whose base is oriented in a temporal direction, what is called the *color-stereo* effect is experienced (Kohler, 1962). Due to the differential refraction of light of different wavelengths, the observer with these spectacles will see colored objects proportionally displaced. Thus, a woman wearing a red blouse might be seen walking across the street with her red blouse following a few feet behind her, mysteriously moving in perfect synchrony with her movements. Even after wearing such spectacles for almost two months, Kohler reports that the visual system failed to adapt to the color-stereo effect.

Similarly, deep sea divers who spend many hours every day under water never succeed in adapting to the effects of the refractive power of water. Since the refractive index of water is greater than that of air, all objects are made to look larger and, therefore, appear closer. Although experienced divers learn to intellectually correct for this in judging distances, their visual systems do not: The magnification is always seen.

Both the color-stereo and the water magnification effects provide evidence that not all systematic transformations of physical information can be offset by recalibrations of the modulatory system. In other words, the operation by which these optical devices transform information is not a member of the modulatory group and, hence, has no inverse. Why might this be so?

The color-stereo effect presents the visual system with a random assortment of displaced images. An observer looking around his world is just as likely to encounter objects of one color as another. As light of various wavelengths is picked-up, the amount of displacement of the images is nearly unpredictable. With no invariance in the transformations presented there is no possibility of a systematic inverse transformation of the modulatory system that might compensate for the arbitrary change in the state of the world.

As Kohler (1962) points out, there is good reason to believe that the visual system has been genetically pre-attuned to utilize a natural color-stereo effect caused by design of the eyes. It has perplexed biologists that the fovea of most animals, including man, lies to one side of the optical axis of the lens system. This misalignment, combined with the eye's natural degree of chromatic aberration, may produce prismatic effects that give rise to a weak color-stereo effect. The functional utility of this effect, however, is not yet understood.

The failure of underwater observers to adapt to the refractive index of water is also probably due to genetic pre-attunement. After millions of years of living in an air filled environment, it seems reasonable to conclude that the evolutionary design of

the visual system has been specifically attuned to the refractive index of air—a global physical invariant like the refractability of light according to wavelength. The visual system can no more adapt to these global physical invariants than it can to the horizon, day-night schedule, terrestrial thermal conditions, texture gradient of the ground plane, or gravity. Therefore, at all levels of analysis, from gross to fine modulatory activities, the principle of cognitive symmetry must be satisfied if normal cognition is to be possible.

The Principle of Perceptual Transitions

A very important hypothesis emerges from close examination of the question regarding how the modulatory states of organisms become attuned to local physical invariants through experience. The principle of cognitive symmetry states the necessary conditions for such attunement but does not provide a sufficient mechanism for explaining how the attunement is accomplished. The basis for the principle by which local physical invariants condition symmetrical modulatory states is Mach's (1902) conjecture that "symmetry carries over into equilibria."

One of the most difficult problems in psychology, and one which approaches the difficulty of the so-called "many-body" problem in physics, is explaining the perceptual analysis of complex events. Why do people see complex events in specific ways when often the information determined by them is ambiguous, and thus specifies many different events? What minimal information is needed to specify adequately the nature of an event? When is such information redundant?

An answer to each question hinges on whether we can predict the structural components of complex events to which observers are most likely to attend. In other words, we must be able to explain how invariant structures in the event induce a symmetry, or equilibrium, of modulatory states in the organism.

Let us say that components with the greatest probability of being attended to (i.e., highest cognitive saliency) possess the greatest *attensity* value. If we had some principle by which we could determine the relative attensity value for each structural component of an event, then we might predict which structures of the event are most likely to be seen first. The structural component with the greatest attensity value that is seen first would be a likely candidate for the anchor or reference point around which less attentive structures might be organized.

