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Could Optical 'Pushes' Be Inertial Forces? A Geometro-Dynamical Hypothesis

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Under various circumstances perceived optical disturbances have been shown to upset an observer's upright posture (e.g., Lee's gliding room), but no principled explanation has yet been proposed that will handle these phenomena. These so-called optical 'pushes' are conveyed by kinematic information but they act on the observer as if they were kinetic forces. Information (geometry and time) and forces (geometry, time, and mass) thus are mismatched in dimensionality—a problem which precludes treating optical pushes explicitly as mathematical functions. Interpreting such apparent forces as cognitive representations of real forces merely exacerbates the problem. Instead an argument is presented that agrees more with ecological psychology's view that such 'forces' must be directly perceived. Specifically, d'Alembert's Principle adopted from classical mechanics provides a way to construe information for optical pushes as functionally equivalent to inertial forces that arise in reaction to impressed forces and thereby resolve the dimensional mismatch problem. Several examples are given to illustrate this approach.

PROBLEMS WITH THE TRADITIONAL VIEW

Consider a cat getting ready to jump to the top of a fence. Many different paths might be initiated to carry-out this ballistic task but only a few will succeed. How then does the cat choose one of these successful paths from among the myriad that fail? To do so, it must set its neuro-muscular control parameters to satisfy the distance and direction to be traversed. Such parameters must also have values that take the cat's mass into account. This is tantamount to saying the cat sees the

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work-to-be-done—that is, detects the information specifying the forces required to move its mass over the target distance. The key question, then, is *how can information that specifies the directed distance to the top of the fence also determine the forces that will get the cat there?* More briefly, how can information be both specific and efficacious with respect to the intended action? We turn our attention to this question

For two millennia theories of mind have been proposed to explain how something insubstantial, like information, can causally interact with something substantial, like mass. Descartes saw the problem as one of interactive dualism, namely, that of determining how a substance that is extended (i.e., matter) could either affect or be affected by a substance that is unextended (i.e., mind). Herein lies the recalcitrant conundrum of dualism.

Modern expressions of mind-matter dualism typically assume the cat makes a mental representation of the fence, of itself during take-off, of the trajectory to be taken, and perhaps, even ‘snapshots’ of itself in flight. To plan the jump, the cat’s brain must possess the ‘machinery’ to preview these mental snapshots like a movie; and, supposedly, in this way, it creates in advance a dynamic representation of the action to be performed. (But who’s to watch this movie? An internal miniature observer?) Somehow, and this ‘somehow’ is left a mystery, the process of mental representation achieves an astounding feat; it somehow causes the brain to dynamically impose just the right neuro-muscular forces ultimately responsible for the cat’s successful action. Thus by mental representation a mysterious link is forged between mind and matter whose nature is yet to be scientifically understood, perhaps, because the problem is ill-conceived. (For discussion, see Shaw, 2003).

Here we see that a profound mystery enters the control sequence at the very beginning where the action is initiated. Under this view, to initiate the proper action, the brain must translate a complex sequence of mental representations (information?) into the complex chain of neuro-muscular forces (control) that produces the cat’s final action. Unfortunately, a ‘mind-to-body’ linkage does not sound credible for two reasons:

First problem: mismatched dimensions. The linkage spans variables that have incommensurate degrees of freedom; on the mental side an information linkage is kinematic (geometry over time) with only two degrees of freedom, while on the material mass side a control linkage is kinetic (force plus geometry over time) with three degrees of freedom. This mismatch of dimensions makes the linkage an ambiguous one-to-many mapping, and hence it is not even a mathematical function (which must be single valued). No function, then no mechanism. It fails to explain how myriad degrees of freedom on the neuro-muscular force side can be precisely constrained by a sequence of mental events on the information side.

Second problem: conservation violation. Even with no degrees of freedom problem, the basis for a credible scientific account is lacking. Assume we reject mind-matter dualism because of its obscurity; we then must solve the linkage’s incommensurability problem. Conservation of energy requires that we count the energy units on both sides of the coupling. This entails a pair of residual problems too important to be ignored. To wit: On the input side, we must ask what kind of energy could do the work of constructing immaterial representations to register mentally the goal-specific information? And, on the output side, what kind of energy can representations bring to bear on the muscle masses to do the requisite control work?

Given their immateriality, one must assume representations draw upon some kind of ethereal psychical energy that augments and guides the physical energy in producing the movement. This psychical energy potential, being arbitrary and unspecifiable, has no definable limits. Such unlimited sources of energy violate the most sacrosanct principle in science—the energy conservation law. The only way to avoid this violation would be to show that the energy gained in creating the mental representation of an action equals the energy lost in producing the action. It is bad enough that no such before-and-after energy measurements have ever been done, but even worse that no one has any idea how to attempt them.

