

Chapter 11

Dimensionless Invariants for Intentional Systems: Measuring the Fit of Vehicular Activities to Environmental Layout

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"When in use, a tool is a sort of extension of the hand, almost an attachment to it or a part of the user's own body, and thus is no longer a part of the environment of the user. But when not in use, the tool is simply a detached object of the environment . . . the boundary between the animal and the environment is not fixed at the surface of the skin but can shift."

—Gibson (1979, p. 41)

"The field of safe travel, it should be noted, is a spatial field but it is not fixed in physical space. The car is moving and the field moves with the car through space. Its point of reference is not the stationary objects of the environment, but the driver himself. It is not, however, merely the subjective experience of the driver. It exists objectively as the actual field within which the car can safely operate, whether the driver is aware of it. It shifts and changes continually, bending and twisting with the road, and also elongating or contracting, widening or narrowing, according as obstacles encroach upon it and limits its boundaries."

—Gibson & Crooks (1938, p. 454, *emphasis in original*)

11.0 Introduction

11.0.1 Aims and Motivations

Ecological psychology holds the belief that theory and basic research must ultimately aspire to practical applications. This follows directly from its primary aim to understand an actor's functional relationship to natural and manufactured environments. Although laboratory research is useful, it is no substitute for observations and measurements in the field. Even highly realistic simulations may be misleading. The safe flight of aircraft (Gibson, 1950) and the much earlier study of the field of safe travel for automobiles (Gibson & Crook, 1938; see above) presages the work we have undertaken here. The explicit aim of the evolving ecological approach is to develop methods of scientific investigation with ecological validity. Methods with ecological validity treat the organism and its environment as a system with variables defined on the *econiche*—environmental variables that make reference to and are scaled to the organism as a perceiver and actor in that environment. Methods are sought to explain, first, how actors achieve success on ecologically significant tasks performed in their natural environments and, second, to extend these methods to architectural and landscaped environments. This contrasts sharply with those methods that merely seek to understand how subjects attain statistical significance on arbitrary tasks performed in the laboratory. In this sense, the ecological approach aspires to treating the laboratory as an extension of the subjects' *econiche*.

More than any other trait, the explicit commitment to strive for both ecological validity and ecological significance in one's research is the hallmark of an ecological psychologist. This commitment qualifies many to contribute as ecological psychologists, even though they call themselves by other names. One needs only share the belief that success on practical problems involving the perceptual control of action is an important, perhaps the most important, way to validate the consistency and the significance of one's theory and research. Thus, the attempt to wed ecological psychology with human factors and human engineering, as this volume tries to do, seems quite natural and overdue.

Our specific goal in this chapter is to present a new approach to the wheelchair navigation problem. How are wheelchair users able to select and follow the best route through cluttered architectural spaces, such as office and factory workplaces, or residential and public spaces? This chapter is, in part, a progress report of an ongoing research and theory

on this topic.

Regarding this problem we have three aims—one practical and the others theoretical. One aim is to offer a practical solution to the problem of measuring the dynamic fit of active wheelchairs to their functional spaces under the constraints of a given navigation goal. The second aim is to place this problem under *intentional dynamics*—a general method of ecological psychology for modeling goal-directed activities. Although we use the wheelchair navigation problem as our focus, these methods and principles should apply to other activities as well. To meet these aims, we explore the use of dimensionless analysis—a mathematical engineering technique for finding dimensionless measures (called π -numbers) of the similarity in the structure and functioning of systems which may appear quite different. We shall need to tailor this technique to certain prospective control problems. This entails our third aim, namely, to show how, in principle, dimensionless measures (π -numbers) may allow us to compare the invariants of the information detected about the layout of the environment relative to an intended goal path to the invariant aspects of the control law that must be applied to navigate the path so as to attain that goal.

A caveat and apology are in order. Our aim is to introduce the steps required to arrive at this conclusion without developing or explaining in detail the mathematics involved. We recognize, therefore, that our explanations are somewhat opaque to the reader, but our aim is to point the way to a solution to this problem rather than to provide such a solution, for such details would go well beyond the space permitted for this chapter. We hope the reader nevertheless is stimulated to help develop the approach sketched here.

11.1 Task Similarity, Intentional Dynamics, and the Prospective Control Problem

The general navigation problem is a prospective control problem; it asks how perceptual information about a future state of affairs—an intended goal—can be used to control a current state of affairs (that is, the current forces) in order to reach that goal. More specifically, it asks how a rule for the perceptual control of action allows an actor to find his or her way through a cluttered environment over a preferred path to an intended goal. We explore the prospective control problem by focusing on adult human actors who locomote through architected environments by wheelchair. The task selected for them is a simple one—to pass without

collision through passageways of various clearances that connect enclosed cubic chambers. We assume that the ceiling, wall, and floor surfaces of these chambers are smooth, flat, level, and with sufficient roughness to minimize wheel slippage. Although choosing this type of actor, environmental situation, and task narrows down the possible solutions to the prospective control problem considerably, it still leaves the major issues of the general navigation problem intact.

Consequently, any steps toward solving this general prospective control problem should help solve similar versions of the problem involving different actors, situations, and tasks. Of course, how general the method can be depends on how similar the tasks are. A formal measure of task similarity would therefore be desirable. Discovering such a similarity measure is one of the major goals of the current project.

What would a generic description of the prospective control problem in navigation tasks involve? One requirement is that it makes clear what variables of the task situation are most relevant to a solution. Should the variables be mechanical, biomechanical, physiological, psychological, or ecological? If the description depends on the first three kinds of variables, then the explanation will be causal. Causal explanations, however, are incomplete because they do not address the roles played by perceptual information, skillful control, and intention in goal-directed activities. On the other hand, psychological explanations are likewise incomplete because they typically ignore forces, masses, and friction. An understanding of the prospective control problem, therefore, involves both causal and intentional variables.

Clearly, an ideal explanation must address how goal-specific information specifies the forces by which actors intentionally control their goal-directed movements. In this regard, there are two problems: the *scaling problem* and the *transduction problem*. The *scaling problem* is a measurement problem at the ecological scale or, more briefly, a problem for *ecometrics*. Ecometrics asks how information at the scale of geometry and kinematics, the scale at which intended goals are specified, might be transformed into force units at the scale of the statics and kinetics of control, the scale at which goals might be accomplished. Taken together, these problems combine to form a more general problem of measuring the similarity of environmental information detected by the actor to the actor's control of the behavior required to accomplish an intended task in that environment. Put simply: We must answer the question of how the information detected is similar to the action produced whenever different actors succeed in solving the same task. This is a natural question for similarity theory, and its natural answer is

to be found in discovery of the appropriate dimensionless numbers.

The *transduction problem*, which presupposes a solution to the ecometrics scaling problem, is a problem for *ecomechanics*. *Ecomechanics* asks how forceless information might be made efficacious in directing control processes so that the actor might reach an intended goal state. When classical mechanics is the study of laws governing *motions* of inanimate bodies and biomechanics the study of laws governing *movements* of biological systems, so *ecomechanics* is the study of laws that govern *actions* (goal-directed movements) of agents (Shaw, 1987; Shaw & Kinsella-Shaw, 1988). Unlike the first two forms of mechanics, the last one involves a special relationship holding between information and control which goes beyond mere force or energy flow descriptions. The details of this relationship have recently been spelled out as a theory that information detection and energy control must be *self-adjoint* in the sense of having mutual and reciprocal quantities (Shaw, Kugler, & Kinsella-Shaw, 1990; Shaw, Kadar, Sim, & Repperger, 1992).

Solving the scaling and transduction problems entails a new approach to prospective control problems, one that is a hybridization of physics, biology, and psychology. Attempts to develop such an approach are being made. A branch of ecological psychology called *intentional dynamics* subsumes ecometrics and *ecomechanics*. The transduction problem of *ecomechanics* and the scaling problem of ecometrics comprise dual aspects of the central problem of intentional dynamics. Hence, the prospective control problem falls naturally under this new discipline. (For an overview of intentional dynamics, see the earlier studies mentioned as well as Kugler & Shaw, 1990, and Kugler, Shaw, Vicente, & Kinsella-Shaw, 1990).

Before addressing the questions raised, we provide a statement of the social motivation for this project and explain the practical significance of its potential success.

11.2 Part I: A Problem with Ecological Significance and a Method with Ecological Validity

11.2.1 Overcoming Barriers

An estimated half-million persons are added to the population of the handicapped each year through illness or injury (Arthur, 1967). Of the estimated 36 million Americans with disabilities (Disabled USA, 1984), many achieve education and, later, employment or self-employment

because there are specially seated workplaces available for them in accessible facilities (for example, homes, schools, offices, and factories). Unfortunately, many other disabled Americans are excluded from these opportunities because standardized design guidelines fall short of the ideal and do not accommodate their particular disability. Current minimal standards sometimes fall short of the ideal for the disabled for two reasons: First, because the practices used to implement the standards may be underconstrained by the current specifications, or, second, because design tradeoffs are used to strike a practical and economical balance between the architectural dimensions needed to accommodate both the disabled and the nondisabled.

A significant reason for these problems is that, at present, it is impossible to measure the dynamic fit of goal-directed activities (e.g., moving wheelchairs) to the environments in which they must be performed. Consider the special case of designing environments for wheelchairs. Only recently have methods begun emerging to determine design guidelines for the fit of such activities performed from relatively stationary chairs and wheelchairs (see Dainoff, 1987, 1991a, 1991b; Abdel-Moty & Khalil, 1989). However, no significant work has yet been done on ways to measure dynamic maneuvers of wheelchairs over accessible routes through functional spaces toward goals. The proposed project offers a way to remedy this shortcoming.

In its specifications, the American National Standards Institute (ANSI, 1986) emphasizes the need to recognize that persons with disabilities that confine them to wheelchairs are no longer "average" persons. They are shorter and wider, rolling instead of walking, being unable to climb stairs. They require ramps or oversized elevators, require more space to turn around, and more clearance under tables and other equipment. Nor can they see over or cross over barriers that others might more easily. Moreover, they require, on the average, more time to egress through cluttered environments (such as furniture arrangements and milling crowds).

This list comprises but a few characteristics that distinguish the wheelchair-bound individual from ordinary individuals. Wheelchair users have fewer opportunities for actions than the nondisabled. Consequently, a question of great interest is how functional spaces for wheelchair users can be designed to afford freedom for them to act in ways comparable to nonwheelchair users. Consider other important characteristics of wheelchair users that make the designing of functional spaces explicitly for them even more critical.

11.2.2 Determining Dynamic Tolerances of Fit

The Uniform Federal Accessibility Standards (UFAS) sets the specification for a clear opening width for doorways. When the approach is head on, the recommendation is for widths of 30 in (76 cm), whereas turning a wheelchair to enter an opening requires greater clear widths. For most approaches, the addition of an inch leeway on either side suffices—making for a minimum clear width of 32 in (81.5 cm). However, to accommodate the likelihood that control of straightline travel of the wheelchair will be imperfect adds at least 2 in tolerance. For instance, to accommodate traversing passageways that are more than 24 in (61 cm) long increases the minimum clear width to at least 36 in (91.5cm) as compared to doorways with unrestricted approaches. For similar reasons, the minimal clear widths of checkout aisles in stores must also be 36 in (91.5 cm). However, the specifications must be even more generous in libraries. Because their greater length makes larger meanders from the straightline path even more likely, aisles between library bookcases require greater tolerances. In this case, the UFAS recommends a clear width of 42 in (106.5 cm) where possible. Where a wheelchair user might meet other wheelchair traffic or pedestrian traffic, then the clear width of passages must be adjusted to even greater tolerances (66 cm and 48 cm, respectively). Ultimately, our project aims at dynamic in-the-field testing to see if such standards remain realistic under a variety of wheelchair velocities, approach angles, and intentions.

These alterations to the specified clear widths of doorways or passageways, as a function of direction of approach, distance to traverse, or type of traffic, are only estimates. Dynamic measures might verify whether they are adequate adjustments to existing codes. Clearly, wheelchair speeds may increase when going down ramps, under emergency egresses, keeping up with traffic flow, and so forth. The possibility of varied speeds in approaching doorways or moving through passageways, therefore, requires dynamic measures to set their clear width tolerances. Wheelchair velocities may vary as a function of the layout of the environment, the circumstances, and the intentions of the wheelchair user.

Physical mechanics dictates that wheelchairs moving at even moderate speeds will have momentum characteristics that make them more demanding on space requirements than slow-moving ones. Momentum influences both maneuverability and control—sometimes in a positive way and sometimes in a negative way—depending on

circumstances. The formula for linear and angular momentum involves the multiplication of mass by velocity. For every unit of increase in speed or in the mass of the wheelchair user, there is a dramatic, multiplicative increase in the minimal requirements for stopping and turning. For this reason only dynamic measures can determine the limits on the safe and comfortable fit of the active wheelchair to architectural spaces.

In preparation for the rest of this chapter, it is well worth rereading the Gibson and Crooks' (1938) quotation given at the beginning of this chapter. Clearly, its full appreciation entails an innovative approach to accommodate the facts of dynamic measurement as discussed.

11.2.3 Limitations of Current Measurement Techniques

The Americans with Disability Act (ADA) provides an important incentive to actualizing the commitment of our society to allow every member of this community to participate, fully, in all aspects of life. Designing for architectural spaces that are accessible and usable by wheelchairs is a fundamental step toward equal rights. Legislation can do no better than to implement the best design guidelines that exist. If these are inadequate, then the resulting standards and codes will be equally wanting. In most cases, current standards have evolved from practical experience, legal precedent, and the intuition of experts. They have not been set nor verified by scientific methods. Without dynamic measurements there is no alternative to current practices. The simplicity of the proposed method can be better appreciated.

As observed earlier, the field needs dynamic measures of active wheelchairs to determine the safest, most comfortable, and efficient paths for traveling between points in the environment. Current methods may measure the total time to travel a fixed path, but not the selection of the path, nor the continuous time accumulated at each point along the path. A few experimental studies investigate wheelchair use in simulated environments. They use treadmills or dummies attached to wheelchairs, rather than human wheelchair users in actual workplace settings. None of the existing methods can make direct and continuous measurements of wheelchair use over paths that connect different areas in the workplace. We now appraise the existing techniques for dynamic investigations of this problem.