Such a principle would allow us to predict the perceptual transitions among modulatory states as a function of the structural development of the event over time. Furthermore, such a principle could be interpreted as providing an explanation of the modulatory "program" by which the perceptual system becomes attuned to invariant information through experience. Discovery of such a principle of perceptual transitions would provide another instance of the successful application of the principle of cognitive symmetry—this time to the domain of "event" perception.

The world is full of various classes of events—events where the symmetry period is degenerative, as in the case of a bouncing rubber ball; events whose symmetry period is repetitive, as when a wheel rolls or a person walks; reversible events, as in opening and closing a door, or the swinging pendulum of a clock; irreversible events, as in burning a forest or bursting a balloon; and "slow" events, such as evolution,

growth, and aging. The symmetry period of an event is specified by all the nonredundant information presented within some region of space-time values. All events have such periods, even non-repeatable and irreversible ones, because the symmetry description is abstract (i.e., a class description) rather than specific to a particular event.

In many cases the symmetry period must be discovered for events which are not spatially or temporally contiguous, and which are, therefore, mnemically rather than causally related. For instance, although we can not see the same egg broken twice in succession, we learn to recognize the symmetry period of the event class by seeing different eggs broken. Similarly, we learn the symmetry period of the human aging process by synchronic examples (i.e., by seeing people of different age levels). In order for such concepts to be learned, we must be able to perceive the invariants of the transformations by which the changes take place.

Local invariants can be distinguished by their dimensionality. The simplest local invariant is a fixed-point property. Since such invariants are defined over points they have a dimensionality of zero. An invariant relation defined over two relatively fixed points determines a fixed line and thus is one dimensional; an invariant area or plane surface is determined by three points and is said to be three dimensional, and so on. The *degree* of an invariant property is simply one more than its number of dimensions (e.g., a fixed-point property is a first-degree invariant).

We are now in a position to formulate the *principle of perceptual transitions* needed to explain event perception: *Global physical invariants have greater attensity than local ones. The attensity of local invariants is inversely proportional to their degree. Thus, the perceptual organization of an event proceeds from globally invariant properties to locally invariant properties according to their degree. In the case of invariant structures of the same degree, the structure most consistent with lower degree invariants will have the greatest attensity.*

Let us now apply the principle of perceptual transitions to predict a well-known perceptual phenomenon. If you place a light on a wheel and roll it laterally across the floor of a darkroom, observers report seeing the light trace out a continuous, scallop-shaped curve called a *cycloid*. A cycloid is an open curve symmetrical around a perpendicular bisector. The termini for the cycloid traced out by the light lie on the ground plane at a distance apart equal to the circumference of the wheel. The height of the cycloid is the diameter of the wheel.

A dramatically different phenomenon, however, is reported when a second light is placed at the center of the wheel. The rolling wheel is now seen to trace out a complex event consisting of two distinct components: The rim light is seen to be orbiting around the hub light while this rotary system translates rectilinearly across the floor.

The puzzle to be explained is why the mere addition of one light can make such a dramatic change in what is seen. Geometrically, the rim light still traces out the same curve as before, but is now seen to trace out a circle when the context of the second light is added. Furthermore, a circular trace is radically different from a cycloid trace: Where a cycloid is an open curve, a circle is closed; where a cycloid has but a single axis of symmetry, a circle has an infinite number of them. Instead

of the translating orbiting system, why isn't a cycloid with a straight line trace through it seen?

In the case with a single rim light, the wheel is seen to trace out a cycloid because that curve is the lowest degree invariant optically specified. A static cycloid is a third-degree invariant, while a kinetic one is a fourth-degree invariant, since it has the added dimension of time. The direction in which the event unfolds and its orientation to the observer is defined within the framework of the observer's spatial reference system. This spatial reference system is defined over the global physical invariants. This is as expected by the principle, since local invariants are always oriented within the system of global invariants which have the greatest attensity.

Now consider how new, lower-degree local invariants are introduced when the hub light is added to the display. The hub light traces a straight line over time, a third-degree invariant. This invariant, according to the principle, has greater attensity than the cycloid, since it is of lower degree. The hub light then becomes the anchor or reference point around which the other components of the event are organized.