Assume there was no breakdown of energy conservation; we would still be left ignorant of how forceless representations can cause forces to act in a certain manner. Unfortunately, this dualism hypothesis sounds as irrational as the so-called 'ether' hypothesis of the 19th century. Both views postulate 'stuff'' with contradictory properties: ether is a kind of 'stuff' substantial enough to support light waves but too insubstantial to impede them; similarly, information is 'stuff' forceless enough to be mental, but forceful enough to direct neuromuscular control of biomass. Mind-body dualism creates a paradox that science cannot resolve. Naturally, we desire a simpler theory, and if not simpler, then one that does a better job of avoiding paradox when made explicit. Does modern science offer any hope in this regard? We believe it does.

THE GEOMETRO-DYNAMICAL HYPOTHESIS FOR PSYCHOLOGY

A fundamental problem that faces psychology is getting rid of the mismatch in dimensions between information and control, as discussed in the cat-jumping-to-top-of-fence example earlier. A possible solution to this fundamental problem resonates to Einstein's claim that physics is basically geometry plus time—what the dean of American Physics, John Archibald Wheeler called "geometro-dynamics." You may recall that in general relativity Einstein showed how forces that direct an object's path might be replaced by curving the space around the object; in this way, moving objects can be made to follow a trajectory that otherwise would have required outside guiding forces. In general, with the formulation of the gravitational field equations, he showed how a forceful stress tensor could be equated with a geometric strain tensor. This is the mathematical basis for the idea that mass and space are dynamically reciprocal, with mass changing the curvature of space and space changing the course of matter endlessly but always in a manner that leaves total energy invariant (conserved).

A similar dynamic reciprocation is reflected in the perceiving-acting cycle where the action taken is guided by current information and the current information is continually changed by the action taken. Mathematically, we say that two variables are in *reciprocal variation* if each is the function of the other *and* the function satisfies some invariant condition (like conservation). If this invariance requirement is violated, then the reciprocal variation is unpredictable (nonlinear) and potentially chaotic. Recognizing this need for the perceiving-acting cycle to satisfy some invariance, we introduced what we called "intentional" dynamics as a geometro-dynamic approach in psychology. Here the invariance condition is the requirement that the perceiving-acting cycle be constrained to paths that lead to an intended goal. Under a goal constraint the information detected becomes goal-specifying and the control of the action goal-achieving. In this way, the perceiving acting cycle, when these two variables are in true reciprocal variation, remains goal-directed as the agent intends; hence the name *intentional* dynamics, or here more appropriately renamed as intentional *geometro*-dynamics.

In what follows, several examples of intentional geometro-dynamical tasks are introduced. They all share the same features: an agent intends to maintain a goal-state (e.g., upright posture or level flight) in spite of disturbing external influences (e.g., optic flow disturbance) whose effects (when noticed) the agent attempts to counter.

As we shall see, the specific character of these corrective responses provides the most convincing evidence for geometro-dynamical effects in psychology.

EXAMPLES OF INFORMATION-BASED GEOMETRO-DYNAMICAL EFFECTS

As an example of geometro-dynamical reasoning in psychology, consider the following case. Pilots sometimes make errors flying aircraft because their control is made faulty by misperceptions of which they are unaware. Here is one such case communicated to us by Rik Warren, an ecological psychologist trained by James Gibson (1979) who has studied flight control extensively. He presented a problem that has bedeviled the field for years but now seems amenable to a geometro-dynamical treatment.

Consider a low flying airplane that passes from a less densely textured ground surface to a more densely textured one, such as flying from a calm seascape (low texture density) to a forested landscape (high texture density). Even when the pilots are told to maintain a fixed altitude, they are often unable to do so no matter how diligent they are. Perhaps even more surprising, they are unaware of their error. Something about flying over surfaces with dramatically different texture densities causes the pilots to change altitude involuntarily as they cross the boundary between the sea and the land. When it is pointed out to them that they failed to maintain a fixed altitude as instructed, they are surprised. Instead of taking responsibility for the error, they often blame the deviation on some unnoticed force acting on the airplane, perhaps, an up-draft or a down-draft.

Furthermore, if on a flight they are asked to watch the altimeter, when crossing the boundary where the texture changes, they report seeing the altimeter jump. Even though there is no updraft (or downdraft), the pilots nevertheless claim they actually felt such a force act on the aircraft. The same misperception of a force also occurs in a flight simulator where there are no wind conditions. The virtual force that is induced by an abrupt change in optical information is aptly called an optical 'push.' We take this optical 'push' to be *prima facie* evidence that the geometro-dynamics hypothesis has valid application in psychology.

To explore the geometro-dynamical hypothesis further, let's look at another optical 'push' situation where optical information might induce a disturbing virtual force.