One might use aerial movies to measure wheelchair paths through natural environments. Unfortunately, aerial techniques prove

impractical for several reasons: One must mount several cameras at heights greater than the ceilings usually available. Filming or taping the wheelchair's path requires an extremely wide angle lens that introduces gross distortions. This makes exact measurements problematic. Finally, the filmed or videotaped record from each camera must be digitized and their data somehow combined—a costly, time-consuming, and tedious job. Current automatic digitizing programs prove very expensive and are not, indeed cannot be, entirely automatic. Alternatively, automatic sonic digitized recordings are possible. This technique mounts sound emitters on the wheelchair and distributes an array of microphones widely over a prearranged path. As the wheelchair moves its location, speed, and direction are recorded. Such devices work best in limited spaces. They are also subject to serious problems of acoustic reflectance in nonstandardized environments. Neither audio nor video techniques work well in environments cluttered with furniture and other architectural barriers. These methods restrict the wheelchair routes to those within range of the camera or emitters. Consequently, they are unable to measure the user's natural preferences for routes that fall outside the predetermined set. These methods are thus less practical, more costly, and more restricted than the method to be proposed.

In short, we know of no current ecologically valid method for "in-the-field" measurements. Such a method is indispensable to the design of functional spaces in which wheelchair users perform daily a wide variety of diverse activities. A chief concern is that although ordinary environments may be sufficiently clear for walkers, they are usually not barrier free for wheelchair users. Hence, design criteria must differ for environments that allow these different modes of locomotion. Dynamic measurement of clear movement through such environments requires the development of new tools and techniques. We have built a prototype of a new measurement instrument and tested its feasibility which we describe next.

11.2.4 A New Technique for Measuring Dynamic Fit of Active Wheelchairs

For this project we have developed a unique tool and associated methods for determining the relevant variables for the fit of wheelchair activities following paths through functional spaces. Following Gibson and Crooks (1938), one of our aims is to discover how users perceptually select fields of comfortable (safe and efficient) travel

within fields of possible travel. A current series of experiments are designed to uncover the informational basis and the stable styles of control by which wheelchair users navigate successfully through doorways and passageways. In the heart of this research is a typical wheelchair that has been modified to gather data online, while running diverse routes through such environments. A prototype of the computerized wheelchair has been built and tested. These exploratory experiments are described later. Here we use the computerized wheelchair to examine a range of doorway and passageway width tolerances under a variety of experimental conditions with different velocities, widths, distances, and intentions. Our ultimate aim is to discover the relevant information variables, control parameters, and values or goals, whose interrelationships define the dynamic field that moving wheelchair users carry with themselves as they move about (Gibson & Crooks, 1938). For convenience, we might call these three kinds of parameters—observables, controllables, and valuables—the *affordables*¹ of the task situation.

This prototype computerized wheelchair has an on-board, laptop computer that reads data from optically encoded accelerometers. The accelerometers are attached to a pair of measurement wheels that have been added to the undercarriage of a standard wheelchair between the large wheels by which the vehicle is steered and driven. This instrumentation allows the wheelchair's changing locations, orientations, and velocities to be automatically recorded at each point along the route traversed (within tolerances of approximately 2 mm for each 100 cm of lineal distance traveled). The existence of this first prototype demonstrates the feasibility of the concept, the soundness of

¹Gibson (1979) introduces the term *affordance* for those objective properties of environments which, taken in reference to an actor, provide opportunities for that actor's actions (i.e., goal-directed behaviors). It is one thing, however, to have an opportunity for action and another to have the means to seize that opportunity; hence, the necessary distinction between affordances as *opportunities to be seized* and effectivities as the means by which such opportunities may be seized. Similarly, it is still another thing for an actor to value one opportunity more than others so that it and not they is intended and acted on—that is, to select, on a given occasion, one affordance as being more valuable than another; hence, the need for the term *valuable* to pick out the intended affordance. For these reasons, we introduce the term *affordables* to refer to the component parameters (observables, controllables, valuables) by which affordances, intentions that select them, and the effectivities that realize them, may be conveniently referred to under a common rubric.

the hardware and software designs, the practicality of the engineering and production standards used, and a reasonable cost-effectiveness.

For these reasons, we believe that the computerized wheelchair has more face validity for the purposes of relatively unrestricted dynamic measurements in natural settings than any other currently available method. Furthermore, in field testing, the first prototype has functioned successfully, although improvements might be made. For instance, in the future we hope to develop a second prototype that will have strain gauges added to the wheel hubs which are connected to the circular handrails. This will allow manual forces applied to the wheels to register the torques applied to each wheel. Because it is these manually applied wheel torques that both drive and steer the wheelchair, these modifications will allow direct measures to be made of both the kinematics and kinetics of the vehicle's navigation through naturally cluttered environments. However, in what follows, we base our analyses solely on the kinematic description of wheelchair activities because these data are all that is currently available to us.

The approach to be taken clearly recognizes that adequate environmental architectures for wheelchair users can not be merely normal designs with add-on allowances for a "special population" of users. Rather, moving wheelchairs require environments with fundamentally different designs than those that accommodate nondisabled walkers. Clear passage through shared functional spaces for the latter is not necessarily clear passage for the former. The economics of design and construction, however, must remain realistic. If minimal standards are too excessive, then costs of construction will be wasteful, perhaps, even exorbitant—setting up but another barrier to society's commitment to provide realistic accommodations for wheelchair users.

To reiterate: Our aim is to develop accurate techniques for dynamic measures of functional spaces for wheelchair activities that avoid intuitive overestimations. Such techniques could contribute significantly to lowering the economic barrier to mainstreaming wheelchair-bound disabled individuals into society at large. Of course, merely having an instrument that is appropriately designed to measure the required dynamic fit is not enough. One must also have a theory that clearly indicates which measurements should be made.

In the next section, we develop the background for the scaling (ecometric) problem and the transduction (ecomechanics) problem. With this background in place, we are better able to formulate an answer to the question of how to measure the similarity of information and

control required for goal-directed actions to be successful.

11.3 Part II: Theoretical Background: Dynamic Fit as an Ecometric Problem

In this section we introduce some of the basic concepts that underlie ecometrics and ecomechanics. Such concepts will be involved in any attempt to solve the scaling and transduction problems associated with actors (e.g., wheelchair users) navigating successful goal paths.

Consider, first, some properties an environmental situation must have to afford an actor navigating through a cluttered environment: The medium that surrounds the actor must permit freedom of movement, and the actor—who may be a walker, flyer, swimmer, or vehicle user—must maintain mechanical contact with a surface of support. For swimming and flying creatures, the medium and surface of support are the same—water and air, respectively. For arboreal and land creatures, the medium is air or water, and the surface of support is tree, rock, ground, floor, stairs, or sidewalk. If the actor is neither arboreal nor aquatic, say a human, horse, or dog, then the surface must be sufficiently rigid to support the actor's weight, have sufficient friction to allow traction, and be sufficiently level to prevent falling over. Environmental properties that afford opportunities for actions—that is, goal-directed activities—are called *affordances* (Gibson, 1979).

Examples of environmental affordance properties are the *graspability* afforded by certain objects, the *supportability* afforded by certain surfaces, and the *edibility* afforded by certain substances. In general, properties of objects or surfaces count as affordances if they provide appropriate structural and informational support for the action capabilities of properly attuned actors, that is, actors who have the means, opportunity, and motivation to carry out the relevant actions. Hence the definition of an affordance necessarily implicates corresponding action skills, called *effectivities* (Shaw & Turvey, 1981; Turvey & Shaw, 1979). Such effectivities determine whether a specific class of actors, for whom information specifying the relevant affordance property is available, can use that information to realize that affordance property, that is, to guide its behavior successfully toward an intended goal.

Table 11.1 shows examples of how affordances, effectivities, and actions have an underlying dimension of abstract similarity. Affordances and effectivities are functionally defined and functionally

similar under the action of a sufficiently skilled organism relative to appropriate environmental structures. The affordance refers to the scale-dependent aspects of the physical situation that support an opportunity for a definite action; effectivities refer to the commensurate means available to the actor for realizing that definite action in that given situation. The actor's intention to act toward realization of that affordance goal in the given situation provides the motive to act. As in a court of law, when all three conditions—means, motive, and opportunity—are met, then we may conclude that the agent has a realistic intention to act. Under this interpretation, an action is necessarily goal-directed and intentional, involving the goal-specific affordance and the intention-specific effectivity.

TABLE 11.1: *The Interrelationship of Affordances, Effectivities, and Goal-Directed Actions.*

AFFORDANCE OF E (opportunity for action)	EFFECTIVITY OF O (means for acting)	ACTION OF O ON E (realizing intended affordance)
graspable	able to grasp	O grasping E
climable	able to climb	O climbing E
catchable	able to catch	O catching E
passability	able to pass through	O passing through E
sit-on-able	able to sit	O sitting on E

A wheelchair is a vehicular tool, that is, a tool that aids locomotion. Hence, whatever similarities carry over from affordance information to effectivity control must do so through the tool that interfaces the actor-as-perceiver with the environment-as-perceived and acted on. But what happens to the affordance description of the environment and the effectivity description of the actor when tools as objects and tools as functions enter the story? We address this important question next.

11.3.1 Tools as Ecological Interfaces

Tools enhance, extend, or restore the action or perception capabilities of humans or animals. They may assist in manipulation, locomotion, or exploration. Tools may extend existing capabilities or restore lost functions either partially or completely. Tools may be simple machines (e.g., levers, ramps, pulleys, springs) or complex tools (e.g., trucks, bulldozers, automatic assembly lines, refineries) that amplify the capacity for work, or they may be devices that amplify information detection (e.g., microscopes, telescopes, sonar, radio, television) or amplify its usage (e.g., computers, libraries). Currently, it is still an open question whether computational tools may also amplify intelligence. They surely carry out, in many ways, the tasks that would otherwise take an intelligent person to do. In short, tools may focus or extend effectivities of humans, thereby providing greater access to affordances in the environment and, consequently, new opportunities for action.

Tools may also serve a prosthetic function by restoring the effectiveness of lost or infirmed sensory abilities (e.g., eyeglasses or hearing aids), or they may restore lost or infirmed action capabilities. Some action prostheses restore manipulatory abilities, such as the artificial hand or arm; others restore lost or infirmed locomotory abilities, such as crutches, walkers, artificial legs, and wheelchairs. Manually driven or motor-driven wheelchairs fall into the category of vehicular tools. Little if any work, to our knowledge, has been done on the formal description of such tools and how they alter the dynamic fit of the disabled user to that environment in which they are typically used. Moreover, achieving empirical validation of the dynamic fit of vehicular tools might be furthered by considering how people navigate other vehicular tools, such as automobiles. We consider such comparisons later. A natural question is how does an object with a tool function relate to the affordances of the environment and the effectivities of the actor? Does the tool act as a mediator between the environment's affordances and the actor's effectivities. Or, if the tool does not sit in the seam between the environment and the actor, does it belong more to one component of the ecosystem than the other?

Figure 11.1 shows the relationship that a tool might have to the user and the user's environment in which it is applied. Notice that the tool might be treated as a mediating device which belongs neither to the organism nor to the environment, but interfaces the two. This interpretation seems to overly complicate the relationship between organisms and their environments and to potentially destroy the

theoretical balance that is needed to keep affordances and effectivities mutual and reciprocal (i.e., mathematically dual) as illustrated in Table 11.1. This theoretical balance is desirable because it allows the relationship between information and control to remain epistemically direct for all the reasons that we have discussed at length elsewhere (Shaw & Turvey, 1981; Turvey & Shaw, 1979; Turvey, Shaw, Reed, & Mace, 1981). Mathematically, this directness in the coupling of the components of the ecosystem allows us to use some powerful theorems from (self-adjoint) information/control theory that permit an economy of description for modeling psychological ecosystems, affordances and effectivities, information and control, and the perceiving-acting cycle that would not otherwise be possible (Shaw et al., 1992; Shaw, Kugler, & Kinsella-Shaw, 1990). There seems to be another alternative which keeps intact our fundamental assumptions.

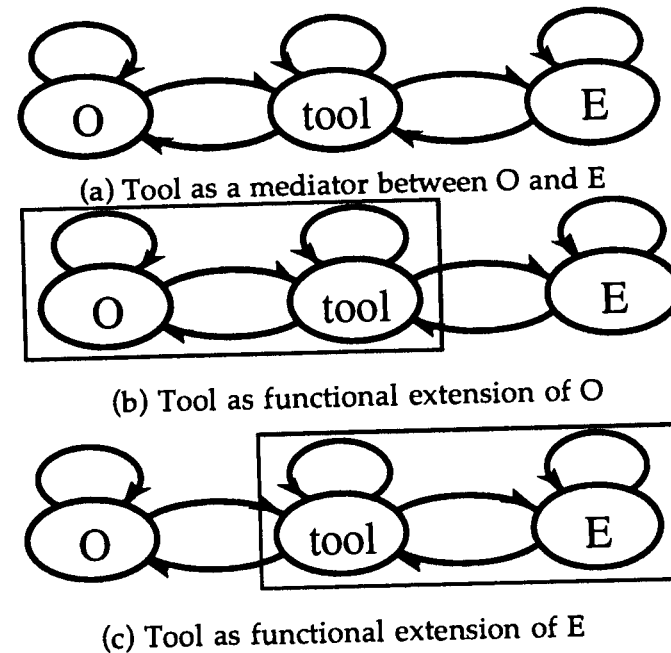


Figure 11.1. Possible ways to partition the role of a tool in an ecosystem: (a) as a mediator, where $E \leftrightarrow T \leftrightarrow O$; (b) or as an extension of environmental affordances, where $(E \leftrightarrow T) \leftrightarrow O$; or (c) or as an extension of an actor's effectivity, where $E \leftrightarrow (T \leftrightarrow O)$.

