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Intrinsic and Required Dynamics of a Simple Bat–Ball Skill

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Three experiments investigated the coordination dynamics of a simple bat-and-ball skill: cyclically striking a ball suspended by a string with a pendular bat. The relative phase ϕ between the bat and ball is dictated by the potential function $V(\phi) = k \sin \phi$ and the difference $\Delta\omega$ in their uncoupled frequencies. For various $\Delta\omega$, ϕ and its standard deviation were measured in the absence of any environmental restraints (intrinsic dynamics) and when the ball had to reach resistive or nonresistive targets at set distances (required dynamics). Results support the dynamical theory of coordination patterns (G. Schöner & J. A. S. Kelso, 1988a, 1988c), particularly the hypothesis that required dynamics are understandable as the addition of terms to the potential governing the intrinsic dynamics.

Ball skills are challenging action–perception problems. Synchronizing movements of the limbs and body with the trajectory of a ball, and intercepting and interacting with a ball to redirect its trajectory, entails the controlling of information detection by action and the controlling of action by detected information. Research on catching skills has advanced understanding of the act's dependence on the availability and type of optical information about the trajectory of the ball (e.g., Peper, Bootsma, Mestre, & Bakker, 1994; Savelsbergh, 1990; van Wieringen & Bootsma, 1989) and the location of the catching hand (e.g., Fischman & Schneider, 1985; Smyth & Marriott, 1982). Research on hitting skills has provided insight into the fit of action and perception, showing how variation in the pick up of optical information about the ball is compensated for by variation in the kinematics of the bat's trajectory and vice versa (e.g., Bootsma & van Wieringen, 1990). Investigations of one particular ball skill, cascade juggling, conducted from a dynamical perspective also have been illuminating with regard to underlying principles (Beek, 1989a, 1989b; Beek & Turvey, 1992; Beek & van Santvoord, 1992). As first defined by Shannon (cited in Horgan, 1990; Raibert, 1986), the (universal) temporal constraint on juggling is given by the following equation:

$$\omega_{ball}/\omega_{hand} = \text{number of hands } (H)/\text{number of balls } (N), \quad (1)$$

where ω_{ball} and ω_{hand} are the average angular frequencies (rad s^{-1}) of ball and hand oscillations, respectively, and N is more precisely the time-average number of balls in the air. As derived and demonstrated by Beek (1989a), the primary spatial constraint on the cascade juggle is

$$D/2r \geq \sqrt{1 + 4(N^2/H^2 \tan^2 \alpha_r)}, \quad (2)$$

where D is the distance between catch and release points, r is ball radius, and α is the release angle. The preceding constraint is satisfied if no midair collisions occur. Experiments on the precisely formulated cascade juggling task have illuminated the role of task constraints (Bingham, 1988; Newell, 1986) in assembling coordinations. For example, researchers have tested a hypothesis shaped jointly by Equations 1 and 2 and the physical requirements for braiding repetitive subtasks into a smooth, stable dynamic. The hypothesis is that this action–perception task is characterized by one or a few fixed points. Specifically, a juggled object must be held for three fourths of the cycle time of the hand ($\omega_{hand}{}^{-1}$) regardless of N, H, frequency of juggling, and the inertial properties of the juggled objects, with two thirds ($\omega_{hand}{}^{-1}$) and five eighths ($\omega_{hand}{}^{-1}$) being the most accessible options (e.g., Beek, 1989a, 1989b; Beek & Turvey, 1992). The three-fourths stable point characterizes five- and seven-ball juggling and dominates the early stages of learning three-ball juggling (Beek & van Santvoord, 1992). The two-thirds and five-eighths attractors apply in skilled three-ball juggling, and unexplained fixed points in excess of three-fourths appear in skilled, but substantially slower, three-scarf juggling (Beek & Turvey, 1992).