A clever experiment done by David Lee at the University of Edinburgh several decades ago illustrates most dramatically the reality of optical pushes (Lishman and Lee, 1973). Assume someone stands in a room whose walls and ceiling are detached from the floor. Further assume that the 'room' (without the floor) swings on a very long cable attached to a high ceiling so that it appears to glide. The result is the room's walls can glide but of course the rooms floor can not.

If the room is swung toward the person (who sees the wall's motion), he will sway backwards; if the room is swung away from the person, he will sway forward.

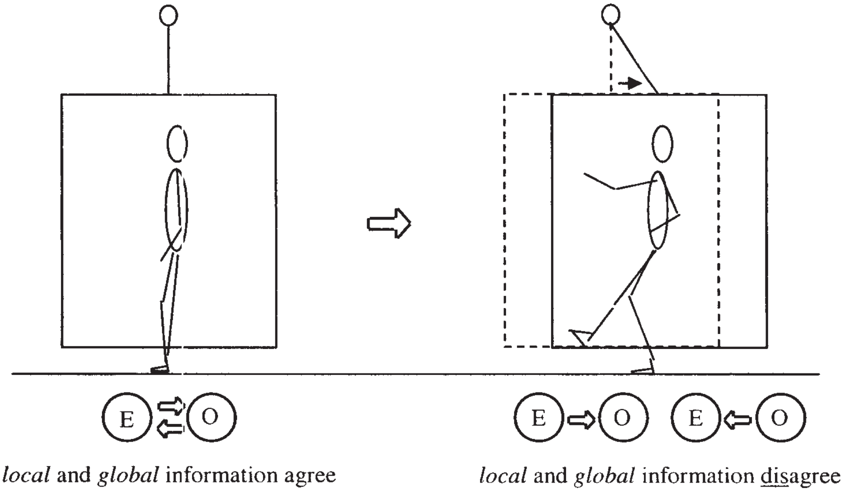

FIGURE 1 Optical 'Push' Created by the Lee Room Swinging.

Note that at no time does the wall touch the person; thus no mechanical force can possibly be responsible for the person's swaying. Also, since the person faces a wall that fills his visual angle, he sees only the wall and nothing else in the environment, especially not the floor. Consequently, we must ask, what couples the movement of the person to the movement of the walls—or, more accurately, what couples the flux in optical information to the agent's neuromuscular control system? Since there is no mechanical coupling between the wall and the person, mechanical forces provide no answer; and, likewise, since no optical functions, as usually construed by projective geometry, can convey forces, a mental representation of such optical flow information must be forceless; hence, traditional psychology has no answer either. The answer must lie elsewhere.

Let's try treating this optical 'push' as a problem for intentional dynamics to see if ecological psychology suggests an answer. Optic flow information from the wall somehow specifies to the person's control system what neuromuscular forces to apply. The information does not cause the person's reaction—information is forceless and therefore cannot be a mechanical cause. Since the information-to-control coupling is forceless, we need an answer to this question: *By what means, metaphorically speaking, does the language of information get translated into the language of control?* A clue comes from a condition we consider next.

As seen in the airplane example earlier, optical disturbances may trigger involuntary reactions from the perceivers. Here we have another example. A movement of the wall so subtle that it goes unnoticed can still induce the person to sway in phase with the wall's movement. Although instructed to stand still without moving, precise (goniometric) measurements at the person's ankle joint show he still sways in phase with the room's movement. The person's involuntary reaction reminds us of the pilot's involuntary change in altitude upon seeing a change in texture. Both are cases of optical 'push.' Also, as the room swings the person sways in phase with the room.

TWO PROBLEMS TO BE OVERCOME

An answer to the challenging question of how information can guide action must address two properties controlling information must have: First, information must *specify* the control needed for the intended action to take place and, second, the information must be *efficacious* in making the control apply in the manner specified. We need a coupling which embodies both of these properties: specificity and efficacy. Let's address the problem of specificity first.

A hindrance to a coupling satisfying the specificity requirement is the dimensional mismatch between information (geometry + time) and control (force + geometry + time), i.e., 2 (kinematic) dimensions versus 3 (kinetic) dimensions. A phase correlation between information and control could account for specificity because correlations are dimensionless numbers; they do not require matching dimensions between the two series, like optical perturbations and to-and-fro postural sways, whose degree of correspondence they measure. But phase correlation, like information, is a coupling concept that is forceless. Although it can measure relationships between two series, it can neither produce nor alter that relationship. This lack of efficacy means phase correlation cannot be a replacement for forceful coupling. Specificity alone is not enough.

A dimensional mismatch also means no mathematical function can be defined for the information-to-control coupling. This is a very serious problem because we know of no lawful couplings in nature that are not mathematical functions (in the sense of being unambiguous, single-valued mappings). Recall that statistics teaches us that correlation does not imply causation. Clearly, a phase correlation involves no forces, say, on either side of the *wall–phase ankle-phase* mapping since it maps one kinematic variable (wall movement) to another kinematic variable (body sway). This specificity solves our dimensional mismatch problem bur leaves untouched the efficacy problem of how a phase-to-phase map can induce control forces that move the person's mass. Now let's look at the efficacy requirement information must satisfy.