We prefer to distinguish between tools-as-objects and tools-as-functions. As objects, tools must be treated as contributing to the affordance structure of the environment. That is, before the tool is used, like any object it has its own affordances, thereby inviting certain actions. But tools-in-use are no longer just objects but play an intrinsic role in extending, replacing, or restoring the user's repertoire of effectivities—that is, the actor's capabilities for realizing affordances of the environment. Tools, therefore, have a *dual* function—as objects within the environment's affordance structure and as components within the actor's effectivity system. Through the dual function, a tool both scales and transduces the information detected and the control executed in the course of the task involving the tool use.

For instance, on the ground a steel rod may afford grasping, lifting, and wielding. But once grasped and properly braced under a boulder, the rod becomes a lever and is part of the effectivity for leveraging the rock causing it to roll down a hill. Hence, to call this object a rod is to distinguish its role in the affordance structure of the environment, whereas to call it a lever is to distinguish its function as an integral part of an effectivity system. There is clearly a logical distinction between the affordance structure of an object that invites a goal-directed function and the effectivity, or goal-directed function, it invites. These concepts should not be confused. The former refers to the environment as a source of goals that an organism might achieve, whereas the latter refers to the organism as a source of intentions that achievement of environmental goals might satisfy. Hence, there is no ambiguity introduced by dually partitioning the tool, first, as a part of the functionally defined environment with affordance properties and, then, as part of the functionally defined organism with effectivity properties, as shown in Figure 11.1b and 11.1c.

Consider the methodological benefits of recognizing the dual functional roles that tools play: As objects they have affordances like any other, but as tools they engage effectivities, thereby permitting actors to perform tool-specific, goal-directed functions. Thus, a tool function of an object contributes, in a unique way, to the affordance-effectivity compatibility underlying an actor's fit to its environment. Furthermore, and here is the main point, because tool use requires a high degree of coordination between perception and action, *tools focus behaviors*. When theoretical description and empirical study might be intractable at the microscale level of the enormous number of degrees of freedom exhibited by neuromuscular acts, both theory and experiment become more tractable when only the macroscale degrees of freedom of

the tool using system is considered.

This suggests a way that one might develop a manageable inventory of solutions to the degrees of freedom recognized by Bernstein (1967). The scientific study of tool functions involved in manipulatory and locomotory acts might identify more tractable analogues of their corresponding, more complicated, unaided goal-directed actions. A study of these "free-action" analogues might provide a theoretical basis for better understanding the underlying effectivities that underwrite them.

If tools promote the fit of actors to their environments in ways more visible than tool-independent behaviors, then studying activities involving wheelchairs may help us understand the coordination requirements for successful locomotory navigation in general. For describing the behavioral interface of the wheelchair to the floor surface is much simpler than describing the free actions of a walker. Of course, so far as specifics are concerned, the study of one is not a substitute for the other. Even so, as related locomotory cases, they are equally interesting examples of how similar systems may exhibit intentional dynamics under different navigational demands.

For these reasons, it would be important to have a way of measuring the degree to which tools promote the common fit of actors to the task environment, as compared to the same actors without the tool. It seems reasonable to assume that use of the same tools by different actors performing the same tasks increases the task constraints so that they become dynamically more similar as intentional systems. We turn next to a discussion of various ways in which systems (actors plus tools) might be similar in performing goal-directed behaviors.

11.3.2 Similarity across Physical Systems

Assume there are two systems, each with the same tool interface with the environment but different masses. If the tools and tasks are similar, then their behaviors should be similar—differing by only a scale factor reflecting their different masses. For instance, imagine two wheelchair users of different weights who must select efficient paths for navigating through a room cluttered with furniture so as to exit through a doorway. If they travel at the same speed (are kinematically similar), the heavier actor (greater mass) will require larger turning radii because he or she will have the greater momentum (mass \times velocity). In spite of the fact that the two cannot travel the same curvilinear paths at the same

velocity, there are still numerous ways that they may perform their tasks similarly. They may cover similar distances to get to the doorway, take similar amount of time to do so, go in similar directions, use similar work (i.e., proportional to their masses), and so forth.

Table 11.2 shows the dimensions involved in the various kinds of similarity that one system might have to another with respect to the paths they follow in navigating through an environment. (Similarity means alike up to a scale factor). Geometric, kinematic, and kinetic path similarity hold for systems that have proportional relationships among state variables involving the dimensions of L, LT, LTF, or DTF, respectively. Likewise, work, impulse, and torque similarity hold if the two systems exhibit proportional path functions (integrals) involving these same dimensions in which each dimension is multiplied by force (i.e., $\int FdL$, $\int FdT$, $\int FdD$, respectively). Specifically, geometric path similarity means the two systems cover similar distances between homologous environmental locations; kinematic path similarity means the two systems arrive at homologous places on a similar temporal schedule; and kinetic path similarity means the two systems apply similar forces to move over similar distances (similar work forces), times (similar impulse forces), and directions (similar torque forces).

Table 11.2: Various Kinds of Physical Similarity Two Systems Might Exhibit.

Similarity over	length (L)	time (T)	angle (D)	force (F)
distance	√			
time		√		
direction			√	
work	√			√
impulse		√		√
torque			√	√

Perceptual similarity involves a common proportionality defined over information variables detected by the systems, whereas actional similarity involves a common proportionality over control variables used by the two systems to produce their respective behaviors. Finally, intentional similarity means the two systems share similar goals (i.e., target parameters, manner parameters, or both). When the goalpaths pursued by the two systems are similar, then we say that they are similar with respect to their intentional dynamics. We now describe how this works.

Table 11.3 indicates the intentional dynamical similarities that two systems might exhibit in their goal-directed performance, in which the physical similarities defined in Table 11.2 refer to measures indicating how the path is generated from the initial condition (start-up) to some final condition. This path may or may not reach the target nor do so in the manner of approach intended. By contrast, the intentional dynamics similarities, defined in Table 11.3, refer to measures indicating how the path ought to be generated to conform to an intended final condition (reaching the intended target in the manner intended). Hence, Table 11.2 refers to measures of the causal generation of the mechanical path indifferent to the goalpath, whereas Table 11.3 refers to measures of the intentional control required to generate the ecomechanical path, that is, the intended goalpath. The former is a "blind push from behind" by

Table 11.3: Various Kinds of Intentional Dynamical Similarity Two Systems Might Exhibit.

type of similarity between systems	information (target parameters)			control (manner parameters)		
	distance to contact (δ)	time to contact (τ)	direction to contact (γ)	work to contact ($F \times \delta$)	impulse to contact ($F \times \tau$)	torque to contact ($F \times \gamma$)
target	√	√	√			
manner				√	√	√
goal-path	√	√	√	√	√	√

forces determined retrospectively by physical law as a function of the past states of the system; the latter is a 'directed pull from in front' determined prospectively by the actor's control law being provided with (perceptual) information about how the observed future states of the system reckon with those intended. For example, *time* as a variable for physical dynamics tells us how much time has elapsed since the system left its initial start-up state given where it is; whereas *time-to-contact*, as a variable for intentional dynamics, tells how much time should elapse before the system reaches its intended final state, given where it is. A similar prospective interpretation should be given to the other "contact" variables.

The importance of extending physical similarity theory to include intentional dynamical similarity is to allow measures of the similarity between intentionally driven systems analogous to the similarity measures possible for causally driven systems. Dimensionless analysis, an outgrowth of dimensional analysis and similarity theory, provides a way to construct measures of physical similarity between the observed characteristics of two systems as well as between a single system's observed and intended characteristics.² Both of these similarities will be of interest to comparing systems with both physical as well as intentional dynamics. The general nature of these measures is discussed next.

11.3.3 Dimensionless Ratios as Measures of Similarity

Dimensionless analysis developed from dimensional analysis: the classic parameter approach introduced by Buckingham (1914) and Rayleigh (1915), systematically developed by Bridgman (1922), reformulated by Drobot (1954) to recognize dimensionless functions, and used by Schuring (1977) to emphasize law-based similarities. Dimensionless analysis serves two important functions for model construction and theory. First, it puts a limiting framework around the model to be constructed. This framework is based on the necessity of any functional relation to remain invariant if the units are changed. A second important function capitalizes on the fact that any physical system can be analyzed into functions of a limited number of

²The notion of ecowork and affordance-effectivity duality as dimensionless quantities, were introduced in a series of lectures on ecometrics given in the late 1970's at the University of Connecticut by R. Shaw, first reported in Warren & Shaw (1981), and given experimental application by Warren (1982).

dimensionally fundamental variables that define its equation of state. The number of independent dimensionless groups, as proven under the π -theorem, is equal to the difference between the number of variables that make them up and the number of dimensions involved (Ipsen, 1960). We explore these two functions of dimensionless analysis in the context of the tasks involving the perceptual control of actions required to perform wheelchair navigation tasks and other locomotory tasks. Precedents for using such an approach in psychology have been set by Warren and Shaw (1981), Kugler, Kelso, and Turvey (1980, 1982), and especially applied to ecological psychology by Shaw and Warren as reported in part by Warren (1982).

The physical dimensions of interest, as discussed earlier, are those of length (L), time (T), direction (D) and force (F). The L, T, D, F system of variables are fundamental in that they can be used to define any other variables, called *derived variables*. Using these fundamental dimensions, the respective functions describing a pair of systems can be compared so as to reveal any similarities that might exist. Such similarities may exist at various levels of analysis with respect to the state equations of the systems

In order for a comparison of the equations for two systems to be legitimate, however, certain formal criteria must be satisfied. One important criterion, discovered by Fourier in 1822, is that such equations be *dimensionally homogeneous*, that is, be expressed in the same dimensions. (Consequently, a solution to the scaling and transduction problems introduced earlier depends on this property holding between the equations describing the information and those describing the control as exhibited by the perceiving-acting cycle involved in a given task.) On initial inspection, however, two equations may not appear to satisfy *dimensional homogeneity* because they may be expressed in different derived variables whose symbols are quite distinct. However, by reducing derived variables to the fundamental dimensions of F, L, D, T (or the corresponding fundamental dimensions, F, δ, τ, γ , for contact variables), it is possible to determine if two systems satisfy this homogeneity property. For example, in Newton's famous law (stated in a form to emphasize its dimensional analysis), $F = MA$, although mass and acceleration appear as derived variables, they can be expressed as a function of the fundamental dimensions so that $A = LT^{-2}$ and $M = F(LT^2)^{-1}$.

The most important progress in the study of similarity among systems resulted in the development of the so-called π -theorem (Buckingham, 1914), one of the great achievements of similarity theory.

This beautiful theorem asserts that any analysis of a physical phenomenon in dimensional terms can always be reduced to a simpler functional relationship among dimensionless variables (Drobot, 1954; Rosen, 1978; Stahl, 1963). An appreciation of this theorem is necessary if one is to understand how systems too complicated to be described in terms of mathematically closed formulae (e.g., differential equations) might nevertheless be compared in terms of more abstract levels of similarity. The worth of this approach was summarized by Stahl (1963) as follows:

"It is reiterated that similarity criteria may always be obtained in a simple manner from governing differential equations, when such equations are available. When clear differential formulations are not at hand much progress can be made by the study of simplified model formulations of the problem, followed by an apparent dimensional 'integration' which gives the pertinent similarity criteria without actually performing the numerical integration process. In other cases there may be no differential equation which appears appropriate and one can choose similarity criteria on the basis of prior experience and direct manipulation of dimensional variables." (p. 369)

This is not the place to go into details regarding this approach, but fortunately several lucid accounts are available that the interested reader might consult (Birkhoff, 1960; Duncan, 1953; Johnstone & Thring, 1957; Langhaar, 1967; Sedov, 1959; and in the area of similarity of biological systems, see especially, Günther, 1975; Rosen, 1978; Stahl, 1963). To give the spirit of dimensional analysis and similarity theory, and to show how dimensionless numbers naturally arise in the former and have application in the latter, consider a classical example—the case of the generalized Reynolds number.

An Example: The Reynolds Number. In 1638 Galileo made initial contributions to similarity theory by noting that static objects, such as pillars, tree trunks, and animal's legs, should scale area of support as a function of volume of the mass supported. Newton went still further by recognizing that laws of nature should be formulated so as to be independent of certain dimensions, such as the overall size of objects. Finally, similarity theory moved to a new plateau of abstraction and generality when it was recognized that one might avoid specific dimensions altogether by composing variables from the ratios of two variables which were not just identical in dimensionality, but whose dimensions canceled. Such variables, when evaluated, came to be

known as "dimensionless numbers" —the most famous of which is the Reynolds number.

Specifically, a variable is *dimensionless* if it is formed by a ratio of two quantities measured in units of the same dimension so that these units cancel out of the numerator and the denominator, leaving a pure ratio. A so-called π -number is a numerical evaluation of a dimensionless variable. The numbers receive their name from the fact that the well-known geometric ratio $\pi = 3.1415$ is itself a dimensionless number, one achieved by dividing the circumference of a circle by its radius, where both are measured in the same units. Hence, the units cancel leaving a dimensionless π -number. Quantities in which this is so are said to satisfy the property of *dimensional homogeneity*.

The Reynolds number, perhaps the most famous example, derives from the Navier-Stokes equation which describes the behavior of fluids. In this context, the Reynolds number expresses abstractly the ratio of inertial forces to viscous forces acting on a small volume of fluid. To be more specific, the Reynolds number is composed in the following way (Here, for convenience, mass, M , is used as a fundamental rather than a derived dimension):

$$vL\rho/\eta$$

in which v is velocity (LT^{-1}), L is characteristic length, η is viscosity ($ML^{-1}T^{-1}$), and ρ is density (ML^{-3}). If we substitute these fundamental dimensional quantities in this formula, it confirms that Reynolds number is dimensionless; namely,

$$(LT^{-1})L(ML^{-3})/(ML^{-1}T^{-1}) = LLLMTL^{-3}MT = MTL^3M^{-1}T^{-1}L^{-3} = [1].$$

(Brackets indicate a dimensionless quantity rather than an integer.)