The next invariant of greatest attensity is the fixed radial distance between the hub light and the rim light which sweeps out a circular area. The areal invariant (e.g., the circle) is a third degree invariant. Since this curve is one degree lower than the fourth degree invariant specifying the cycloid, it is seen next. And finally, a translating rotary system is a fourth-degree invariant. But since it is consistent with more of the lower-degree invariants (e.g. the circle and the line of translation) than the fourth-degree cycloid, it is seen while the cycloid is not. Thus, a translating rotary system is the event of greater resultant attensity than a repeating cycloid. If valid, this analysis solves the Gestaltists' problem of what is meant by "good" form (Prägnanz).

We contend that the principle of perceptual transitions will apply algorithmically to predict the perceptual organization of any event, regardless of its intrinsic complexity.

It is important to point out that this principle, although algorithmic in its application by the human perceptual systems, nevertheless has an algoristic basis. The role of the epistemic who enters, as expected, in the form of a cost variable—what might be called the psychological analog to the principles of "least effort." This principle asserts that natural phenomena develop along lines of least resistance. Without the assumption of this algoristic principle at the basis of cognition, invariants of lesser degree would have no logical priority over those of greater degree. Hence the principle of perceptual transitions would not be derivable from the principle of cognitive symmetry.

The principle of perceptual transitions, if it continues to prove its mettle, may qualify as the first precise example of an invariance law in cognitive psychology. This is only half the story. An explanation must also be given for the second half of the adjunctive expression for invariance laws in psychology, $\psi > \phi$. The algoristic basis also imposes structure on physical information, to produce local and global invariants over and beyond those that can be explained by physical law alone.

LOCAL AND GLOBAL INVARIANTS OF PSYCHOLOGICAL INFORMATION

Currently, little that is precise can be offered regarding the manner and extent to which psychological states directly impose structure on physical information (i.e., about $\psi > \phi$). Consequently, in an attempt to round out the chief problems faced by cognitive theorists, we shall offer a few tentative speculations.

Finding Seams in the World of Energy Flux

A person who has dropped a gold coin in his yard one evening returns the next morning to search for it among the glistening, dew laden grass. By selectively attending to all yellow glitterings he easily finds it. A copy editor rapidly scans through pages of print, scarcely comprehending what is read, but accurately circles in red mistakes in spelling and punctuation. A ballet master comments critically on minute flaws in the performance of his prima ballerina that go unnoticed by the admiring audience. An eminent guest conductor kindly chides the slightly off-key, off-tempo performance of the home orchestra and directs them to the performance of their life. The trained linguist detects the subtle accent of a well-schooled emigrant who has been told by other natives that he speaks with no accent. The disembarking passenger sees his wife's face within the large crowd.

In all such examples, none of the properties detected are in any way specially noted within their context of ambient physical information. We see or hear better what we notice, whether or not it is set apart by any obvious energy accents.

The perceptual partitioning of energy distributions by highly selective modulatory activities attuned by experience is what Gibson (1966) has called "the education of attention." As argued earlier, the objectification of either phase or amplitudinal characteristics of ambient energy distributions is a negentropic process, requiring information producing processes. It requires cognitive processes that raise the attensity level of a particular property of physical information over and above the average attensity level of the total available information.

Normally, the probability of an individual property being attended to over its background energy level is a function of the "signal-to-noise" ratio. The apparent function of selective attention is to enhance the signal-to-noise ratio of certain properties through selective modulation. There are only two ways this might be achieved: Either the information must be directly modulated at the energy source, or the transmitted information must be differentially modulated on reception. The latter case requires no unusual assumptions beyond the hypothesis that cognitive processes detect, filter, and, perhaps, amplify information. The former case, however, apparently strains scientific credulity by suggesting a bizarre conclusion, namely, that cognitive processes somehow act directly upon the physical sources of information—a hypothesis apparently invoking a mysterious mental force that acts at a distance.