GEOMETRO-DYNAMIC EFFICACY

If we could but eliminate the person's mass from the coupling equation, then the need for forceful control would also be eliminated, and like specificity, efficacy could also be kinematic. Unfortunately, an elementary argument shows why this problem of dimensional mismatch is not easily resolved. On the information side, the wall's mass can be ignored since only kinematic variables enter into the definition of optic flow. However on the control side the person's body mass cannot be ignored since kinetic variables enter into the definition of its postural sway—the angular acceleration of the person's body mass. The mismatch between the information specific to wall movement and the control determined by it is clearly revealed in this mapping schema:

$$\textit{acceleration} \rightarrow (\textit{mass} \times \textit{acceleration})$$

This mapping is not a function, which requires a one-to-one mapping, but a one-to-many. Rather it is what mathematicians call a 'germ,' an ambiguous *few-to-many* mapping. Contrary to the cognitive psychologist's strategy, replacing the forceful control variable with a representation of that control doesn't help, for it just complicates the mapping and makes it more obscure:

$$\text{acceleration (geometry + time)} \rightarrow \textit{representation} \text{ (geometry? symbols?)} \rightarrow \\ \textit{control} \text{ (mass + geometry + time)}$$

We will need to look elsewhere for an answer.

Let's try a different tack. While the phase coupling approach does not match dimensions over the *information* → *control* mapping, there is an approach which makes information kinetic rather than kinematic—an approach which matches information dimensions with control dimensions. It is called *d'Alembert's Principle*, and is so fundamental that all forms of mechanics, whether Newton's, Hamilton's, or Lagrange's have been derived from it. Could intentional geometro-dynamical effects, at least in the simplest cases, also be derived from this fundamental principle? Let's reconsider the Lee swinging room experiment in this light.

In that experiment, the participant's intention is to maintain a steady upright stance in spite of any optical disturbances in the perceptual information created by swinging the room (Figure 1). An intention intuitively entails a goal-directed action for its satisfaction. Stated in words, the action rule entailed must be something like this: *If, while you are standing upright with feet firmly planted, you perceive a global optic flow, that means your posture is changing from the upright, then move so as to re-aright yourself.* To perceive a departure from an upright posture, there must be either haptic information or optical information or both. Here we are only interested in the optical disturbance involved in the movement of the 'room'; since the floor on which the person stands never moves, no haptic disturbance is felt—a fact that participants in the experiment apparently ignore, allowing visual information to be in control.

A global transformation of the optic flow information specifies ones self-movement; by contrast, a local optical transformation specifies that something else has moved. The movement of the wall causes a global radial optical expansion that is *centrifugal* when the wall is moved toward the person standing and *centripetal* when moved away from the person. Either of these global transformations on optic flow is information that specifies to the person that she is falling over. Of course, if the person were falling over in either direction, she would experience the same global optic transformations. It is important to emphasize that motion between two objects in isolation is relative; this means the observer in the room, who sees only the moving wall, has no way to distinguish the wall's motion from his own. This fact is central to understanding the application we wish to make of d'Alembert's Principle.

An external observer would be in a position to see that it is the room that has moved and not the person. This observer would also see the motion of the room as a local transformation of her optic array specific to the context of the larger environmental situation. Thus, not being presented with a global optical transformation, the external observer standing outside the room does not herself sway. The person standing in the room sees the motion of the wall as a global optical transformation—information specifying that he has moved rather than the wall. Seeing himself as having moved, he naturally attempts to counter that departure from the vertical. But since the person is already standing straight up, the correction takes him into an oblique posture. In this way, a person's intention to stand upright is shown to depend on the information which controls his posture. We might say the information 'pushes' the person from the upright, and the person's reaction is to try to readjust.

A caveat. Although an agent's act may be reflexive, in our view, it is no less intentional for being so. It may have been instilled by evolutionary adaptation or learning rather than a conscious choice. In other words, acts need not be consciously chosen to be intentional; they need only be goal-directed. Intentional acts contrast with those caused by external forces because the latter cannot be end-state determined by goal-specific information while the former can. This is in keeping with philosophy where being *intentional* is characteristic of anything that refers beyond itself in its current situation to something else (perhaps itself) in another situation. A chosen response is intentional if it refers beyond itself not because it is chosen. Thus goal-directedness is just a particular example of intentionality.