The use of dimensionless numbers in science is exemplified by applying the Reynolds number in biology. For instance, it has been shown that hemodynamical systems (blood flow) are similar across the circulatory systems of a variety of animals because they all share fundamental similarities with hydrodynamical systems in general (Stahl, 1963). In general, a rather large set of invariant dimensionless numbers have been found that seem to hold for complex multivariable biological systems. These numbers specify relationships of these otherwise diverse systems that are independent of specific parameters such as size. One theorist goes so far as to claim inductively that "behind the complexity and astonishing variety of forms and functions quantitative criteria of

similarity for all living systems have been disclosed" (Günther, 1975, p. 660).

11.3.4 Dimensionless Measures of Similarity Between Information and Control Processes

In the same spirit, the Gestalt psychologists claimed that abstract structure can be recognized, even though transposed from one situation to another (Koffka, 1935). Similarly, Gibson (1979) has spoken of invariant information as being both "timeless" and "formless," that is, as being independent of the specific parameters of a given environmental situation. Recent evidence suggests that information underlying perception of environmental structure must be very abstract. If what comes in on the perception side is to be of use to what goes out on the action side, then there must be a similarity of goal-specific information detected to the control which is guided by that information. For example, a cat must detect the target information, the direction and distance to be jumped, in order to land on a perch, say, the top of a fence. In the information picked up both from the environment and from its own body in that environment, there must be a reasonably precise specification of the torque required to aim itself and the impulse forces required to do the necessary work of transporting its body mass to the target. These general requirements for any successful action by any actor can be summarized by using the concept of affordables introduced earlier.

The observables present in the perceptual information specify the affordance goal to be selected (e.g., the fence's perchability). The affordance's observables are then transduced into ecometrically scaled, ecomechanical values of control parameters governing the biomechanical degrees of freedom, or controllables. If these action-control parameter values define a successful goal-directed function, or effectivity, for accomplishing the intended goal (e.g., jumping to the fence), then they comprise the set of values, or valuables, sought. More generally: If the mapping of the observables over the dually scaled controllables allows an actor to transduce its energy into the intended valuables, then an opportunity for action is successfully seized. This is the condition that allows the affordance and the corresponding effectivity to be duals, and as such it is guaranteed that the information and control are sufficiently similar so that the former can be both transduced and scaled into the latter.

Indeed, a target's observables must be sufficiently abstract to be

invariantly transposed from the particular form of information they must take to be detected to the particular form they must take to be the controllables that allow a successful action toward that target. This is the basic assumption of ecometrics (Shaw & Kinsella-Shaw, 1988). Mathematically, this assumption entails that a "flow" of information and the "flow" of control over the perceiving-acting cycle together define a *dynamic invariant*, or what physicists call a *conserved quantity*. A formal argument can be made that this generalized abstract quantity, called the *total action potential* (Appendix A), must be conserved if one is to explain how action skills acquired in one situation can generalize to other similar situations (see Shaw et al., 1990, for details).

Furthermore, the conserving of this total action potential whenever a goal-directed behavior is successful has been mathematically shown to entail that an inner (scalar) product invariant (Appendix B) must hold between the dynamic flow of goal-specific information and the dynamic flow of the control of goal-relevant energy expenditures (for details, see Shaw et al., 1992). In other words, the information detected over the path to a goal must determine the path over which the actor controls its manner of approach to the goal—otherwise, either the target moved toward is not the intended target or the manner of approach is not the one intended. In either case, if there is an error in information detection or control, the path traveled will not be the intended goal path.

Proving the existence of a conserved quantity, or dynamic invariant, as a condition for success of goal-directed behavior might be called *the fundamental theorem of ecometrics*. Showing empirically that this theorem does in fact hold in a variety of experimental contexts for different actors is very important for motivating research into systems that exhibit intentional dynamics, that is, goal-directed behaviors. For the theorem asserts that an abstract similarity must hold over the perceiving-acting cycle whenever an actor successfully uses perceptual information to achieve an intended action goal—a statement tantamount to the claim that a dimensionless quantity exists which is a kind of *intentional dynamical invariant* and may be constructed over the relevant task affordables. These are clearly π -numbers at the ecological scale rather than an arbitrary physical scale, as illustrated in the following example.

Consider two cats who satisfy their respective intentions equally well by jumping to the top of a fence. Let one cat be more massive than the other. If we construct the equations for each performance (analogous to Reynold's equations for flow), the control and information variables should be similar (just as the inertial and viscous forces were similar). The pair of resulting ratios of these similar quantities will

produce a π -number that is the same for both cats, being indifferent to the difference in their masses. The total action for a given cat comprises certain functions of the information detection and energy control that maintain an invariant relationship just in case the cat's behavior is successful. When the two cats' behaviors satisfy similar goals, then other similarities should hold as well. For instance, the parabolic paths through the air to the fence top should be geometrically similar in shape and kinematically similar in the time to traverse the paths; likewise, the work-to-contact with the fence top should be kinetically similar, and so forth. The fact that they also have the same targets and similar manners of approach to those targets and hence similar goal paths should be reflected in the dimensionless ratio. Hence, the similarity of their goal-directed tasks with respect to both ordinary causal dynamics and intentional dynamics should be revealed by their respective π -numbers being identical. The degree to which these numbers are not identical provides a measure of the dissimilarity of their actions. This is the promise of dimensionless analysis that we wish to explore in tasks involving different actors performing the same or similar locomotory tasks.

Our research project, therefore, ultimately aims at determining the empirical validity of the fundamental theorem of eometrics, as sketched earlier. As a means for exploring this thesis of intentional dynamics, there is no problem more scientifically interesting nor socially significant problem than wheelchair navigation. In the next section, we outline the method of attack on this problem and present some recent encouraging results. It should be clearly noted, however, at the outset that although these results obtained encourage the use of π -numbers, they also indicate that the current use has been too limited to solve the prospective control problem in its most general form (e.g., locomotory navigation toward a goal through a cluttered environment). Consequently, after reviewing the promising but limited results of current research by ourselves and others, we shall discuss the mathematical foundations of π -numbers. Our goal will be to suggest the ways of removing these limitations so that π -numbers might be generalized to measure similarity of any actions by systems operating at the ecological scale under the aegis of intentional dynamics.

11.4 Part III: Recent Investigations of the Fit of Actors to their Environments

11.4.1 π -numbers of Different Orders

Anthropometry is the study of the design of environments, furniture, tools, and equipment in units proportional to the intrinsic measurement of the human body (Panero & Zelnik, 1979). The architect Corbusier employed anthropometric principles, on a large scale, in the design of cities and buildings and, on a smaller scale, in the design of furniture and equipment. The famous Bauhaus initiated exploratory investigations into such design principles in the 1930s. Since then, the anthropometric approach has become an integral part of all design disciplines, especially architecture, human factors, and human engineering. Anthropometric π -numbers are those dimensionless ratios based on body-scaled measurements that play a role in the design and evaluation of environmental structures, furniture, tools, and equipment. Clearly, anthropometric π -numbers constitute an important class of geometric π -numbers. Very recently the empirical investigation into the validity of such geometric π -numbers has grown in popularity.

Specifically, in our laboratory and elsewhere, we use π -numbers to express similarities between energy control and information detection. As argued earlier, under the basic eometric theorem of intentional dynamics, such similarities should exist whenever actors must make compensatory motor adjustments, given goal-specific information, to carry out successful goal-directed activities. Such compensatory motor adjustments, under the control of perceptual information, specify the actor's fit to the relevant environmental structures. This is the pragmatic meaning of the perceiving-acting cycle (Gibson, 1979).

A decade ago, as his dissertation, Warren carried out a now famous study exploring the kinetic basis for anthropometric (geometric) π -numbers (Warren, 1982, 1984). He showed that for people of different heights the perceived optimal stair design was specific to the person's individual height defined as a function of leg length. (Here the measure of optimal stair design is the riser-to-tread ratio taken relative to the climber's leg length.) Nevertheless, the ratio of leg dimensions to stair dimensions for all people (riser height in cm/leg length in cm = r/l)—short and tall ones—were identical ($r/l = \pi_p = .88$). These people then had to climb a variable motorized stairmill. The stairmill ran at speeds requiring a wide range of step frequencies (30 to 70 cycles/min), while the subjects climbed as comfortably as possible. Warren obtained an

important result. Subjects produced an invariant optimal work measure ($\pi_a = .26$, as measured in calories/kg-cycle by oxygen utilization methods) only for those designs that matched the dimensionless number obtained from their original perceptual judgments.³ This research showed convincingly the existence of anthropometric π -numbers that hold predictably over both perceptual and action measures (π_a and π_p). How general are these π -numbers?

Konczak, Meeuwse, and Cress (1992) investigated stair designs with maximum riser heights. They asked whether leg strength and hip-joint flexibility provide additional relevant constraints on both the perceptual judgment and action capability of subjects regarding such stairs. Thus, their work generalized Warren's conditions. Warren measured only anthropometric factors, omitting the effect of different dynamic conditions (such as different speeds) and used only young adults of the same approximate ages. Konczak et al. (1992), by contrast, showed that younger and older adults differ in the perception of maximum climbable riser height. In other words, adults can accurately perceive the relative limitations and, presumably, the change in their action capabilities that aging brings. Specifically, their analysis showed that one's action capability (in stair climbing) is subject to multiple biomechanical factors (that is, kinetic π -numbers). These factors, taken together with anthropometric constraints, are better descriptions of the action capability than the anthropometric constraints (geometric π -

³The action π_a -number, on first consideration, may not seem to be truly dimensionless as it must be in order to be a π -number since it involves a kinetic quantity (minimum energy expenditure per vertical meter measured in units of calories/kg-m) taken in reference to a geometric quantity (leg length). However, π_a is found by, first, plotting energy expenditure, E_d , as a function of riser height, R , and, then, taking R_o , the riser height value at which E_d is minimal, as numerator and leg length, L , as denominator to obtain $\pi_a = R_o/L = .26$ for all people regardless of height so long as the stair design is anthropometrically optimal for them. This was Warren's (1982) method. A more general method exists for computing π -numbers from dimensionalized measures that are dimensionally inhomogeneous. If one takes the measures in two different situations, as over learning trials for an individual, a test-retest, or over different individuals, dimensionless ratios can be constructed from measures that fail to satisfy the property of dimensional homogeneity. Let the two measures taken in the first situation be k_1 and k_2 , dimensionalized as MLT and LT , respectively; and where k'_1 and k'_2 are the same measures taken on a different occasion. We can then form a dimensionless quantity by taking their cross-ratio as follows: k_1MLT/k_2LT .

numbers) taken alone.

By using π -numbers, these experiments make an exciting discovery. They show that the control of energy expended for action (kinetic π -numbers) is formally similar to the information available through visual perception (geometric π -numbers). But where is the dimensional homogeneity required? It must be supplied in some way, for it seems that people can "see" the work to be done, the impulse forces to be scheduled, and the torques by which to direct their behaviors toward selected goals in the manner intended. Perhaps, the underlying ecometric commensurability required for this to be so is a similarity relationship existing between something felt and something seen. Could it be the similarity between the "felt effort" experienced during the act of climbing the stairmill and the information detected in seeing the stair design? Is this not the source of the commensurability of haptic information with visual information required for their dimensional homogeneity? We return to this point later.

Other approaches have also successfully constructed π -numbers for different activities than stair climbability. Each of these is constructed from a ratio of an anthropometric variable with an environmental variable, defining what we call an *ecological π -number*. In principle, such ecological π -numbers may be anthropometrically based (geometrical π -numbers), biomechanically based (kinematic π -numbers), bioenergetically based (kinetic π -numbers), or intentionally dynamically based (ecological π -numbers). Researchers have identified a range of π -numbers in different scientific domains. In psychology, following Warren (1982, 1984), Mark (1987) found geometrical π -numbers, whereas Lee (1974) and Todd (1981) found kinematic ones. As pointed out, in at least one case these numbers have been validated both for perceptual judgment and for physiological measures of action under normal conditions. These findings suggest that the human perceptual system is a fine measuring device for different dimensions of locomotions, such as climbability (Warren), sittability (Mark), catchability (Todd), brakingability (Lee), and passability (Warren & Wang, 1987).

In addition, we have extended this methodology to wheelchair passability studies. We review these results next.

11.4.2 Wheelchair Passability Studies

With the exception of the work on driving (e.g., Gordon, 1966) or braking automobiles (Lee, 1976), or flying and landing of airplanes (e.g., Warren, 1991), research on the dynamic fit of actors to their

environments has been restricted to the investigation of natural biological motions, that is, movements unassisted by prosthetic or vehicular tools. Humans, and even some animals, use tools successfully as substitutes to increase their innate or learned abilities. The success of these tool-using activities naturally suggests the possibility that π -numbers might be used to construct scales that measure the dynamical fit of human activities to their functionally (affordance) defined spaces.

The General Hypothesis: Function Unit Scaled Actions. In the initial research, we restricted our efforts to the careful measurement of some of the chief dynamic variables that affect the observed value of π -numbers for wheelchair passability with respect to doorways and passageways. Our working assumption is that such π -numbers will be most stable when the behavior engaged in is most comfortable. The orientation we take to this problem is a functional one and was anticipated by Gibson (1979). (Please reread carefully the extracted quotes at the beginning of is chapter.)

What one derives from these passages is that the measure of fit of an actor to its environment is not rigid but plastic and functionally variable; the actor's egospace, or field of safe and comfortable action, whether it be manipulation or locomotion, is not restricted to a fixed region of space-time. Because a tool extends the actor's body so that the boundary between the animal and the environment is not fixed at the surface of the skin but can shift. Similarly, this action field is not stationary relative to the environment but moves with the actor, shifting and changing, continually, bending and twisting, and also elongating or contracting to accommodate obstacles that encroach on it and limits its boundaries. Thus, no dimensionless constants may be based merely on static structural units of measure but must be based on functionally defined units.