In the current research, we took a dynamical perspective on an elementary member of the class of ball skills that involve implements. Our purpose was twofold. First, we wanted to provide a test field for the dynamical theory of coordination patterns developed by Schöner and Kelso (1988a, 1988b, 1988c). This theory is significant because it

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aims to incorporate within a single formulation the fundamental dynamics of a particular coordination (i.e., intrinsic dynamics) and the modulation of those dynamics by environmental and informational factors (i.e., required dynamics). Second, we wanted to lay the groundwork for a model experimental system, amenable to systematic dynamical analyses, that could be used to advance the understanding of the basic principles underlying bat–ball coordination. In the child's game of repetitively hitting a ball against a wall after a bounce, the physical properties of the ball, paddle, wall, and floor and the physics of collisions dictate an elaborate pattern of forces that must be addressed by the child if he or she is to succeed in executing, under perceptual control, a succession of complete cycles. In the theory advanced by Schöner and Kelso (1988a, 1988b, 1988c), the child addresses the complexity of forces defined at the level of physical dynamics through collective variables that are specific to the bat–ball task and defined at the level of coordination dynamics (see also Schöner, 1994). Therefore, a model experimental system would do well to capture the essentials of a single-participant, bat–ball task of this kind. An example is our experimental arrangement, with ball and bat pendulums, depicted in Figure 1. In the depicted task, the participant is instructed to hold a bat vertically with his or her right hand and to swing it like a pendulum in a comfortable, continuous way. A ball hanging by a cord from a bar is to be paddled so that it too swings smoothly and repetitively, like a pendulum, within the stipulated boundary conditions given by the instructions and the design of the apparatus.

The task depicted in Figure 1 may be considered among the simplest of bat–ball skills. In general, such skills lie toward the open end of the closed skills–open skills continuum as defined originally by Poulton (1957) and developed by Gentile (1972). That is, they are performed in an environment that is highly unpredictable and constantly changing, such as in lawn and table tennis. By contrast, a "closed skill" is performed in an environment that is stationary or certain. The behavior of the ball in Figure 1 is highly predictable, requiring a fairly simple cycle-to-cycle adjustment to maintain the coordination pattern. On Poulton's (1957) continuum, the bat–ball skill of Figure 1 and its prototype, the child batting a ball against a wall, are closed except for an important source of variability specific to their execution. In the prototype task, uncertainty arises from fluctuations in the force vectors (the forces of contact are not always parallel to the planes normal to the bat, floor, and wall). Similarly, in our simple bat–ball task, fluctuations can occur in the direction of contact between bat and ball. For our goals of evaluating the dynamical theory of coordination patterns within the context of bat–ball skills, and establishing a model experimental system for the study of such skills, a task that is almost closed (with experimentally controllable dynamics) rather than one that is open fully (with experimentally uncontrollable dynamics) provides the most appropriate setting.

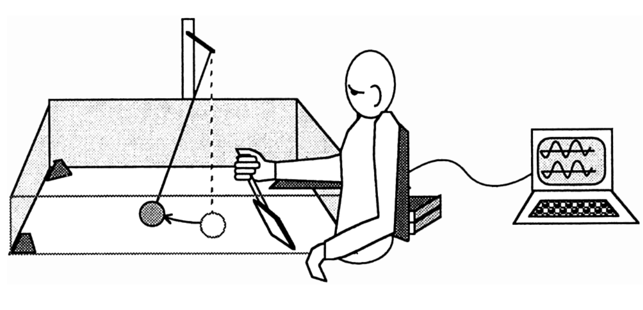

Figure 1. In a simple bat–ball skill, the participant rhythmically bats a ball pendulum with a bat pendulum inside a movement digitizing cubicle. Lengths and masses of the bat and ball pendulums can be manipulated.

The Order Parameter Hypothesis

An important first step in the analysis of our simple bat–ball skill within the dynamical theory of behavioral patterns (Schöner & Kelso, 1988a, 1988c) is the identification of relative phase—the difference between the phase of the ball and the phase of the bat, $\phi = (\phi_{\text{ball}} - \phi_{\text{bat}})$—as a collective variable or order parameter (Haken, 1983). A general observation about the behaviors of complex systems is that they can be modeled successfully in certain circumstances using one or a few macroscopic quantities. These particular quantities or observables provide succinct indexes of the spatiotemporal details of the system's behaviors and are the essential (relevant) quantities for expressing the stabilities of, and changes in, the behavioral patterns exhibited by the system. In the spirit of the term introduced initially by Landau (1936) to describe "degree of order," these quantities are referred to as *order parameters*. Within the general set of arguments making up synergetics (i.e., an interdisciplinary field directed at cooperative phenomena in nonequilibrium, open systems), the notion of order parameter has come to mean a quantity that is created by the cooperation of the individual components and that in turn governs the behavior of these components (Haken, 1975, 1983).