TAKING STOCK

Let's take stock of the facts presented up to now. Earlier we reviewed several examples in which perceived optical disturbances also act like forces *if that information is intentional*: in the case of the airplane pilot who, on 'seeing' the change in surface texture, is compelled to move the control stick causing an unnoticed change in the plane's altitude. A counter move of the stick is intentional because it is corrective relative to the goal of maintaining a intended altitude. Likewise, in the Lee gliding room case, the person adjusts his posture as a function of the optical disturbance produced by the wall's movement and counter-moves to restore her posture to the intended upright position. From these examples (and many others that might be cited), we concluded that an optical disturbance can act as a kind of 'force'—an optical 'push' if it conflicts with an intended goal-state. In these cases, the motion of the geometry of the world acts as information that specifies to the person that he should move his arm or his body. We must ask again how this information brings about a perceiver's response.

We can dispel one problem. There is no mystery how the person moves his body or his limb, for the applied force is self-produced by neuromuscular control that draws on the person's metabolic resources and the intention to conform to an intended goal-state. What remains mysterious is how that kinematic information produces properly scaled kinetic forces involved in particular goal-directed activities. Previously, we have called this the fundamental problem of ecological measurement theory, or more briefly *ecometrics* (Shaw and Kinsella-Shaw, 1988).

Let's take a close look at d'Alembert's Principle—a principle that resolves the dimensional mismatch while satisfying both the specification and efficacy conditions as well. To understand this principle, we must first appreciate the concepts of inertial frame, non-inertial frame, and inertial force.

D'ALEMBERT'S PRINCIPLE AND INERTIAL FORCES

There is a class of inertial forces which includes, in addition to the inertial force arising from accelerated translation, the centrifugal force, the Coriolis force, and, surprisingly, even gravity—as Einstein showed in his arguments for the Equivalence Principle which equates gravitational mass with inertial mass (recall Einstein's famous elevator thought experiment). In Newton's original analysis inertial forces were omitted and only the impressed forces considered. Newton's laws of motion only apply to frames of reference in which a body remains at rest or moves uniformly at a constant speed when no forces are impressed upon it. This is called an *inertial frame of reference*. The frame itself need not be at rest—it can be moving at a constant speed relative to another frame of reference.

In all these cases, *an optical 'push' arises whenever an abrupt change in optical structure occurs that transforms the person's inertial frame of reference into a non-inertial frame*. Put simply: Optical pushes arise from information specifying frame discrepancy. In the rest of the paper, by establishing its physical foundations, we shall see if this hypothesis has merit.

Newton's laws as originally formulated however do not apply to objects in non-inertial frames, that is, in frames that are accelerating. But they may be reformulated so that they do, as shown by the French physicist, Jean le Rond d'Alembert (1717-1783). His principle states: *When any object is acted on by an impressed force, an inertial force is produced as a reaction. In keeping with the Principle of Virtual Work, the resultant of this impressed force and the inertial force is zero.* In other words, when a car is at rest or moving uniformly at a constant velocity, no impressed forces act on it or its driver. However when the driver depresses the accelerator, the car's motor impresses a force on the car that accelerates it. At the same time, in reaction to the impressed force, an equal but counter-directed inertial force is produced that acts on the driver pushing him back against the seat.

As observed earlier, Newton's laws only apply to objects in inertial frames; therefore they do not apply to the accelerating car—a non-inertial frame. But by invoking d'Alembert's Principle, Newton's laws can be generalized to cover this case too. Behind this principle was a simple but brilliant insight, which is clearly revealed in four steps. (For a general discussion, see Lanczos, 1970):

First Step: We start with Newton's Second Law of motion which asserts that mass multiplied by acceleration equals an impressed force, the familiar, $m\mathbf{A} = \mathbf{F}$.

Second Step: Rearrange the equation as follows: $\mathbf{F} - m\mathbf{A} = 0$

Third Step: Define a new vector, $\mathbf{I} = -m\mathbf{A}$. This is called an inertial force. Notice, this is a counter force; its sign is the opposite of the sign on the impressed force vector.

Fourth Step: We can now reformulate Newton's Law as $\mathbf{F} + \mathbf{I} = 0$.

The third step looks a bit trivial, being nothing more than giving a new name to the negative product of mass x acceleration. In fact, it allows the expression of an important principle in the next step. In Newtonian mechanics, the concept of a system being in equilibrium entails the nullification of all impressed forces acting on it. Static equilibrium applies to objects not in motion. With this reformulation of Newton's law, d'Alembert showed us how to generalize the concept of equilibrium to objects in motion. To make this generalization required a brilliant insight—d'Alembert had to see that inertia itself is a force that can be included with impressed forces to make up the total effective force of the system, i.e., effective in the sense of summing to zero. This now allows us to extend any criterion for a mechanical system being in static equilibrium to a moving mechanical system being in dynamic equilibrium.