In the same vein, we wish to argue that a wheelchair, as a vehicular tool, may be considered an extension of the user's body; moreover, it carries with it a dynamically plastic field of safe and comfortable travel—a field that functionally tailors its boundaries to fit whatever constraints that might arise from the actor's dynamics or from the environment. This means tool changes the functionally defined action capabilities of the user, or more briefly, the user's *effectivities* by enabling the user to do things in a way that otherwise could not be done (Flascher & Shaw, 1989). The success of tool using activities naturally raises the possibility of generalizing body-scaled relationships to the environment so that anthropometric measures might be generalized to ecological measures. When the former are geometrically defined, the

latter must be defined as units of measure of the functional fit of the user to the environment—a fit that usually is dynamically scaled. Hence, we call such measures *function unit scaled*. Such measures are to be contrasted with extrinsically defined, static anthropometric measures of fit. But what difference should we expect tool use (e. g., wheelchair use) to make on the calculation of π -numbers?

As indicated earlier, a tool can be defined as an environmental structure the control of which aids an actor in achieving a goal-directed activity. As shown in Figure 11.1, we are interested in π -numbers that are identical when the two systems, S_1 and S_2 , are similar, have similar tool interfaces, T and T' , with similar environmental structures, E and E' (recall Figure 11.1). That is, where $S_1: O+T \rightarrow E$ yields π and $S_2: O' + T' \rightarrow E'$ yields π' , then we expect π , ideally, will be identical to π' . For example, where Warren (1982) found identical π -numbers for people of different body-scales climbing stairs of different but anthropometrically proper designs, so, for the reasons given earlier, we would expect that the π -number yielded by one wheelchair user passing through a doorway of proper clearance will be identical to the π -number yielded by another user in a different wheelchair passing through a different but properly designed doorway.

Several studies of passing through apertures (that is, clear widths) revealed the possibility of a universal ratio of body size and minimal passable gap size. Humans as well as frogs perceive an aperture as affording passage if it is at least 1.3 times their body width (Ingle & Cook 1977; Warren & Whang, 1987). As pointed out, Warren (1984) found that the perceptual category boundary between climbable and unclimbable stairs corresponds to a critical riser height as a function of riser height/leg length ratio. Interestingly, Warren was able to compute a critical, dimensionless number describing this relationship.

By analogy, our laboratory uses π -numbers to discover control similarities among different wheelchair users who pass through the same openings at different velocities or through different openings at the same velocities. Warren's research showed convincingly a π -number that holds predictably over perceived dimensions of the environment (a stair design) can be used to predict the minima of a kinetic measure (oxygen consumption). We try to generalize these measures even further so that ultimately they might include intentional dynamic π -numbers.

In pursuing these practical goals, we assume that:

1. The dynamic fit of a wheelchair user is a function of the

- efficiency, comfort, and safety associated with carrying out a specific task in a given environmental setting.
2. That geometric, kinematic, kinetic, or intentional dynamic π -numbers provide scaling coefficients that can index the dynamic fit of wheelchair activities to the functional environment.
 3. That critical values of dynamic π -numbers can be defined and used in qualifying guidelines for the design of wheelchair environments.

11.4.3 Passability by Walkers, Automobiles, and Wheelchairs

Ecologic π -numbers and their generalized forms provide measures for characterizing the functional architecture of spaces designed to accommodate active wheelchairs. The experiments reported next suggest the main themes of future investigations. The main purpose of these preliminary experiments was twofold:

1. To assess whether passability is body scaled (i.e., indexed by a geometric π -number) or whether it is dynamically scaled (i.e., indexed by a kinematic π -number).
2. To compare passability π -numbers for walking, car driving, and wheelchair using and other activities.

We investigate these questions and methods in both the laboratory and the field. Future endeavors will focus more broadly on characterizing accessibility routes through functional workspaces of different designs (offices versus homes).

Experiment 1. Automobile Passability Judgments. (Flascher, Shaw, Carello, & Owen, 1989).

Method. Ten subjects (all experienced drivers) were asked to judge the minimum clear width passable by a car. Using the limits method borrowed from psychophysics, subjects judged aperture clear widths regarding the maximum width of their own car and a smaller or larger experimental car. Subjects made judgments under six conditions for each size car from a distance of 12 m: with subjects (1) sitting in their own car; (2) sitting in the experimental car; and (3) standing outside. Hence, there were $2 \times 6 = 12$ conditions in all.

Results. We calculated the dimensionless π -number for passability by dividing the judged minimal passable gap by the target car width. The passability π -number in this task was 1.22. Judgment position had no influence on judgment $F_{(2,9)} = .747$; however, there was a difference between the judgment for the experimental and own car $F_{(1,18)} = 7.008$, $p < .05$). The π -number for the subjects own car (1.24) was larger than the π -number for the experimental car (1.20).

Conclusions. These results support the assumption of a functionally defined scale for the perception of passability, of which body-size scale (that is, geometric dimension) is a limiting case when no dynamics are involved. Even though the passability ratio for cars (1.22) was different from that for walking (1.3), they seem close enough so that differences may be accounted for by different task demands due to different modes of locomotion.

Experiment 2. Dynamic Choices of Wheelchair Passability (Flascher et al., 1989; Flascher & Shaw, 1989).

Method. Each of four subjects, while sitting in a wheelchair, was confronted over 14 trials with a row of 11 apertures using a forced-choice method. The apertures were ordered by size (either ascending or descending order from the subject's point of view). Subjects were asked to roll themselves along a line parallel to the aperture row, stop when they saw the smallest passable gap, and then race through the chosen aperture as fast as they could. Apertures were varied in 2 cm increments to create 7 ranges: 59–79, 61–81, 63–83, 65–85, 67–87, 69–89, and 71–91 cm. Each subject approached each row from the left and from the right in a counterbalanced design. The size of the passed aperture and the direction of approach and speed of passage were recorded.

Result. The average passability π -number in this task was found to be 1.18.

Conclusions. From these results it can be concluded that in active selection as in passive judgment the same conclusions as in Experiment 1 are warranted.

Experiment 3: Perceived Passability Judgments for Walkers and Wheelchair

Users. (Flascher & Carello, 1990).

Method. Fourteen naive subjects were asked to judge the minimum aperture width that was passable under two different styles of locomotion: walking through or wheeling through in a wheelchair. Using the limits method, subjects judged aperture passability under four different conditions (two in each style) from a distance of one eyeheight: standing or sitting in a wheelchair. None of the subjects had ever used a wheelchair before.

Results. The passability π -number for wheelchairs in this experiment was found to be 1.22 and for walking 1.12, $F_{(1, 13)} = 8.564$, $p < .05$. There was no significant difference between the judgment positions (standing or sitting; $F_{(1, 13)} = .521$), and there was no interaction effect, $F_{(1, 13)} = 2.966$.

Conclusions. In this case, a direct comparison between the perceived widths for wheelchair passability and walking reveals a significant difference in π -numbers.

Discussion: How General are π -numbers?

In these experiments we were able to identify a passability π -number for car driving (1.22), for wheelchair using (1.22 and 1.18 for different tasks), and for walking (1.12). The discrepancy in these results generally suggest a need to understand perceptual scaling in terms of function units (dynamic π -numbers) rather than merely in terms of body dimensions (static π -numbers), as has previously been the case (e.g., Warren, 1982).

Although a significant difference was found to hold between dynamic π -numbers for walking and for wheelchair wheeling, one cannot argue for a basic difference in their perceptual characteristics. It now seems likely that the value of a π -number is changed (possibly reduced) with experience. For instance, in the first experiment we found that subjects scaled differently to their own cars than they did to an experimental car (which was a different size from their own, being either bigger or smaller). In the wheelchair experiments, none of our subjects had any prior experience with wheelchairs, so no conclusions could be drawn regarding learning and the possible reduction in the size of the π -number. In future research, of course, the hypothesis might be tested by comparing novice versus expert users. We intend to do so.

On the one hand, these experiments confirmed the usefulness and general applicability of the π -number methodology. On the other hand, they suggest that limits might be placed on the generality of these numbers. In other words, the assumption that a passability π -number might be a fixed universal constant now seems less likely. However, this limitation, if proper, still does not reduce their usefulness as the basis for a generic function scale that might be used for measuring the perceived or actual fit of actors to their environments (i.e., their use as ecological π -numbers). Recent experiments support this appraisal.

An important issue is whether the π -number of passability is "body scaled" or function-unit scaled (where the "body" scale may be defined for geometric dimensions of a wheelchair, automobile, or walker). In this regard, tasks involving different geometric ratios between diverse bodies and apertures, relatively close π -numbers for passability were nevertheless found. This presumably resulted from the tasks placing similar functional requirements on relatively independent body dimensions. Consequently, it seems likely that functionally scaled π -numbers may exist rather than geometrically scaled ones, as originally thought (Warren, 1982, 1984). However, to ratify this conclusion, further investigations across a wide range of task categories are called for to determine the exact relation of perceptual π -numbers to action π -numbers.

The experiments discussed also suggest that, although π -numbers are useful for gaining a handle on a problem, they should be treated as functional and dynamic variables that may change rather than as fixed universal constants—what we later shall call "dimensionless functions". Nevertheless, over a wide variety of modes of locomotion and passability conditions, π -numbers provide useful measures for both specific dynamic conditions (e.g., comfort mode or maximum speed) as well as for limiting cases (e.g., minimum passable gap size, maximum riser height, etc.). The differences in π -numbers over different tasks further suggest that, because of their abstractness, they may provide a task-dependency measure of great generality. Such a scale could be used to index the differential properties affecting the degree of fit of different types of behavior to different kinds of situations. In this way, one might scale the dynamic fit of wheelchair users to their environments, whereby making reference to effort and fatigue to which they may be subjected in obtaining or maintaining the respective fit.

These findings suggest that the investigation of the various π -numbers identified should be extended in new directions. For instance, dimensionless analysis should be carried out in a wide variety

of work-spaces. To do so, however, requires moving beyond π -numbers to more abstract descriptions of dimensionless quantities. We need these more general mathematical procedures for coupling the affordables intentionally—that is, for coupling the function units in perceptual information (observables) to the dual function units in the control of actions (controllables) as constrained by the goals (valuables) intended.

For dynamic tasks, dynamic π -numbers are most relevant. Thus, a method is needed to determine how expert actors (e.g., wheelchair users) select those routes that are minimally clear for travel from among those that are not, and from among the minimally clear routes how they select those that are also most safe, efficient, and comfortable. Only in this manner will the most adequate design principles emerge. Our future project has as its chief aim to propose such mathematical methods whereby continuing to develop and evaluate the instrument we have for making the required dynamic measurements.

11.5 Part IV: Generalized Dimensionless Analysis

11.5.1 Overcoming Traditional Failures to Address the Ecometric Scaling Problem

Earlier we discussed the seminal experiments on stair climbing conducted by Warren in his dissertation carried out in our laboratory. This research showed convincingly the existence of anthropometric π -numbers that hold predictably over both perceptual (information) and action (control) measures—the use of geometric π -numbers to index kinetic efficiency and comfort. However, to reach into the realm of intentional dynamics, even more general dimensionless concepts than these must be used.

By using π -numbers to express stair-climbability, Warren's experiments made an exciting discovery. They showed that the control of energy expended for action is formally similar to the information available through visual perception so that actors can *see* geometric design information as if scaled in terms of energy demands. In the second phase of the same experiments Warren validated these perceptions. Here subjects actually climbed stairmills of designs more or less like the stairs judged in the first phase of the experiment. Although actors of different heights found different stairmill designs easier to climb, measurements showed they expended the least amount of energy on those stairmills that they had selected a priori in the first

phase. Hence the actors were solving the ecometric scaling problem. Because, as a function of anthropometric fit, geometric information (the stair design) constrained the kinetic activity of climbing, we can also conclude that the actors were solving the ecomechanics transduction problem as well.

Earlier we suggested that the ecometric commensurability underlying Warren's climbability task was a similarity between something felt and something seen. We speculated that the similarity needed holds between the "felt effort," experienced during the act of climbing the stairmill, and the information detected in seeing the stair design before climbing. Hence, haptic information must somehow be commensurate with geometric visual information. This suggests a need for understanding at a more general level how ecological π -numbers work.

An ecological π -number is specific to a rule for the perceptual control of action as defined for a particular affordance goal (e.g., stair climbability, aperture passability). Like any other π -number, it must satisfy the physical property of dimensional homogeneity mentioned earlier. Ecological psychology typically treats information and control as being dimensionally inhomogeneous, thus creating the problem of how kinematics can specify kinetic values. That is, information is treated kinematically (e.g., L, T), being defined over dimensions of geometry and time; whereby control is treated kinetically (e.g., F, L, T), being defined over dimensions of force (or mass), length, and time. For example, we speak of visual information as kinematic, as in the case of time-to-contact with a target (Lee, 1974). Yet, such force-free information somehow specifies forceful control, as in the application of braking forces to avoid colliding with the target seen. Runeson has identified the problem of how kinematic information specifies dynamics (kinetics) as the search for the KSD principle (Runeson & Frykholm, 1983). Clearly, there is no dimensional homogeneity under this interpretation. In contrast, here we argue that the KSD principle is, in fact, a DSD principle with kinetic based information detection specifying kinetic-based energy control. Thus, dimensional homogeneity is satisfied, and ecological π -numbers may be constructed. Before presenting this DSD, view as one way to rid ourselves of the incommensurability problem, consider another way. The traditional approach attempts to avoid this incommensurability problem by taking a different tack. But, as we shall see, this approach has major shortcomings.

In standard information theory, as developed by Shannon, the

concept of information is dimensionless, being measured in "bits." (Shannon & Weaver, 1949) (*Bits* are binary units obtained by taking the logarithm to the base 2 of the total number of exclusive disjunctive choices possible under a given choice-set of items.) Likewise, from the development of abstract machine theory we have a dimensionless concept of control. Consider Turing's notion that state transitions a machine makes (reading symbols on tape, writing symbols on tape, erasing tape, moving tape right, moving tape left) are treated as determined by rule-directed symbol manipulations independent of the forces required to control a real machine. Just as information variables are not bound to the semantic dimensions of a message's content, so are control variables not bound to the physical dimensions of a machine's plant components. By both being dimensionless, the problem of dimensional inhomogeneity is avoided from the start by assumption.