Must we affirm the "modulation-upon-reception" view and reject out of hand the "action-at-a-distance" view? Or can the two views be reconciled under the scope of a theory of a higher order invariance law which succeeds in banishing mysterious forces from psychology, as Einstein's relativistic space-time invariance law banished from physics Newton's mysterious gravitational force without banishing gravity as a

useful concept? Perhaps the direct cognitive modulation of events can be preserved as a useful concept in a similar manner. Such a notion might prove useful in explaining several puzzling psychological phenomena.

Psychological Information

A person who speaks English can hear pauses and breaks in the acoustic stream of speech signals that clearly separate phoneme, morpheme, phrase, and sentence boundaries. A foreigner who speaks no English will not be able to hear where these same boundaries occur. For a native speaker the segmentation problem for English speech is solved perceptually, while for a non-English speaking person the problem is not solvable at all, no matter how carefully he listens. Consequently, whatever allows a native speaker to solve perceptually the segmentation problem can not be attributed solely to discontinuities specified in the acoustic stream.

This is a good example of the distinction between psychological information and physical information. The fact that as the foreign speaker learns English he also comes to perceive the speech segments indicates that whatever acoustic invariants signify their existence, their modulation requires attunement through experience. Hence they are local invariants and probably not due to genetic preattunement of the species. By contrast, however, if it is true that young infants require no learning in order to perceive the distinction between speech and nonspeech sounds, then one might expect the acoustic invariant for speech to be due to genetic pre-attunement, i.e., to be a global psychological invariant.

The crucial question for cognitive theory is how acoustical contours become nonlinearly modulated to create perceptual boundaries where none exist in the physical information. Similar processes seem to be operative in visual perception as well.

Usually we think of the world as composed of rigid objects and sharp edges. A wave function characterization of the ambient optical energy available over a given period of time provides only a probability distribution of continuous energy spectra. Due to the diffraction of light by rough corners of objects, reflection by many faceted, multicolored surfaces, and refraction by a dust filled, gaseous medium, no sharp optical contours truly exist. Like acoustic waves, optical waves interfere in both additive and subtractive ways to create standing as well as kinetic four-dimensional convolutional waveforms.

The fact that so-called "optical illusions" exist confounds any claim that what humans see is directly based on the available optical information. In the Herring illusion, parallel straight lines are seen to warp curvilinearly away from each other over a fan-like pattern of straight lines; in the Müller-Lyer illusion, lines of equal length appear to be different in length; and in the Necker cube illusion, a three dimensional cube is seen to reverse, although it is well known that a single perspective is not sufficient to specify a unique solid shape.

Perhaps what is in the light, as in the speech signal, is something *more* than the laws of physics place there. But what and how? There seems to be a real possibility that many geometric illusions can be explained by the fact that the perceptual systems of organisms have been designed by evolution to function discretely and with finite capacity.

Machines that "see" illusions. The claim that the retinal image contains all the information necessary for vision can be shown to be false on many accounts. The existence of the geometric illusions mentioned above provides one obvious source of evidence that no simple relationship holds between the optical information projected into the visual system and what is in fact seen.

Moore and Parker (1972) have designed a machine in the form of a computer program which "sees" the Müller-Lyer illusion in the sense that it computes approximately the same discrepancies in length between the two figures as reported by human observers. The fact that such illusions of length have been shown to hold for other species (e.g., the Jastrow illusion on chickens by Révész, 1934), suggests that they may be due to a very general property of nervous systems (e.g., their discrete functioning).

Although we cannot go into the mathematical details of Moore's model here, it can easily be shown that such geometric illusions as the Müller-Lyer do not occur if the model is provided an infinite number of continuous computational states. However, in order to implement the model on a digital computer with a finite number of discrete states, the perceptual function by which the figure is processed by the model had to be both finitized and discretized. It was found that the strength of the illusion varies directly with the coarseness of the samples of the figure taken, and inversely with the number of samples processed. Thus it seems likely that the illusions arise for humans and other animals for precisely the same reason, namely, the finite discreteness of their perceptual processes.