Inertial forces are experienced daily by those of us whose bodies are carried along with a variety of accelerated frames—automobiles, trains, buses, airplanes, swings, carnival rides, horses, or rocket ships to the moon. The origin of these 'unimpressed' forces is the tendency for objects to resist change of their motion or state of rest, in accordance with Newton's Second Law, which asserts that a force is anything that accelerates a mass, i.e., $F = mA$. Inertial forces differ from impressed forces in how they are produced. An inertial force is created by the accelerating frame moving out from beneath the objects it contains—temporarily leaving them behind—until the train's impressed force drags them along as well.

ILLUSTRATING THE ORIGIN OF AN OPTICAL PUSH

Here is one of our favorite cases—one personally experienced by most of us. Consider the case of Alice standing in the aisle of a train, facing forward, holding a full cup of coffee, when, suddenly, the train lurches forward. As expected, Alice's posture is jarred backward and the coffee is spilled, following which she reacts by up-righting herself. There are two kinds of forces involved in this minor calamity—an *impressed force* applied by the train's engine to accelerate its considerable mass, and an *inertial force* jerks Alice's body and sloshes her coffee. Strictly speaking, since this latter force arises from the inertia of the subsidiary masses—the person and the liquid—it is not an impressed force and, for many, not a true force in Newton's sense. Thus such 'forces' are variously called 'pseudo-forces,' 'fictitious forces,' 'apparent forces', or 'inertial forces.' We have chosen to use the term 'inertial forces' throughout this paper since it is most accurate and most descriptive.

Next, consider Bob who is in a similar situation but his train does not move; rather Alice's train on the adjacent track does. If through the windows of his own train car Bob, who is also holding a full coffee cup, sees the other train move, a remarkable thing happens; he also spills his coffee. Understanding this remarkable phenomenon would help clarify what information is, and suggest how (in at least one way) it can embody both specification and efficacy—how information can specify to a non-moving observer an accelerating frame of reference, and how that same information can cause that observer to experience an inertial force. Applying d'Alembert's Principle to both Bob and Alice's situation is elucidating.

The law of relative motion dictates that if Bob perceives Alice's moving train as stationary, then he must experience his own stationary train as moving at the same speed as hers but in the opposite direction. Unexpected perhaps is that the information for his relative motion causes him to experience a corresponding inertial force that is as real for him as Alice's was for her. Also somewhat astounding is that kinematic information Bob detects can bring about in him a kinetic experience in the absence of any impressed force. This experience of an apparent inertial force is so real for Bob that he reacts by trying to realign his posture even though no such correction is needed. Consequently, it is his unnecessary response that upsets Bob's coffee. This contrasts with the case where Alice's train actually upsets her coffee by its abrupt acceleration.

Clearly, if Bob were blind he would feel no optical 'push' while a blind Alice would still feel the inertial force induced by her train's impressed force. Instead, through his well-planted feet Bob would experience no motion. Obviously, optical disturbances alone cannot induce inertial forces; in addition, a person must perceive his frame to be accelerating—whether it is proximal (his train) rather than distal (Alice's train).

Box 1 (above) exhibits the relevant facts in their familiar mathematical form. To make clear the content of these formulae, let's express them in both words and pictures (Figure 2): Alice in her train experiences its acceleration (non-inertial frame) as an inertial force that causes her to spill her coffee (Equation 1, Figure 2(I)). Meanwhile Bob in his stationary train (inertial frame) experiences no acceleration and spills no coffee (Equation 2). On a later occasion, while watching Alice's train through the windows of his own train car, Bob misperceives its jerk forward as being a jerk of his own train backward which he experiences as an inertial force, causing him to react reflexively and spill his coffee (Equation 5, Figure 2(II). Note, where Alice spills her coffee because her train *actually* moves, Bob spills his coffee because his train's apparent movement induces in him a neuromuscular reaction. An optical 'push' is thus a perceptually induced self-produced movement (Equation 3).

Our aim is to use d'Alembert's Principle to explain how optical 'pushes' arise. But here we run into a problem. Bob's equation is incomplete. Physically speaking, because no impressed or inertial forces actually acts in Bob's case, the first two terms in his equation must be null. Before d'Alembert's Principle can be applied, these missing terms must somehow be supplied. Indeed they are—by Bob's perception of Alice's train. The exact nature of the information of the missing terms is clarified if we introduce another train case—call it Tom's train—one that is the exact mirror reflection of Alice's and putatively embodies the kind of information Bob needs (Equation 2, Figure 2(III).