Does not the existence of dimensionless descriptions imply the widest possible theoretical generalization across a wide class of analogous systems because their specific semantic and physical details can be ignored? So why not build π -numbers directly from these classical vacuous notions of information and control? If only dimensionless variables go into the ratio, then there can be no problem of their inhomogeneity. But is a deeper problem of incommensurability really avoided by this tactic?

Unfortunately, ignoring the dimensional variables and attempting to work directly with dimensionless ones does not solve the commensurability problem, so that information can scale control constraints. Because information and control variables are not bound to qualitative dimensions, their scale is left free floating, undefined, and hence potentially incommensurate. For instance, the information value of any nested (fractal) pattern is infinite (undefined) unless an ad hoc, extrinsic measurement constraint is applied to fix the level of analysis. Likewise, the control of state transitions in a nested state space is undefined until the level of control is extrinsically constrained, say, by an ad hoc programmer's ploy or physical limitations of the machine or organism.

How, then, can we be sure that the ratio of information to control is the appropriate one for the perceiving-acting cycle involved? An answer to this question comes in two parts: First, the concept of π -numbers must be generalized to a mathematical level of abstraction sufficient to allow information and control to be treated commensurately if indeed they can be. Second, it must be shown that under the intentional dynamics approach, one is, indeed, justified in treating them

commensurately. Such commensurability would guarantee dimensional homogeneity of these dynamical processes over the perceiving-acting cycle; thus, dynamical π -numbers might then be actually constructed to reflect the degree of success of attempted goal-directed behaviors.

11.5.2 Higher Order Dimensionless Invariants

To illustrate the generalization process needed to pursue to get higher order invariants, it is useful to scrutinize further the hydrodynamic example already introduced with the Reynolds number. As mentioned earlier, one specific Reynolds number is specific to a family of flows, namely, to flows in which the ratio of inertial forces to viscous forces is the given Reynolds number. In the literature these flows are called *geometrically similar steady state flows*, referring primarily to the constant velocity of the flow along a given section. It is important to note that flows characterized by the same Reynolds number are often called *dynamically similar flows*. In other words, a Reynolds number is deemed necessary and sufficient to capture dynamically similar, steady flows.

An analogy can be drawn between a geometrical π -number for a wheelchair "flowing" through a passageway and a Reynolds number for a liquid flowing through a passageway. However, the analogy is not simple; although in the former case we refer to a geometrical π -numbers, in the latter case we refer to a different, more general, dynamical π -number. Consequently, a closer look is needed to clarify what is meant by dynamical π -numbers as a generalization of geometric π -numbers. Earlier studies on climbability and passability revealed a geometric π -number which was specific to the "minimal" dynamics involved. In other words, subjects were instructed to apply a controlling force (analogous to inertia force) to maintain a normal, comfortable, and constant speed. Each subject belonged to a population with specific attributes affecting the effort required to maintain that comfort mode activity (similar to overcoming viscous forces). From attempts to control (inertial forces) one's movement while remaining in a comfort mode (overcoming viscous forces), a Reynolds number interpretation seems to follow naturally.

Using terminology from hydrodynamics, we might consider the "normal speed" to be a steady-state mode of motion. However, these steady-state modes are ideal and, in that sense, represent only a minimal dynamical solution. We say ideal, because real flows are not constant but change as their boundary conditions change. For instance, the flow velocity of a river is not invariant but changes as the river's cross-section

changes, or the speed of a car changes according to the changing road conditions (e.g., roughness), even if road width remains the same. Thus, for the general case of flow in natural settings, the term dynamical means continuously changing velocity, or continuously changing viscosity, or whatever. For our "dynamical" examples, the analogy sought is between a Reynolds number for real rather than ideal dynamic flow, that is, for flow that is continuously changing between cross-sections and also in time at the same cross section. Thus, similarly, the passability π -number must also be capable of reflecting change between cross-sections and in time for each cross-section as a function of changing environmental and subject conditions.

In Figure 11.2 we have an example of how the Reynolds number's effect on the mode of flow depends on the environmental boundary condition that is present. For flows of equal velocity, equal viscosity, and so on, but different boundary conditions (roughness of the pipe), different frictional values arise causing a bifurcation into flows exhibiting different modes of turbulence. Notice how the Reynolds number range (R) of the joint critical-transition region increases as friction (f) decreases. This is a function of roughness on the boundary of the flow (which effects the growth of the boundary layer and therefore the size of the region containing the inviscid core). The analogy to intentional dynamics is straightforward: Intentions, defined on the initial condition, not only select target parameters, defined on the final conditions, but select manner parameters as well which, in turn, determine the manner of flow toward the target from the point of initial intent. The value of this manner parameter is a boundary condition, like roughness, which selects the particular path over which the actor moves. In fluid mechanics, the path that minimizes fluid action is that which allows flow to remain most laminar, just as in ecomechanics, the goal-path that minimizes the actor's need for self-control (while maximizing target direction information) is that which allows behavior to remain most comfortable.

Consequently, the most natural generalization is from dimensionless invariants that are constant π -numbers (constant dimensionless functions along a steady flow) to nonconstant dimensionless functions (along varying flow). At this level of generality, instead of using invariant dimensionless constants (π -numbers) to express the dynamical similarity between different flows (fluid particles or moving vehicles), we should use π -functions—that is, invariant dimensionless functions. A mathematical strategy for achieving the generalization process is heuristically outlined next.

11.5.3 Searching for the Laws Responsible for Dynamical

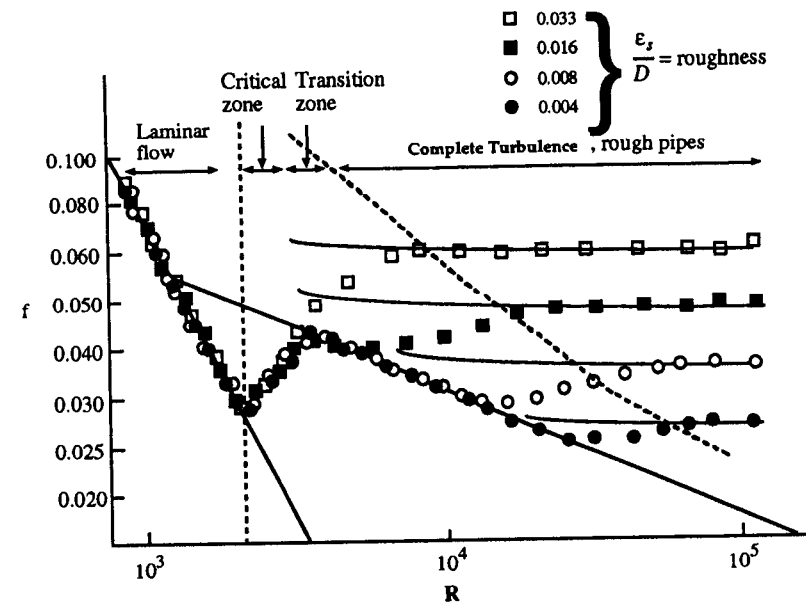


Figure 11.2 The effect of the environmental condition of roughness on flow as a function of Reynolds number.

Similarities

The heuristic outline sketches a popular technique by which the laws underlying dynamical similarities might be discovered and characterized. (For more detail and definitions please consult the appendices.)

Step 1: Identify the dynamical dimensional space (so called base space) in which the laws governing the phenomena might be formally expressed. The dimensional space in most cases captures a certain amount of redundancy in the measured variables used. The derived variables of velocity, acceleration, or momentum can be expressed in terms of the fundamental M (or F), L , T dimensions. In most cases, the dynamics of

this dimensional base space is expressed in differential equations.

Step 2: Characterize the redundancy of the dimensional base space that expresses the dynamical similarities of interest. This redundancy can be characterized in terms of the symmetry structure of the base space. Symmetry structure is best characterized by the continuous Lie group of infinitesimal differential operators derived from the original differential equations modeling the relevant system dynamics (for details see Shaw et al., 1990; Stephani, 1989). The Lie group symmetry technique is especially useful for simplifying the analysis of complex dynamic systems because it reveals the participating laws (invariant subgroups) responsible for creating the symmetry structure of the base space in the first place.

Step 3: Using the Lie-group technique, factor the dynamical space into its invariant subgroups. These will identify the participating laws. In other words, by unfolding the factor structure of the Lie-group description of the base space, we reveal the laws (invariant subgroups) that are operationally identified with the base space redundancies (symmetries). By doing so, dimensionally homogeneous, invariant functions are obtained (see Appendix A).

Step 4: By using Drobot's more general reformulation of Buckingham's π -theorem (Buckingham, 1914), obtain π -numbers (if the dynamics is simple) or invariant functions (if the dynamics is complex). Invariant functions are those functions that do not change when the system of measurement is changed. Thus, similar properties remain similar over two systems, even if one of the systems is rescaled. Generally, a two-step procedure is required to obtain invariant functions: One first constructs dimensionless constants and then generalizes them to the dimensionless invariant functions. There is a more direct way to obtain invariant functions. If one uses Drobot's reformulation of the π -theorem (Drobot, 1954; Kasprzak, Lysik, & Rybaczuk, 1990), then there is no need to construct the dimensionless constants first.

It is worth noting that Drobot reformulated the π -theorem so as to obtain invariant functions that are directly defined over the π -numbers of Buckingham's π -theorem (see Appendix A for details): From Buckingham's π -theorem we are guaranteed that it is always possible to reduce the number of independent parameters in a problem by compacting the dimensional parameters into a set of dimensionless parameters. From Drobot's reformulation and generalization of this

theorem, we are also guaranteed that the Buckingham procedure will work for more abstract dimensionless functions. (Usually the more recent reformulation is given without crediting Drobot with the generalization; e.g., Gerhart & Gross, 1985, p. 364, whose treatment we follow.)

Assume any physical process governed by a dimensionally homogeneous function whose argument consists of n -dimensional parameters, such as

$$x_1 = f(x_2, x_3, \dots, x_n),$$

where the x 's are dimensional variables. Then there exists an equivalent function whose argument consists of a smaller number ($n-k$) of dimensionless parameters, such as

$$\pi_1 = F(\pi_2, \pi_3, \dots, \pi_{n-k}),$$

where the π 's are dimensionless groups constructed directly from the x 's. The amount of reduction in the parameter set, signified by k , is usually equal to, but never more than, the number of fundamental dimensions involved in the x 's. Here the dimensionless parameters are constructed as products of powers of the dimensional parameters (see Appendix A for details).

Essentially this important theorem asserts that a complex system with redundant structure can be made nonredundant and therefore simplified. This amounts to finding the smallest number of parameters for describing a system by discovering the minimal generator set for all the paths through the system's state space.

Step 5: Model the way and degree to which two systems have similar intentional dynamics (i.e., goal-directedness). Drobot's dimensional space can be interpreted as a fiber bundle space. This suggests that differential geometry can be used to provide a fiber bundle interpretation of the invariant functional associated with the flow of the relevant potential (i.e., the generalized Hamiltonian representation of the total action potential). The similarity between the two systems can then be identified with that connection in the fiber bundle which gauges the flow of information on one manifold to the flow of control on the dual manifold. A third lower dimensional manifold, called the *quotient manifold*, defines a connection in the fiber bundle which carries the scale factors for relating the two flows. The parametrically simpler flow of the

generalized action potential on the quotient manifold is obtained through group factorization of the flows on the other two manifolds (see Appendices A and B).

Earlier we asked how one can be sure that the ratio of information to control is the appropriate one for the perceiving-acting cycle involved? It was suggested that an answer to this question comes in two parts: first, that the π -number concept must be generalized to a level of abstraction sufficient to allow information and control to be treated commensurately; steps 1-5 suggest an approach to answering this first part of the question. The second part requires showing that an intentional dynamics approach justifies treating information and control flows as being commensurate. Intuitive and formal arguments supporting this claim have been made by us elsewhere (Shaw & Kinsella-Shaw, 1988; Shaw et al., 1990; Shaw et al., 1992). These earlier arguments were the primary motivation and justification for the generalized dimensionless invariant story told here.

11.6 Conclusion: The Explanatory Value of Dimensionless Analysis

This chapter has been organized around the notion of the π -number and its generalization to dimensionless functions. The search for these dimensionless parameters promises the following benefits to psychological theory: (a) π -numbers and their generalizations can be used as the basis for intrinsic measures that relate on a single scale quantities that might otherwise be taken as unrelated. For example, they might show intrinsic measurement relationships holding between environmental variables and organismic variables, affordances and effectivities, perceptual and action variables, and so forth. We have concentrated on how π -numbers might be used to reveal intrinsic ecometric relationships between information and control variables. (b) π -numbers and their generalizations provide means to reduce less tractable problems involving systems with high-dimensional state spaces to problems involving systems with low-dimensional state spaces. (Recall that the reduction in complexity of state space will always be proportional to the number of relevant invariants, or π -numbers, discovered.) Finally, (c) through the use of principal π -numbers (or principal π -functions), that is, π -numbers based on laws, the functional relationship of psychological variables can be shown to imply lawful rather than arbitrary bases for explanations. This should have the effect of moving psychology into close alignment with other

law-based sciences, such as physics and biology. (Readers interested in pursuing this point might consult Kugler & Turvey, 1987; Rosen, 1978; Schuring, 1977). Our main concern in this chapter has been to suggest ways in which dimensionless analysis might serve the interests of ecological psychology by providing a formal interpretation of intentional dynamics. We return to this issue next.

11.6.1 Reducing Intractable Degrees of Freedom

The intentional dynamical approach to ecological psychology is a search for laws that relate perception to the control of goal-directed actions. Since Noether's pioneering work in applying group invariant solutions to characterize the classical conservations as symmetries (see Goldstein, 1968), one assumes that the discovery of such lawful invariants will help reduce the degrees of freedom of intractably complex systems to something more tractable. The search for invariants is the search for fundamental symmetries in the relationship between energy control and information operations as they interact to form the perceiving-acting cycle. How might this search proceed?