It can be argued that $\phi = (\phi_{\text{ball}} - \phi_{\text{bat}})$ is an order parameter for the task depicted in Figure 1 for at least two reasons. First, it captures the relation between the bat and the ball: The spatiotemporal configuration of the bat and ball pendulums specifies ϕ and, conversely, ϕ specifies the spatiotemporal ordering of the bat and ball pendulums. Second, within a bout of batting, ϕ is relatively invariant over changes in the variables that describe the behavior of the bat and ball, such as their angular velocities and accelerations, and over changes in the underlying muscles, such as changes in their lengths and tensile states (see Haken, Kelso, & Bunz, 1985; Haken & Wunderlin, 1990). Allowing that ϕ is the essential quantity for capturing bat–ball coordination in our simple bat–ball task, it then becomes necessary to derive an understanding of the basic dynamical rule that it obeys. In general terms, the dynamics of pattern formation in a dissipative system of multiple degrees of freedom will have the following form:

$$\dot{\phi} = L(\phi, c, N), \tag{3}$$

where $\dot{\phi}$ is the first-time derivative of the order parameter, c is a control parameter (e.g., the stiffness of an oscillator or the imposed frequency requirement for a coordination pattern), and N is noise. There may be more than one control parameter and more than one form of noise. The strategy to be pursued in deriving L for our bat–ball task is the strategy developed within synergetics and applied with great success to interlimb coordination dynamics (Haken et al., 1985; Kelso, 1994; Kelso & Jeka, 1992; Schöner, Haken, & Kelso, 1986).

It has proved to be the case that, in many situations, the equations for order parameters are of the form

$$\dot{\phi} = -\frac{dV}{d\phi}, \tag{4}$$

where $V(\phi)$ is a so-called *potential function* (Haken, 1983). An analogy can be drawn easily between $V(\phi)$ and the idea commonly expressed in mechanics of a particle in a well that travels downhill to an equilibrium point. If $\phi(t)$ is a solution of Equation 4, then

$$\frac{d}{dt}V[\phi(t)] = \frac{d}{d\phi}V[\phi(t)]\cdot\frac{d}{dt}\phi(t) = -\{f[\phi(t)]\}^2 \leq 0. \tag{5}$$

In words, Equation 5 says that $V(\phi)$ is always decreasing along the solution curves of the differential equation $d\phi/dt = f(\phi)$ and therefore can be construed as its "potential function" (Hale & Kocak, 1991). The equilibrium points of Equation 4, those for which the left-hand side of the equation goes to zero, are the extreme points of $V(\phi)$.

Obviously, the first step in determining L for the bat–ball task is to identify $V(\phi)$. Physics dictates that the bat–ball system of Figure 1 should have a single, most attractive state at $-\pi/2$ rads. Mode-locked states (Period 1 limit cycles) occur at the driving frequency, and the power absorbed by the damped ball pendulum is maximal when its velocity is in phase with the driving force (e.g., Den Hartog, 1956). Therefore, resonance occurs when the bat is 90° ahead of the displacement of the ball; hence, the global attractor: $\phi = (\phi_{\text{ball}} - \phi_{\text{bat}}) = -\pi/2$. The preceding fact does not change if ϕ is replaced by $\phi + 2\pi$. It can be assumed, therefore, that $V(\phi)$ is a periodic, asymmetric function—$V(\phi) \neq V(-\phi)$ or, synonymously, $V(-\phi) = -V(\phi)$—with minima at $-\pi/2$ and $3\pi/2$. The positive 2π-periodic sine function satisfies the preceding description so that, to a first approximation, $V(\phi)$ for the bat–ball task of Figure 1 can be expressed as

$$V(\phi) = k\sin\phi, \tag{6}$$

with the coefficient k interpreted as a control parameter determining the depth of the potential well. What Equation 6 provides is an interpretation of the relation between bat and ball in the coordinated state of repetitive batting. Inserting Equation 6 into Equation 4 yields the following motion equation:

$$\dot{\phi} = -k\cos\phi. \tag{7}$$

As an approximation to L, Equation 7 is insufficient in two important respects. First, it is noiseless (see Equation 3). The bat's motions are governed by the participant, a complicated hierarchy of many subsystems operating at a multiplicity of time scales faster than that of ϕ. These internal degrees of freedom on the side of the batter may be considered the source of a Gaussian white noise process ζ_t, with characteristics $\langle\zeta_t\rangle = 0$ and $\langle\zeta_t\zeta_{t'}\rangle = \delta(t - t')$, and strength $Q > 0$ (see Haken, 1977, Section 6; Schöner et al., 1986). Because of the noise, ϕ will fluctuate within the potential well defined by Equation 4, such that when the potential well is shallow, "movements" of ϕ will be greater than when the potential well is steep.