Put differently, the empirically established functional efficacy of the perceptual coupling *is* the information content of an optical 'push'—a content that is kinetic

BOX 1: Summary of the mathematical argument that an optical 'push' originates from d'Alembert's Principle

(I) $\grave{\boldsymbol{F}}_{\mathrm{A}} + \acute{\boldsymbol{I}}_{\mathrm{A}} + \grave{\boldsymbol{c}}_{\mathrm{A}} ?\ 0$ (Alice's case)

(2) $\overleftarrow{\boldsymbol{F}}_{\mathrm{T}} + \overrightarrow{\boldsymbol{I}}_{\mathrm{T}} + \overleftarrow{\boldsymbol{c}}_{\mathrm{T}} ?\ 0$ (Tom's counter-case)

(3) $0 + 0 + 0 ?\ 0$ (Bob's case with eyes shut)

(4) $0 + 0 + \overleftarrow{\boldsymbol{c}}_{\mathrm{B}} ?\ 0$ (Bob's corrective response)

If $\overleftarrow{\boldsymbol{F}}_{\mathrm{T}} \rightarrow \overleftarrow{\boldsymbol{F}}_{\mathrm{B}}$ and $\overrightarrow{\boldsymbol{I}}_{\mathrm{T}} \rightarrow \overrightarrow{\boldsymbol{I}}_{\mathrm{B}}$, then

(5) $\overleftarrow{\boldsymbol{F}}_{\mathrm{T}} + \overrightarrow{\boldsymbol{I}}_{\mathrm{T}} + \overleftarrow{\boldsymbol{c}}_{\mathrm{B}} ?\ 0$ (Bob's Tom-like informational coupling to Alice)

Bob's corrective response, $\overleftarrow{\boldsymbol{c}}_{\mathrm{B}}$, is also an impressed force that produces its own opposite and equal inertial force, $\overrightarrow{\boldsymbol{I}}_{\mathrm{B}}$. The addition of this inertial force term to Equation 5 allows Equation 6 to balance and thereby to satisfy d'Alembert's Principle:

(6) $\overleftarrow{\boldsymbol{F}}_{\mathrm{T}} + \overrightarrow{\boldsymbol{I}}_{\mathrm{T}} + \overleftarrow{\boldsymbol{c}}_{\mathrm{B}} + \overrightarrow{\boldsymbol{I}}_{\mathrm{B}} = 0$ (d'Alembert's Principle satisfied)

Note: In Box 1, Equation 1 expresses the content of Figure 2(I), Equation 2 that of Figure 2(III), and Equation 5 that of Figure 2(II).

and not just kinematic. Under this view an optical ‘push’ receives it kinetic character from being a geometro-dynamical phenomenon as proposed, and as such, Bob’s forceful corrective response matches the dimensionality of the apparent inertial force. Hence this geometro-dynamical interpretation dispels the mystery of the optical ‘push.’

Alice’s train’s abrupt forward motion produces a counter but equal inertial force that upsets her posture and hence her coffee cup (Equation 1). In seeing Alice’s train move forward, Bob sees his own train, like Tom’s (Equation 2), as moving backward. On the other hand, if Bob were blind there would be no effect on him by Alice’s train, and all terms in his equation would be null (Equation 3). Equation 4 expresses an anomalous case where, in the absence of any disturbing force, Bob inexplicably jerks his body and upsets his coffee. The mystery of Bob’s anomalous response is removed by recognizing that by perceiving Alice’s train, Bob establishes an informational coupling which acts *as if* it were forceful (as in Tom’s case, Equation 2). This informational coupling specifies an apparent inertial force—an optical ‘push.’ One way the functional efficacy of this informational coupling might be represented mathematically is by identifying the first two terms in Equation 2 with those of Equation 4 as shown in Equation 5.

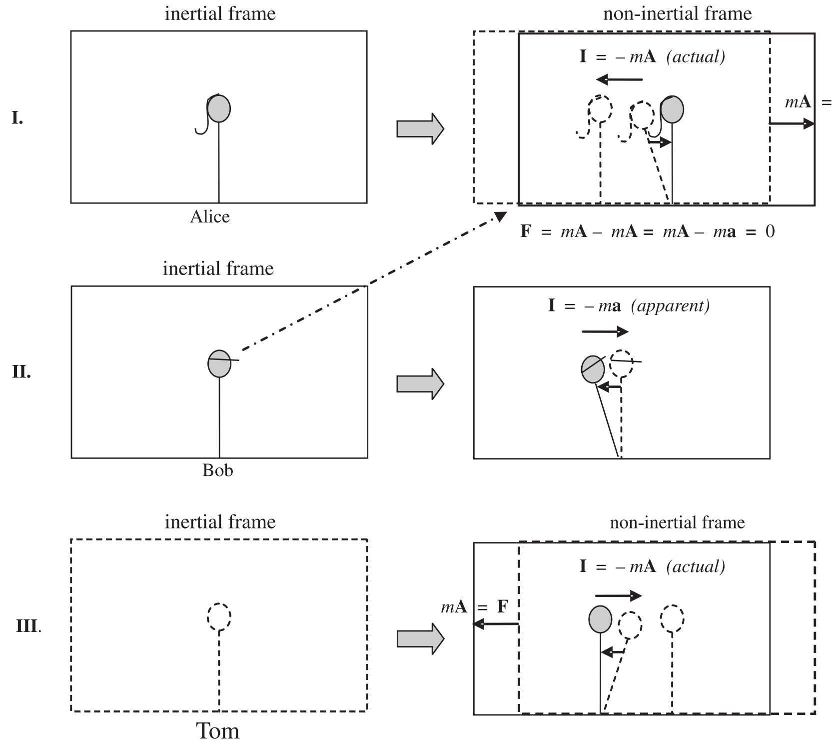

FIGURE 2 An illustration of d'Alembert's principle applied to information and control