Although algebra is needed to solve problems, geometry is used to clarify them. An explicit geometrical setting for this approach is provided by the notion of a *quotient manifold*, M/G , where a manifold M is "divided" by the invariants of some regular group of transformations. For our purposes, these invariants will derive from a "compounding" of control and information detection operations into groups of operations. The coupling of these groups is expressed in their common (dual) action at conjugate points along a goal path on the ecological manifold. (In the Appendices we show that this is a gauge manifold defining a connection in the fiber bundle, and that it is of lower dimension than the control and information base manifolds that it relates.) Why is the ecological manifold, obtained from coupling the information and control manifolds, simpler (more tractable) than either manifold taken alone? The answer lies in showing that the ecological manifold can be constructed as a *quotient manifold* from these dual groups characterizing the two base manifolds. Each point on the dual quotient manifold will correspond to an orbit of each group.

Two main points should be made regarding the relevance of these remarks to the current approach: First, the Lie-group descriptions of the control and information detection operations capitalize on the existence of such invariant group solutions. When the goal-path is successfully traversed by the perceiving-acting cycle, we reveal the major group

invariant that all the orbits (possible paths) share. Any other quantities that commute with a dynamical invariant also reduce the complexity of the original manifold, M . The discovery of such invariants greatly reduces the complexity of the original information and control manifolds, when treated separately, to a simpler quotient manifold, M/G , when treated jointly. From this joint treatment a single ecological field manifold is derived on which the orbits of the separate group operators are coupled into a single dual prescribed goal path. The motivation for constructing the quotient manifold is but a more geometric way to present the algebraic approach pioneered by Buckingham in his famous π -number theorem (see Olver, 1986, for details of this relationship).

The use of π -numbers helps to resolve the crucial problem facing psychological theorists—how to reduce the complexity of a given system to the lowest possible level. For instance, the intentionally dynamical systems to which we have referred may be difficult to model because they have equations of state that posit a very complicated, high dimensional state space. This problem has been referred to in control theory as the “curse of dimensionality” (Bellman & Dreyfus, 1962), in action theory as the “degrees of freedom problem” (Bernstein, 1967), and in ecological psychology as “intractable nonspecificity” (Shaw, Turvey, & Mace, 1982). The original manifold, on which the system’s goal-directed behavior is defined by superposing information and control paths, may be extremely complicated. The use of dimensionless numbers reduces the dimensionality of this manifold to a new description using quotient manifolds. This move significantly reduces the degrees of freedom of the original dual state-space manifold. Hence, the search for law-based ecological π -numbers and their generalizations is clearly mandated.

11.6.2 Affordances as Dimensionless Invariants

Gibson held that perceptual information was scale-free, consisting of formless and timeless invariants (Gibson, 1979). Affordances are compounded from invariant relationships among environmental variables taken in reference to organisms as acting perceivers. Information is specific to these invariants of invariants. That affordances are dimensionless invariants is exemplified by Lee’s generalized tau numbers specifying time-to-contact (i.e., surface contactability) and the tau derivatives specifying the marginal values for braking to avoid hitting objects (the negative affordance of collidability)

or accelerating to intercept objects (interceptability) (Lee, personal communication, July 1992). Affordances as scale-free properties hold across species. The invariant ecometric properties that make one surface, say a bridge, afford support for one creature, say an elephant, makes another surface, such as a leaf, afford support for another creature, say an ant. The affordance of supportability is an abstract dispositional property which, by definition, must be scaled to the animal in question. Thus, the ratio of elephant-relevant properties to the bridge will reveal a value that is dimensionless and equivalent to the ratio of ant-relevant properties to the leaf. Warren (1982, 1984) showed that the climbability of stair designs by actors of different size nevertheless yielded the same π -number. Lee’s tau-derived π -number for a collidability is the same regardless of the different distances and velocities of approach; hence, it is the same, regardless of the type of animal approaching or the type of surface being approached.

In all cases of affordances, supportability, edibility, collidability, graspability, passability, or whatever, the affordance is nonspecific or scale-free. It picks out an equivalence class of actor/environment ratios which, if dimensional homogeneity is satisfied, are dimensionless numbers or dimensionless functions. Not to see this is to miss one of Gibson’s most elegant insights. For if affordances were merely specific measures of an individual animal’s fit to its environment, there would be as many affordances of a given type as there are individual animals and individual environmental situations. Gibson aspired to a general theory of perception that could be applied to all kinds of creatures in a variety of ec niches, and not one restricted to particular environmental situations, individuals, or species. This generality of the affordance concept allows ecological psychology to lay claim to being the study of the lawful aspects of perception and action.

Effectivities by which affordances are realized are no less scale-free than the affordances to which they are dual. Mathematically, the isomorphism that takes an affordance into the corresponding dual effectivity and vice-versa is called a *duomorphism*. The fundamental ecometrics postulate asserts that the duomorphism which, by definition must hold between affordances and their effectivities, guarantees that the measurement of goal-specific information and of the control of goal-directed action must be the same. (Mathematically, they must share Lie-group orbits that are similar up to isomorphism.) Hence, by the group factorization theorem, the same ecological dimensionless invariants always exist when similar goal-directed actions are successful—regardless of the goal, the creature, or the environmental

situation.

The ratio of an affordance to its dual effectivity will yield a dimensionless invariant and have a value specific to the degree of success in attaining a goal—a value independent of the semantics of the situation. Information detection and energy control are operations that coordinate the effectivities with the affordances they serve. The intentionally coordinated perceiving-acting cycle can, therefore, be thought of as a "gauge" group with orbits that run through the environment and the organism thereby generating a goal path. If intentional dynamics is as lawful as we suspect, then modeling complex systems with such dynamics at the ecological scale will yield the simplest explanations possible, and the curse of dimensionality will have been lifted.

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Appendix A

Concepts and Theorems Required for Dimensionless Analysis and to Build a Fiber Bundle Geometry

Here we present the basic definitions and theorems used in this paper. We restrict ourselves to a pure mathematical language. The discussion follows the major steps of dimensional analysis as expressed by Kasprzak et al. (1990). Furthermore we also used Shaw et al. (1990) and Shaw et al. (1992).

Definition: Group $G = (G, *)$ is an algebraic structure on a set G with an operation $*$ called multiplication if

(a) for any $a, b \in G$ there is a unique $c \in G$

$$ab = c$$

(b) for any a, b , and $c \in G$

$$a(bc) = (ab)c \quad (\text{associativity})$$

(c) there exists $e \in G$, for which

$$ea = a \quad \text{for any } a \in G$$

(d) for any $a \in G$ exists $a' \in G$

$$a'a = e$$

Furthermore if for any $a, b \in G$

$$ab = ba, \quad \text{then the group is commutative (Abelian).}$$

Definition: We call G_1 group an *invariant group* under transformation $s \in G$ if for any $a \in G_1$

$$G \supseteq G_1 \quad \text{and } sa = as.$$

In other words, s is a commutator of G .

Definition: We call S subgroup an *invariant subgroup* of group G if for any $a \in G$ and any $s \in S$

$$sa = as.$$

An invariant group is sometimes called a *normal subgroup*.

Definition: If N is a normal (invariant) subgroup of G , then aN $a \in G$ classes are *compatible classes*. That is, if $a \neq b, a, b \in G$, then $aN \cap bN = \emptyset$.

Definition: If N is a normal (invariant) subgroup of G , and $a \in G$, then we can define the multiplication of aN *compatible classes*:

$$aN \diamond bN = abN \quad \text{for } a, b \in G.$$

These classes with this multiplication constitute a group which is called the *factor group* of G with respect to N . The standard notation of this group is G/N .

Example: If G is the quaternion group $\{\pm 1, \pm i, \pm j, \pm k\}$, and $N = \{1, -1\}$, then G/N is the Klein group $\{1, i, j, k\}$. The Klein group is commutative and $ij=k, ik=j$ and $jk=i$.

Definition: Group G is called a *Lie group* if it has the structure of both a manifold and a group. To be more specific, G is a Lie group if

- the manifold is a topological space
- the group multiplication is a continuous operation on the manifold.
- differentiations (infinitesimal transformations) are also defined on the manifold.

Definition: A G Lie group is called a *Lie algebra* if the infinitesimal operators are represented by vector fields and satisfies a *Lie bracket operator* defined as

$$[B, A] = (BA - AB),$$

which provides us another derivative (Lie-derivative) on the manifold. The bracket product satisfies the Jacobi identity, that is

$$[A, [B, C]] + [B, [C, A]] + [C, [A, B]] = 0.$$

If the bracket product vanishes for any $A, B \in G$, then the algebra is said to be *commutative*, because $AB = BA$.

Definition: The M/Q equivalent classes of M with regard to a G transformation is a *quotient manifold*, if M is a smooth manifold and G is a local group of transformations (e.g., infinitesimal differential transformations). Any class of M can be identified for any $x \in M$ by the orbits of G passing through x .

Example: Let M be a manifold of an arbitrary physical system with $Z_1, Z_2, Z_3, \dots, Z_s$ fundamental physical quantities, and G be the corresponding group of scaling transformations $a_1, a_2, a_3, \dots, a_s$. Then the so-called π theorem (see later) for this physical system provides an example of the significance of the M/G quotient manifold.

Theorem: Let A be an operator of a second-order differential equation and X be the generator of a group of local (point) transformations. Then the differential equation of operator A is invariant under the symmetry operation dX/dt if and only if

$$[dX/dt, A] = kA.$$

Example: Kepler's problem (Stephani, 1989, pp. 96-97): motion of planets around the sun is governed by

$$\ddot{x}^i = -Mx^i/r^3$$

which satisfies the conditions of the theorem for the following 5 point symmetry transformations:

$$X_n = a_{n,m}^k \left(x^m \frac{\partial}{\partial x^k} + \dot{x}^m \frac{\partial}{\partial \dot{x}^k} \right) \quad n, m, k = 1, 2, 3$$

$$X_4 = t \frac{\partial}{\partial t}$$

$$X_5 = t \frac{\partial}{\partial t} + \frac{2}{3} x^m \frac{\partial}{\partial x^k} - \frac{1}{3} \dot{x}^m \frac{\partial}{\partial \dot{x}^k} \quad m, k = 1, 2, 3$$

The first three generators X_n constitute the three-dimensional rotation group according to the spherical symmetry of the gravitational field of the sun. The invariance of temporal translation expressed in X_4 reflects the stationarity of the gravitational field. X_5 is an implicit expression of Kepler's third law (spatiotemporal relationship of any two orbits of planets):

$$t_1^2/r_1^3 = t_2^2/r_2^3$$

In sum, these invariant transformations express the underlying spatio-temporal symmetry structure of the gravitational field of the sun.

Definition: Assume that the sets X, Y have a group G defined on them, and there exists an $A: X \rightarrow Y$ mapping, then we call A an *invariant function* if for any $x \in X$ and for any $g \in G$ $A(\phi(g)x) = \psi(g)A(x)$.

Without proof we present some of the important properties of an invariant function.

- Function A maps orbits into orbits.
- A can be decomposed into the sum of invariant functions on the orbits.
- The invariance property of function A is equivalent to the commutativity as shown in Figure 11.A1.
- The symmetry value of an invariant function (Figure 11.A1) cannot be smaller than the symmetry of the argument.

On the basis of these properties there is a method to simplify the determination of an invariant function (for more details see Kasprzak et

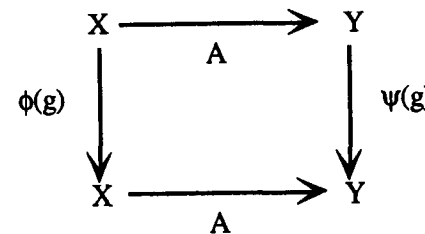


Figure 11.A1: Diagram for an invariant function.

al., 1990).

Definition (by Drobot): An algebraic structure $P = (P, \bullet, []^\circ)$ (P is the base set on which two operations, " \bullet " = group multiplication" and " $[]^\circ$ " = real power function" are defined) is called a *dimensional space* if

- (a) (P, \bullet) is a commutative (Abelian) group.

(b) The power function satisfies the following identities:

For any $a, b \in R$ and $X, Y \in P$

$$X^{a+b} = X^a \cdot X^b$$

$$(X \cdot Y)^a = X^a \cdot Y^a$$

$$((X)^a)^b = X^{ab}$$

$$X^1 = X$$

(c) $P_0 = R_+ \in P$

(d) On $P_0 = R_+$ the group multiplication $\bullet =$ multiplication of real numbers, and

(e) The powers of elements of the subset $P_0 = R_+$ are identical to the ordinary powers of the numbers.

Definition: In a dimensional space every system of dimensionally independent elements is called the *base* of this space.

Definition: $Z = F(Z_1, Z_2, Z_3, \dots, Z_s)$ is called a *dimensional function* of $Z_1, Z_2, Z_3, \dots, Z_s$ if $Z, Z_1, Z_2, Z_3, \dots, Z_s \in P$.

Definition: Z is called a *dimensionally homogeneous and invariant function* of $Z_1, Z_2, Z_3, \dots, Z_s$ if

(a) for any $\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_s \in P_0$ and $Z_1, Z_2, Z_3, \dots, Z_s \in P$ there exists an $\alpha \in P_0 = R_+$ for which

$$F(\alpha_1 Z_1, \alpha_2 Z_2, \alpha_3 Z_3, \dots, \alpha_s Z_s) = \alpha F(Z_1, Z_2, Z_3, \dots, Z_s).$$

(homogeneity condition).

(b) for each dimensional transformation Θ

$$F(\Theta Z_1, \Theta Z_2, \Theta Z_3, \dots, \Theta Z_s) = \Theta F(Z_1, Z_2, Z_3, \dots, Z_s).$$

(This guarantees the invariance of a dimensional function, that is, the independence from the system of units used.) Note: These are the only functions relevant in physics, and in that regard we do not want to make intentional dynamics an exceptional case.