Second, Equation 7 is limited to instances of our simple bat–ball task for which the intrinsic dynamics of the two components are identical. Specifically, Equation 7 assumes that there is no competition between the oscillating bat and the oscillating ball; the frequency at which the bat and ball are coupled in the coordination pattern is the frequency that each would oscillate at in the uncoupled state. For the general case, the uncoupled frequencies (or eigenfrequencies) of the bat and ball oscillators will not be identical and the achieving of any smooth repetitive batting will involve frequency competition. This frequency competition will act as a detuning of the coordination dynamics expressed in Equation 7. The simplest assumption about the form of this detuning term is that it is the (arithmetic) difference Δ between the uncoupled frequencies, that is, $\Delta\omega = (\omega_{\text{ball}} - \omega_{\text{bat}})$, where ω is frequency in rad s^{-1}. In the mathematical modeling of the coordination dynamics of biological oscillators, such as the array of central pattern generators said to make up the spinal cord of the lamprey eel, the preceding definition of the detuning term is standard (see Cohen, Holmes, & Rand, 1982; Kopell, 1988; Rand, Cohen, & Holmes, 1988). It also is the common interpretation of frequency competition in models of the dynamics of human interlimb coordination (e.g., Amazeen, Sternad, & Turvey, 1996; Jeka & Kelso, 1993; Kelso, DelColle, & Schöner, 1990; R. C. Schmidt, Shaw, & Turvey, 1993; Sternad, Turvey, & Schmidt, 1992; Sternad, Amazeen, & Turvey, 1996; Treffner & Turvey, 1995, 1996).

Taking the preceding considerations into account, Equation 7 can be elaborated to conform more closely to the essential structure of Equation 3:

$$\dot{\phi} = \Delta\omega - k\cos\phi + \sqrt{Q}\zeta_t. \tag{8}$$

In words, changes in the bat–ball coordination pattern are due to (a) the deterministic influences of frequency competition of magnitude $\Delta\omega$ and cooperation manifest as a nonlinear coupling of strength k and (b) the random influences of stochastic forces $(Q)^{1/2}\zeta_t$ arising from degrees of freedom at time scales shorter than that of ϕ.

Formulations such as Equation 8 constitute a frequently used modeling strategy for addressing the cyclic interactions making up rhythmic organizations at many scales and in many different complex biological, chemical, and physical systems (e.g., Murray, 1990; Strogatz & Mirollo, 1988). Models of the kind expressed by Equation 8 are general, requiring knowledge of the observed oscillations of the system but not the particulars of the processes producing the oscillations. The models do not aim to capture the internal structure of the oscillators involved and are not meant to be useful to the study of how oscillations originate. Their usefulness lies in studying the collective behavior of a system of oscillators whose substrate (e.g., neuronal, muscular) and modes of interaction (e.g., forcing functions) are either largely unknown or poorly understood. Specifically, Equation 8 is directed at the coordination dynamics of the simple bat–ball skill of Figure 1, not its physical dynamics (although the latter may affect the former).

Stable Bat–Ball Coordinations and Their Fluctuations

Allowing that Equation 8 reasonably approximates L for the simple bat–ball task depicted in Figure 1, it can be used to predict significant features of the coordination. To begin with, plotting the right-hand side of Equation 8 (ignoring the noise term) against ϕ permits the determination of those values of ϕ at which the time derivative of ϕ goes to zero. These zero crossings identify the stationary values of ϕ for given values of $\Delta\omega$ and k. These stationary points, also referred to as "fixed points" or "equilibrium points," can be stable (attractors) or unstable (repellors) according to whether the slope at the stationary point is negative or positive, respectively; that is, whether

$$d\dot{\phi}/d\phi < 0 \tag{9}$$

or

$$d\dot{\phi}/d\phi > 0. \tag{10}$$

Analytically, the stationary values of ϕ can be calculated from the equation that follows simply from Equation 8 by setting the left-hand side to zero and ignoring noise:

$$\phi = \text{arc}\cos(\Delta\omega/k). \tag{11}$$

Equation 11 has either zero, one, or two principal solutions depending on whether the key quantity, the ratio of the absolute eigenfrequency difference to coupling, $|\Delta\omega|/k$, is greater than, equal to, or less than unity. (Because the cosine function always has absolute value less than or equal to unity, there will be no solutions if $|\Delta\omega|/k > 1$.) Thus, the bat and ball in the task depicted in Figure 1 will exhibit phase and frequency locking if the frequency competition between them is sufficiently small compared with the coupling between them. If no such phase-locking solutions occur, then the bat-and-ball system should exhibit phase drift (i.e., no smooth, continuous, repetitive batting should occur). Elaborating on the phase-locked solutions, Equations 8 and 11 indicate that $\phi = -\pi/2$ when $\Delta\omega = (\omega_{\text{ball}} - \omega_{\text{bat}}) = 0$, $\phi > -\pi/2$ when $\Delta\omega > 0$, and $\phi < -\pi/2$ when $\Delta\omega < 0$. That is, when the uncoupled frequency of the bat is lower than that of the ball, the stable bat–ball coordination should be tending toward inphase; conversely, when the uncoupled frequency of the bat is higher than that of the ball, the stable bat–ball coordination should be tending toward antiphase.

An important understanding of the fluctuations in ϕ follows from recognizing that the inverse of the absolute value of the derivative in Equation 9 is a time, specifically that it is the time taken for the system to return to the stable stationary state after a small perturbation. This time is referred to as the relaxation time, τ_{rel} (Gilmore, 1981; Schöner et al., 1986; Schöner & Kelso, 1988a). Thus,

$$\tau_{\text{rel}} = \left(\frac{d\dot{\phi}}{d\phi}\right)^{-1}. \tag{12}$$

Clearly, τ_{rel} is a measure of the degree of stability of a stationary state. It can be shown (e.g., Gilmore, 1981; Schöner & Kelso, 1988a) that the standard deviation (SD) of ϕ, $SD\phi$, is quantified as

$$SD\phi = \sqrt{Q\tau_{\text{rel}}/2}. \tag{13}$$

Equations 12 and 13 imply that the steeper the slope of a trajectory through the stationary point, the shorter the return time to that point following a perturbation and the smaller the magnitude of $SD\phi$. Returning to Equation 8, the stable stationary point is at $\phi = -\pi/2$ when $\Delta\omega = 0$. Numerical analyses reveal that τ_{rel} and, therefore, $SD\phi$ (given that $SD\phi$ is proportional to τ_{rel} according to Equation 13) are least at $\Delta\omega = 0$ and become larger as $\Delta\omega$ deviates from 0 with constant k.

Overview of the Experiments

In the experiments to be reported, $\Delta\omega = (\omega_{\text{bat}} - \omega_{\text{ball}})$ was varied by manipulating the length of the ball pendulum and the mass and length of the bat pendulum depicted in Figure 1. For simplicity, k was assumed to be constant over different $\Delta\omega$ and over specific environmental demands, such as a set target distance to be achieved by the batted ball. Environment-specific coordination requirements were expected to modify the potential function, Equation 6, and thereby to introduce new forcing terms into the coordination dynamics, Equation 8. The conditions on the bat–ball task were (a) comfortable, repetitive batting without restriction, investigated in Experiment 1; (b) a rigid planar board placing a "hard" (mechanical) restriction on the amplitude of the ball's oscillation, investigated in Experiment 2; and (c) a visually perceived goal line placing a "soft" (informational) restriction on the amplitude of the ball's oscillation, investigated in Experiment 3. The expectation from the dynamical theory of coordination patterns (Schöner & Kelso, 1988a, 1988b, 1988c) was that the proposed dynamics for the bat–ball coordination task depicted in Figure 1, namely, Equation 8, would figure prominently in the behavior of ϕ not only in condition (a) but also in conditions (b) and (c).