Another important property of Moore's model is that it is able to analyze complex figures into their structural components. By assigning a weight to types of discrete samples it is possible to enhance various levels of substructure in a complex figure. In this way, various attentivity values can be differentially assigned by a very general principle to all significant properties of a figure. Thus the model is able to attend differentially to various parts of a complex figure. At Minnesota we are currently elaborating these features of this model to test the application of the principle of perceptual transitions to see if the model will "see" simple events as human observers do.

Thus, there does seem to be evidence that perceptual modulation of physical information might create new information that has psychological significance. Just as invariant physical information comes in two varieties—local and global—so does invariant psychological information. The invariance law relating physics and psychology has closure. In closing we would like to discuss the consistency we feel this view of cognitive theory has with Gibson's theory of direct perception.

THE ADVANCING SIEVE OF TIME

An impression may have been conveyed by our claim that since to some extent cognition, no less than physics, structures what is known, then natural phenomena must be subjective rather than objective. This is in no way a necessary interpretation. Since the fundamental view of this chapter rests on an epistemological assumption that whatever is known is known directly, we favor a "direct" realism as argued for by Gibson (1967). This is not to argue, however, that what is known is everything, but only that in so far as knowledge is possible at all, it is

neither constructed by subjective processes nor merely passively imposed upon the knower by physical processes. Instead, to summarize, what is known is a complex interaction of what is, how it is known, and who knows it. A simple analogy may provide a concrete illustration of the relation among these things.

A radio is constructed to tune in certain wavelengths and not others. To pick-up a particular station, its tuner must be in an appropriate state-configuration. The station's transmitter induces a voltage change in its antenna which propagates through the air and induces a voltage change in the receiver's antenna. The receiver then amplifies the received signal.

When you consider the fact that a large number of stations are broadcasting at once and that each induces a complex pattern of voltage changes in the receiver's antenna, it is amazing that the radio can be tuned to detect just one of them. Although the selective attention of the human perceptual system may be more precise, it may not operate much differently.

What "direct" pick-up means in the case of the radio is that the entire active circuit of the radio is in a resonant state. There is a direct transduction of invariant wave form characteristics of the energy transmitted (say from the voice of the announcer) that is preserved in the resonant state of the receiver. By terming this process a *transduction* we mean to distinguish it from a *translation* which necessarily involves "coding" steps. Such coding steps mediate the reception of information by interpolating symbolic representations of the energy forms somewhere between the event and the final detection state of the receiver. At the United Nations a human interpreter performs such a "coding" step for the foreign representatives. In computers this is done by a compiler.

Neisser (1967) speaks of the problem of the Höfding step in perception: how an input gets together with a stored trace. If perception, however, is a direct transduction by the perceptual systems as Gibson suggests, no such problem need be solved. Moreover, if no coding steps mediate the modulation of perceptual information, then no recoding of stored traces or retrieving of information is required; again, no Höfding step necessarily occurs. This difference between a theory of direct perception and a theory of mediated perception is exactly the difference between semantic directed processing and syntactic directed processing (Jenkins et al., 1974).

The evolution and maturation of the cognitive systems can be likened to the process of designing a radio to pick up certain stations and not others. The effect of experience on the cognitive systems is analogous to tuning in a particular station at a particular time. Now, if we make the radio portable and move it through the environment, stations at different locations with similar wave-lengths fall into equivalence classes, so that the setting on the tuner begins to take on the abstract properties of a concept. (It is instructive to attempt to elaborate the analogy to see when or if it breaks down.)