The mathematical argument presented in Box 1 is more easily understood when pictured (Figure 2 (I-III). From the argument summarized in Box 1 we now see how information becomes an inertial force—an optical “push“ to which an agent (e.g., Bob) might give a controlled response. In this way, information can indeed meld efficacy with specification to create a perception-based kinetic effect. As hypothesized in the introduction, information under the right circumstances can be more than mere kinematics but can be raised in dimensionality to the status of kinetics. Moreover, with information matching force in dimensionality, the desired information-control function is mathematically allowed. The general form of this geometro-dynamical function can be captured by a formal analogy.

Assume there are two sites A and B separated by a distance each with a potential for producing similar forces, F_A and F_B. Assume at Site A a force is produced along with information that specifies its character such that if information is made available to the Site B potential, then a force F_B conforming to F_A‘s information can be produced at Site B similar in character to the force F_A produced at Site A. In a sense, the force produced at Site A is ‘teleported‘ to Site B by the intervening information. This teleportation is synonymous with what we mean by an optical ‘push’ being the expression of a geometro-dynamical process. This formulation essentially treats optical “pushes” as being the *ground* of an analogy that connects the topic (an exemplary case) with vehicle (another exemplary case):

F_A stands to *Site A* as F_B stands to *Site B.*

The ground of this analogy tells us what the topic and vehicle have in common. In the case of the optical push the ground is whatever is invariant about the information at Alice‘s site (the topic) and at Bob’s site (the vehicle). This common information across the distinct sites is what real inertial forces and optical pushes share. Under d’Alembert’s Principle the ground of the analogies exemplifying inertial forces and those exemplifying optical pushes are if not the same indistinguishable. Currently nothing more fundamental has been proposed than this formal analogy which ties psychology to physics.

CONCLUSION

This brings us to the conclusion sought: An optical ‘push’ is indeed a real force—but of the inertial rather than impressed variety. It can be perceptually induced by information specific to an accelerating frame whether that frame acts directly on the perceiver’s body mass or not. Hence there are two ways an inertial force might arise, either from an impressed force or from information about an impressed force. This follows from the evidence that ones body mass frame and ones perceptual frame need not be the same. This is a consequence of applying d’Alembert’s principle to agents who can produce forces to counter inertial forces arising from either a force impressed on the observer’s body or from information specific to an impressed force detected by the person.

Most importantly, with the applicability of d’Alembert’s Principle to explain optical ‘pushes,’ the two problems raised in the introduction are resolved. The first problem was that of finding a way to match the dimensionality of kinematic information with that of kinetic control. The geometro-dynamical interpretation of d’Alembert’s Principle solves this problem by showing how two kinetic potentials can be analogically coupled over a distance by information. The second problem was to avoid solutions to the first problem that violated the principle of energy conservation. The analogy presented above, suggests that an important consequence of d’Alembert’s Principle being applied to our phenomenon is resolution of the second problem as well. For systems balanced under reciprocal variation (as the analogy suggests) have equations whose simultaneous solution equals zero and are therefore conservative. In short, the geometro-dynamics strategy, armed with d’Alembert’s Principle, gives an account of optical ‘pushes’ that avoids the dimensionality and conservation problems in a most natural way.

Broadly speaking, our thesis asserts that construing information and forces under geometro-dynamics will help erase logical difficulties introduced into psychology and physics by acceptance of the mind-matter dichotomy. Our specific aim has been to show how at least one class of fundamental psychological phenomena, optical 'pushes,' can be explained as geometro-dynamical effects. Furthermore, we expect that geometro-dynamic methods, like d'Alembert's principle, might help mend the schism between psychology and physics caused by their general acceptance of mind-matter dualism. If so, then the adoption of the geometro-dynamical strategy in psychology would have a most salutary philosophical consequence. It would loosen the grip of representational realism that has for so long misdirected our efforts and produced so much contentious debate.

Although reformulating this geometro-dynamical strategy for psychology in all likelihood will be difficult, convincing psychologists to give up superfluous mediating cognitive constructs will no doubt be even more so. Nevertheless, showing that some phenomena in psychology invite geometro-dynamical interpretations suggests ecological physics is just one more expression of an underlying cosmological physics in which psychology and physics may share common ground.

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