Experiment 1

The child repetitively hitting a ball against a wall via a bounce off the floor is exhibiting what might be termed a *behavioral pattern* (e.g., Schöner & Kelso, 1988c). This pattern is a functional coordination in which many physical, biological, and psychological components relate in an ordered fashion to achieve a particular goal: the cyclic contact of bat with ball and ball with wall. Central to the dynamical theory of behavioral patterns are three propositions (see Schöner & Kelso, 1988c). First, on any level of analysis (kinematic, muscular, neural, and below), a behavioral pattern is characterized by a low-dimensional collective variable or order parameter. Second, observable behavioral patterns—with the qualifier *observable,* meaning that the patterns are reproducible and stationary over a certain time scale—are mapped onto the attractors of the dynamics of this order parameter. Third, there are certain parameters (often one, sometimes a few) that act on the collective dynamics nonspecifically or indirectly, meaning, roughly, that there is no formal resemblance between these parameters and the resultant stationary states. Such parameters are the control parameters identified in Equation 3.

Essentials of the behavioral pattern of batting a ball against a wall are retained in the simplified behavioral pattern, depicted in Figure 1, of swinging a bat pendulum to propel a ball pendulum. For this behavioral pattern, the order parameter is relative phase (between the bat and the ball), the attractors are defined by the zero crossings of Equation 8, and the control parameters are undefined but are expected to be task contexts that affect indirectly the magnitude of *k*. Experiment 1 was conducted to evaluate $\phi = (\phi_{ball} - \phi_{bat})$ as the relevant collective variable or order parameter and Equation 8 as its dynamics.

A key feature of Experiment 1 was that the constraints on the bat–ball task were held to a minimum. Participants were asked to make repetitive contact with the ball pendulum by comfortably swinging the bat pendulum. No specific restrictions were imposed on the participants with respect to either the amplitude or frequency of the bat's motions or the amplitude and frequency of the ball's motions. Rather, the participants were required to organize their movements simply in accord with the dynamics arising from the oscillatory tendencies of the bat and the ball. In principle, this requirement should yield the baseline dynamics of bat–ball coordination. This kind of baseline dynamics is called *intrinsic dynamics* (Schöner & Kelso, 1988a, 1988c), a phrase that refers to the dynamics of the behavioral pattern in the absence of any specific behavioral requirement.

A second key feature of Experiment 1 was the use of bat and ball pendulums that were unequal in their uncoupled frequencies. There were four values of $\Delta\omega = (\omega_{ball} - \omega_{bat})$: −2.78, −2.64, 0.389, and 1.163 rad s^{-1}, with the uneven distribution of values dictated, in part, by the physical constraints of the apparatus. Given these magnitudes of $\Delta\omega$, specific predictions about the intrinsic dynamics of the behavioral pattern can be made. For an arbitrary and constant choice of *k*, the predictions of stationary states and their respective stabilities by Equation 8 are presented in Figure 2. The left-hand panel presents ϕ against $\Delta\omega$, and the right-hand panel presents τ_{rel} against $\Delta\omega$ (recalling that $SD\phi \propto \tau_{rel}$). The depicted qualitative predictions of Equation 8 for the particular $\Delta\omega$ values of Experiment 1 can be expressed succinctly as the following:

1. For $\Delta\omega < 0$, $\phi < -\pi/2$; for $\Delta\omega > 0$, $\phi > -\pi/2$.
2. The larger the value of $|\Delta\omega|$, the greater the deviation of ϕ from $-\pi/2$.
3. The plot of ϕ against $\Delta\omega$ intercepts $\phi = -\pi/2$ at $\Delta\omega = 0$.
4. The larger the value of $|\Delta\omega|$, the greater the magnitude of $SD\phi$ ($\propto\tau_{rel}$).

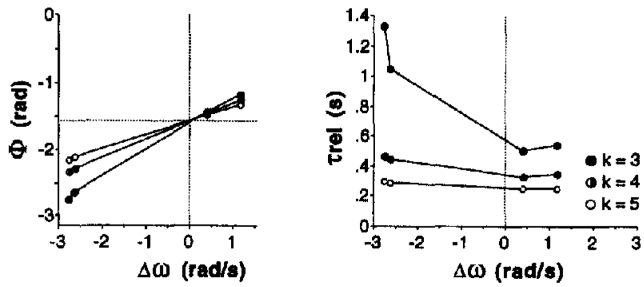

Figure 2. Predicted stable relative phase (left) and predicted standard deviations in relative phase (proportional to τ_{rel}; right) from Equation 8 as a function of $\Delta\omega$ at three parameter settings of *k*.