Various significant properties of the environment "broadcast" invariant wave forms which are directly transduced by the cognitive system, if it is properly tuned to modulate them. How the attunement process is adaptive, whether genetic or through experience, presumably is explained by the law of cognitive symmetry. It is important to emphasize that the invariants of information do not need to be

"symbolically represented," "stored," or "imaged" in the cognitive structures of the organism before, during, or after this processing. If they are, it is for reasons beyond those that need be satisfied for perception. Rather, the invariants of information that relate the sources in the environment with the knowing organism are the same invariants throughout the transduction; otherwise they would not be "invariants."

It is accurate to say that the event perceived is "spread" out over that portion of the space-time continuum in the neighborhood of the observer (Russell, 1948). In no sense does an event exist independently of the invariant information which identifies it. The two concepts are logically co-extensive and spatio-temporally coterminous. The invariants are directly modulated (i.e., transduced); hence that portion of the event defined over those invariants must be also.

The wave function characteristic of a modulated event is different from that of an unmodulated event, since from the standpoint of physics the event in each state is defined by a different probability distribution predicting what information is potential. This view of events as probability distributions over a certain region of space-time poses a philosophical paradox for the direct realist; perceived or known events present actual rather than merely probable information to an appropriately attuned knowing agent.

If knowledge is assumed to be probable then science becomes hypothetical rather than adjunctive. Given the earlier arguments for an adjunctive logic of natural and invariance laws, the view one holds regarding knowledge is not a question of abstract metaphysics, but a pragmatic question of choosing the most appropriate logic in which to express one's commitment to one's ideals. Since we hold both scientific and personal knowledge to be obtainable, and concur with Leibniz in postulating an underlying symmetry to nature, the very existence of events adjunctively implies the existence of knowledge that is compatible with them. And as we tried to show, this compatibility relationship, due to the invariance postulate, implies that knowledge is obtainable. But how can the concepts of potential information and actual knowledge be theoretically reconciled within the manifest scientific framework?

The argument we wish to make is analogous to the hypothesis proposed by Sir Lawrence Bragg, who hoped to reconcile wave and particle views in physics. He suggested, "Everything that has already happened is particles, everything in the future is waves. The advancing sieve of time coagulates waves into particles at the moment 'now' [in Clark, 1971, p. 420]."

It is not just the advancing sieve of disembodied time which distills from potential physical information actualized events, but the embodiment of mind which modulates invariant knowledge from these events. It may just be that neither the speech segments in the acoustic stream, nor the hard optical edges which define the shapes of objects, nor the intensified objects enhanced during a search, nor the subtlest ecologically significant nuances of any event, have any existence until carved out of the energy distributions by a goal-directed algorith.

The greatest difficulty in accepting the above notion derives from the commonsense assumption that events are somehow "out there" while knowledge of them (e.g., concepts, percepts, images, etc.) is somehow "in the head." The line

between the experience of objective and subjective phenomena is held sacrosanct. But this dualism is no longer supported by physics, psychology, or even mathematics.

Percepts, concepts, memories, ideas, and other contents of mind usually considered private and subjective, are in fact as much "out there" as particles, stones, tables, and stars. Acceptance of invariance laws for psychology means the placing of psychological, physical, and biological phenomena on equal footing, within a framework of an objective reality that favors none of them, but accommodates them all. The challenge for cognitive theory is to grasp the full implications of the statement that ideas are not in the mind, nor objects in the world, but that both are in the meeting of mind and matter.

But surely the direct effect of "thoughts" on "matter" is an exceptional conclusion to say the least. However, if the above arguments are valid, then the search for invariance laws that govern psychological phenomena leads to the first "bizarre" conclusion of our science. The wise theoretician will suspend judgment on this issue since there are no valid *a priori* grounds on which hypotheses can be rejected just because they shock scientific sensibilities. To reject them out of hand by ignoring the need for invariance laws in psychology may jeopardize the hope of discovering the polyphasic laws required for the science.

Indeed, the direct interaction of cognitive variables with the wave functional character of events may prove in the long run to be one of those "very small effects," like the effect of light on matter, upon which new sciences are founded.

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