Theorem π : If Z is dimensionally homogeneous and invariant function depending on $z_1, z_2, z_3, \dots, z_m$ among which the first $z_1, z_2, z_3, \dots, z_n$ are dimensionally independent and the rest $z_{n+1}, z_{n+2}, z_{n+3}, \dots, z_m$ are dimensionally dependent arguments, then

$$Z = f(\pi_1, \pi_2, \pi_3, \dots, \pi_k) \prod_{i=1}^m Z^a.$$

where $k = n - m$.

Definition: A *fiber bundle* is a pair of manifolds (E, M) and a projection $Pr: E \rightarrow M$. M is called a *base space* on which a dynamic structure (field) is defined. For any $m \in M$ $Pr^{-1}(m)$ is called the *fiber* over m . In most cases fibers have further geometric structure. For instance, if they are vector spaces then we call $Pr: E \rightarrow M$ a *vector bundle*.

If the fibers are the cotangent spaces on the manifold M , we call it the *cotangent bundle*, T^*M .

If the fibers are the tangent spaces on the manifold M , we call it the *tangent bundle* TM . Physicists call it velocity space (see Figure 11.A2: Examples of different fiber bundles). A fiber bundle can also be viewed simply as a bunch of fibers projected from the same base space.

Definition: A *Hamiltonian* is a function H , defined in phase space (q, p) .

$$H(p, q) = \sum_{i=1}^n \dot{q}(q, p) p_i - L(\dot{q}(q, p))$$

In other words, a Hamiltonian is represented by the vector field

$$v = \frac{\partial H}{\partial p} \frac{\partial}{\partial q} - \frac{\partial H}{\partial q} \frac{\partial}{\partial p}$$

on a cotangent bundle (q, p) . The cotangent bundle is its natural structure.

Definition: A *generalized Hamiltonian*, G , is defined in a generalized (complexified and compactified) phase space

$$\{(Q_1, \dots, Q_n; P_1, \dots, P_n), i(Q^*_1, \dots, Q^*_n; P^*_1, \dots, P^*_n)\}$$

where Q and P are the generalized (target-specific information) coordinates and generalized momenta (manner-specific movements), respectively, and both are observable in the exterior frame. In the interior frame, accessed by the complex operator, we have dually Q^* and P^* —the generalized (manner-specific information) coordinates and

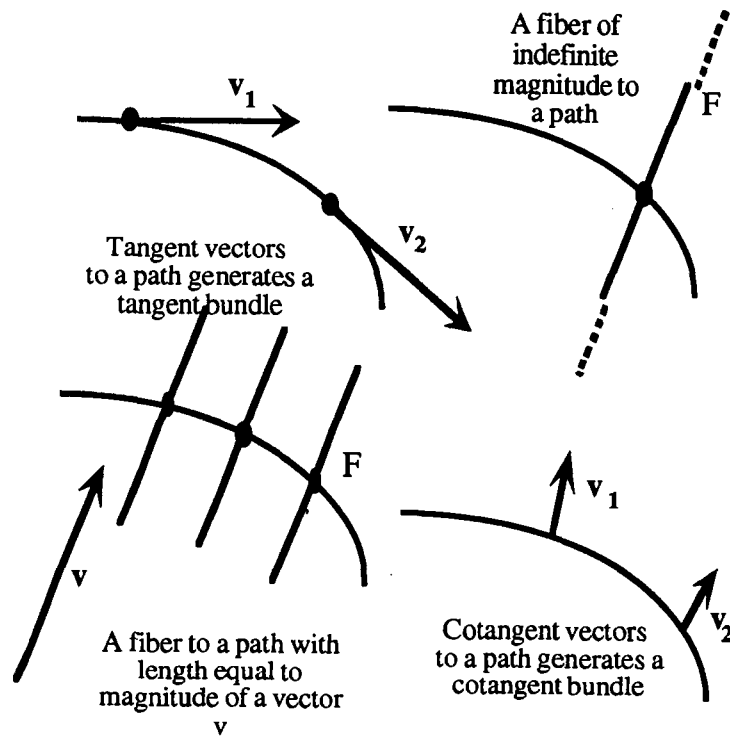


Figure 11.A2: Examples of different fiber bundles.

generalized momenta (metabolically produced control impulses), respectively. The generalized Hamiltonian, G , therefore, is defined (complexly) adjointly over the exterior, and interior phase spaces by

$$G(\dot{Q}, P; Q^*, P^*) = \left\{ \sum_{i=1}^n \dot{Q}(Q,P)P_i - L(\dot{Q}(Q,P)) \right\} + \left\{ \sum_{i=1}^n \dot{Q}^*(Q^*, P^*)P_i^* - L(\dot{Q}^*(Q^*, P^*)) \right\}$$

Just as an ordinary $H(q,p,t)$, over a time invariant path in phase space conserves the time-forward flow of total energy (kinetic + potential energy), so the generalized Hamiltonian, $G(Q, p; Q^*, p^*; +t, -t)$, conserves the relationship between time-forward and time-backward flows of energy and information that comprise total (goal-specific) action. Total action, G , is the sum of total control information, $(Q, Q^*, +t, -t)$, and total useful energy $(P, P^*, +t, -t)$ over both frames (i.e., the interior and exterior phase spaces). (Shaw et al. 1991, pp. 607-608).

Appendix B

A Sketch of the Fiber Bundle Geometry Needed for the Intentional Dynamics

A fiber bundle geometry is the appropriate mathematics in which to express a potential solution to the ecometric (information) scaling problem and the ecomechanic (energy) transduction problem. Using the generalized Hamiltonian and its natural fiber bundle structure, we can define two different fiber bundles on both the control field and the information field construed as base spaces:

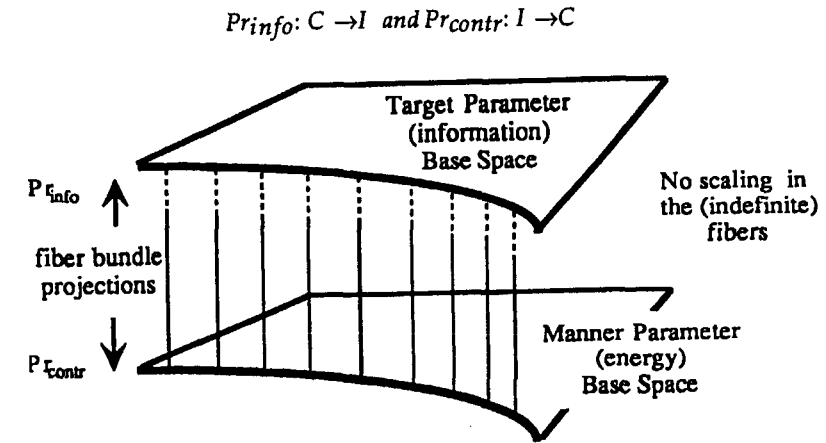


Figure 11.B1: Indefinite Ecological Fiber Bundles without Gauging.

Definition: A fiber is defined as a projectivity from one vector field over a base space, containing information flow paths to another vector field over another base space which contains control paths.

Fibers are parallel to one another and independent unless connected by some function. For instance, a goal-path integral which scales information detection flow paths in one base space to energy control paths in another base space. The ecometric scaling problem is to discover the definite magnitude (gauging) to place on the fiber defined over the information field. The ecomechanics transduction problem is to discover the definite magnitude (gauging) to place on the fiber defined

over the control field. These two gauging procedures have no necessary correspondence or commensurability to each other unless the intended action is successful, then the gauging determines the same curve—the goal path (see, e.g., Fig 11.B2.).

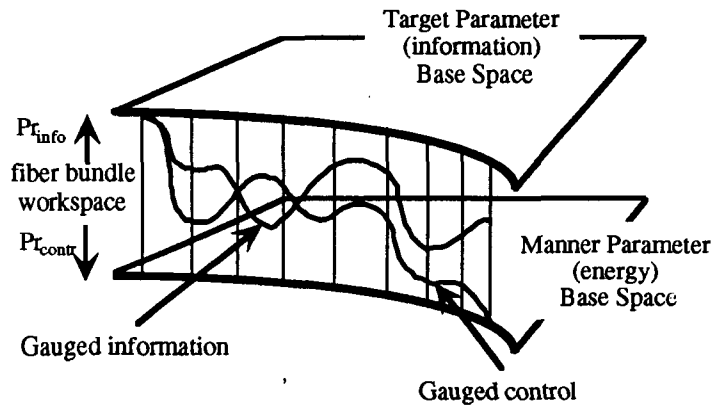


Figure 11.B2: Ecological fiber bundles with non-ecological gauging.

Definition: Assuming the proper gauging in two fibers the function that maps values of one fiber onto the value of adjacent fibers is called a connection in a fiber bundle (Fig 11.B4.).

In the case of navigation by a vehicle the connection is a vector space, that is, the tangent space defined over the *contact elements* to the fibers (i.e., defined as the direction of the curve as it crosses the fiber). The *goal path* in an ecological workspace is a connection in a fiber bundle that *gauges* (or scales) one region of a field to another region of the same or different field. We use a connection to relate the flow of information to the flow of energy within the same field or across different fields, such as, across the interior (organismic) field and the exterior (environmental) field.

Definition: When the connection is itself a field it is called a gauge field.

Definition: Ecological physics is the study of the gauge field between energy flow fields and information flow fields that couples an environment and organism into an ecosystem.

Definition: *Intentional dynamics* studies the properties of this dual-information-control gauge field for goal paths through a workspace.

Definition: *Goal paths*, therefore, are curves of dually compact points of contact elements defining a gauge field coupling the interior and exterior fields. Each point along the path is a dual pair of numbers—one number specifying the location of the fiber in each base space, and the other number specifying the value of the projection along the fiber to the path, moreover:

—When the projection pushes the value forward from the information base space to the curve, then it is the gauge value for solving the ecometric scaling problem.

—But when the projection pulls the value back from the curve to the control base space, then it is the gauge value for solving the ecomechanics transduction problem. Together they determine the gauging of the information and control flows to the goal path by means of the complex involutorial group of reciprocity maps characterizing the perceiving acting cycle (as described by Shaw et al. 1990).

In other words, if we treat each fiber as unity, then the connection (curve) partitions each fiber into complementary magnitudes (length) whose ratio is proportional to the scaling of the first base space to the second space, and whose reciprocal ratio is proportional to the scaling of the second base space to the first. This is shown below.

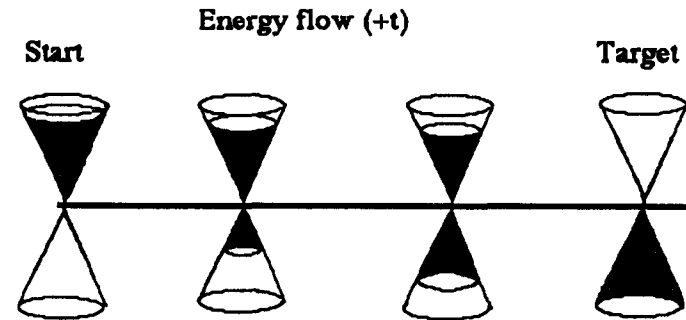


Figure 11.B3: The information-energy potential along a goal path in the ecological workspace exhibits an inner-product invariant relationship.

The dual cones represent projectivities (fibers) from a goal-path in the environmental workspace. Each fiber is two dimensional (a cone)

consisting of a tangent vector to the goal-path and the magnitudes of a dual-potential specific to each goal-path point (The two base spaces are suppressed). The upright cone, incident at a goal-path point, represents a quantity of energy potential, whereas its dual inverted cone represents a quantity of information potential. The fact that the sum of the control and information complements remains constant over the goal-path integral illustrates the meaning of the *inner-product invariant*. Notice the mini-max relationship between the dual cones.

The points in either field's base space are dually compact because they each hide reciprocal degrees of freedom (or constraint) that reside in the other field's base space. These hidden degrees of freedom are represented by the fact that the fibers incident at each point along the goal-path are projections from each base space.

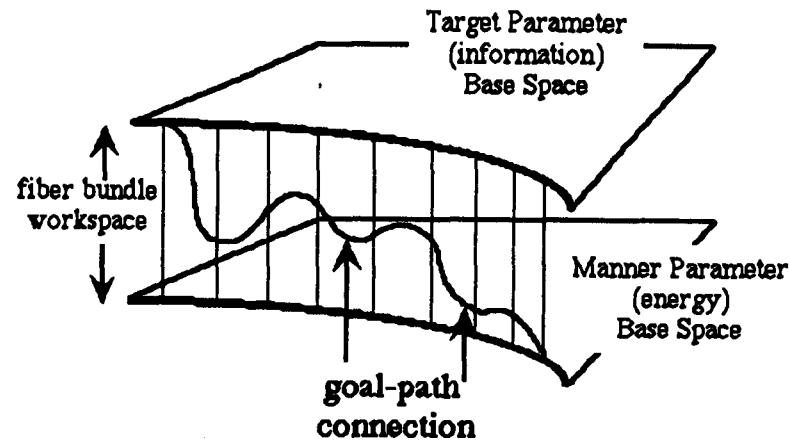


Figure 11.B4: The goal-path connection with an ecological gauging in a fiber bundle workspace.

The three most important properties of intentional dynamics:

- 1 Since the goal-path connection (curve) partitions each fiber into complementary magnitudes of energy control and information detection, then their inner-product remains invariant over the goal-path integral. This dynamical invariant is to be expected if generalized action is a conserved potential.

- 2 This inner-product invariant, therefore, provides us with a metric criterion against which to measure the degree of success of an intentional system in reaching its goals.
- 3 Also, because these energy and information fiber bundles exist at each point in the workspace, then the coupling between the two fields is both "soft" (transactional, or informational) and "hard" (interactional, or energetic) (in the sense of Kugler & Turvey, 1987, and Shaw & Turvey, 1981). Thus each point along a goal path in the connection has conjugate values in the control and information fields vis-a-vis the fibers projecting from each of the dual base spaces