The task depicted in Figure 1 has several variants. For example, a given spatial separation between the axis of the bat and the axis of the ball defines one environmental context for the behavioral pattern of coordinated bat–ball oscillations. With respect to the child batting a ball against a wall with an intervening bounce, axes separation in Figure 1 is perhaps analogous to the distance of the child from the point of bounce (assuming that a variant of the game requires that the ball bounce within a specified distance from the wall). The question is whether this particular environmental context acts significantly on the dynamics of Equation 8. Intuitively, increasing the axes separation means larger motions of either the bat or the ball or both, but this amplification need not have any effects on the phase-locked behavior of the two oscillators. If relative phase is an appropriate order parameter for the simplest bat–ball skill, then the dynamics of relative phase should remain essentially constant over the differences in the behavior of the component oscillators (e.g., different amplitudes, different maximum velocities) resulting from different separations of the axes. Accordingly, a third key feature of Experiment 1 was the manipulation of the distance between the bat and ball axes. At issue was whether Predictions 1–4 would be invariant over this manipulation.

Method

Participants. There were 16 participants in this experiment, with 8 participants per group. Four participants were graduate students and 14 participants were undergraduate students at the University of Connecticut. The undergraduate students participated in partial fulfillment of a requirement for the introductory psychology course.

Apparatus. The participant sat to one side of a wooden box (see Figure 1) with inside horizontal dimensions of 80 × 80 cm and vertical dimension of 66 cm. The box housed a Sonic-3 space digitizer (SAC Corporation, Westport, CT) and, by attachments to its upper edges, the supports for the participant's right arm and the ball to be batted. The coordinate system for the motions of the bat and the ball, and for the positioning of the rotation points of the bat and the ball, had its origin (0, 0, 0) at a point located at the rear, right corner of the box relative to the participant (see Figure 1). The side of the box along which the participant sat was parallel to the side defining the x-axis. The side perpendicular to the side adjacent to the participant, and to his or her rear, defined the y-axis. The z-axis was perpendicular to the horizontal plane formed by the x- and y-axes.

One end of a 50-cm aluminum rod was fixed to the upper edge of the box on the participant's side parallel to the x-axis. Along the rod was attached a wad of soft material that was to serve as a support for the wrist. The coordinates of the wrist when positioned on this support were (21, 75, 72 cm). These coordinates defined the rotation point for the bat pendulum. A similar rod attachment to the upper edge of the participant's side of the box provided support for the ball (in Figure 1 it is shown attached to the side opposite the participant, for convenience). The upper end of the cord (with the ball at its lower end) was positioned at either (38, 75, 82 cm) to produce a distance (Δaxis) between the ball's axis and the bat's axis of 17 cm or at (32, 75, 82 cm) to produce a distance of 11 cm. These coordinates defined the rotation points for the ball pendulum.

In order to reduce errors induced by sound reflection in the operation of the sonic motion analyzer, eggshell-shaped sponges were used for insulation. These were attached to all interior surfaces of the box and to an 80 × 80 cm board positioned 34 cm above and parallel to the top edges of the box and with the same room x- and y- coordinates of the box.

Materials. A bat pendulum and a ball pendulum, each of variable dimensions, were used for the experiment as illustrated in Figure 1. Their overall dimensions were dictated significantly by the configurational constraints of the motion recording apparatus, the box in which it was housed, and the participant's seated position relative to the box and the supports for the ball and the hand-held bat. The hand-held bat pendulum was composed of an aluminum rod 1 cm in diameter that was inserted into the cylindrical wooden grip 2.5 cm in diameter and 12 cm in length. The total length of the rod and grip was 48 cm. At the lower part of the rod was attached an emitter as part of an ultrasonic motion-recording device. To alter the eigenfrequency of the bat pendulum, a 500-g steel ring was fastened (on half of the trials) at the end of the rod by means of a set screw. The bat aspect of the bat–pendulum setup was a rectangular block of wood 5.2 cm in width and 13.7 cm in length that provided a planar surface for striking the ball. The mass of the rectangular block was 114.9 g, which included the 62.4-g metal set screw used to attach the block to the aluminum rod. The rectangular batting surface could be slid up and down smoothly on the rod to match the length of the ball pendulum. The ball pendulum consisted of a rubber ball (plus emitter) of 8.5 g in mass and 1.4 cm in radius. An electrical cord with a sonic emitter was inserted through the center of the ball so that the tip of the emitter would point downward during the ball's oscillations. This emitter provided a means of recording the ball's motions. The length of the ball pendulum (i.e., the length of the electrical cord between the ball and the point of attachment to the support rod) was either 30 or 40 cm.

To quantify Δω depends on knowing the eigenfrequencies of each of the two systems: bat and ball. Because the suspended ball is a simple pendulum (see Figure 1), the eigenfrequency of the ball pendulum is $\omega_{ball} = (g/D)^{1/2}$, where g is the constant acceleration due to gravity and D is the distance from the center of mass of the ball to the axis of rotation of the ball. The eigenfrequency of the bat pendulum is the eigenfrequency of the simple equivalent gravitational pendulum, $\omega_{bat} = (g/Le)^{1/2}$, where Le is the equivalent simple pendulum length and g is the constant acceleration due to gravity. Le is calculable from the magnitudes of component parts of the hand-and-bat pendulum, that is, the grip, shaft, added mass, bat, and hand (the averaged hand mass was calculated as 0.006 × average body mass; see Kugler & Turvey, 1987).

There were four values of $\Delta\omega = (\omega_{ball} - \omega_{bat})$: $\Delta\omega = -2.78$, $\Delta\omega = -2.64$, $\Delta\omega = 0.389$, and $\Delta\omega = 1.163$. These four Δω values resulted from varying appropriately (a) the mass of the bat pendulum and the position of its batting surface and (b) the length of the ball's string. The irregularity of the distribution of Δω values was due to the limited ranges of variation permitted by the bat construction and the configuration of participant and apparatus.

Trajectories of the bat and ball pendulums were collected using a Sonic-3 space digitizer. Sound "sparks" were issued from the emitters at the tips of the bat and the ball at a rate of 90 s^{-1}. The digitizer calculated the distance of the emitter from the best three of the four microphones positioned on the floor to form an x, y, and z Euclidian coordinate grid. This digitized information was stored on an 80286-based microcomputer using MASS digitizer software (Engineering Solutions, Columbus, OH). By using a peak picking algorithm (explained in the section on data reduction) and other programs, it was possible to calculate the angular measures of position, velocity, frequency, and relative phase of the bat and ball pendulum movements.

Procedure. Each participant was seated relative to the apparatus as depicted in Figure 1. The participant was instructed to swing the bat pendulum comfortably, watch carefully, and to make contact with the ball pendulum in as consistent an oscillatory fashion as possible. The participant also was instructed to use only the wrist in swinging the bat pendulum and to avoid using either the arms or the fingers to control the movement. Movements tended to occur in a plane parallel to the participant's sagittal plane. Some practice swings were allowed, but the participant was given no feedback regarding his or her performance. When the participant seemed to show consistency in doing the task, the experimenter began the recording of the oscillatory motions. Each trial lasted 30 s. The overall experiment took about 60 min.

Design. A mixed factorial design involving one between-subjects variable (Δaxis) and one within-subject variable (Δω) was used. There were five repetitions for each of the four values of Δω, presented in a randomized order within a block. For one group of 8 participants, Δaxis = 17 cm; for the other group of 8 participants, Δaxis = 11 cm.

Data reduction. The digitized displacement, time-series data obtained from the motions of the bat and ball pendulums were smoothed using a triangular moving average procedure with a window size of 5 points. Then, each trial was subject to software analysis to determine the relative phase angle ϕ and its variability, $SD\phi$. The analysis proceeded in five steps. First, a peak picking algorithm was used to determine the time of maximum forward extension of the pendular trajectories. From the peak extension times, the frequency (Hz) of oscillation for the nth cycle was calculated as

$$f_i = 1/[(\text{time of peak extension } n + 1) - (\text{time of peak extension } n)]. \quad (14)$$

The mean frequency of oscillation for one trial was calculated from these cycle frequencies. Second, the midpoint of oscillation was determined from the displacement data by taking the mean across each trial's maximum and minimum points; data subsequently were subtracted from this midpoint to obtain the measure of adjusted displacement. Third, to calculate the normalized velocity of each pendulum, the velocity of each pendulum was divided by the mean angular frequency of each trial. Fourth, the normalized velocities of the bat and ball trajectories were divided by the adjusted displacement data to determine the phase angles ϕ_{bat} and ϕ_{ball} according to

$$\phi_{ij} = \arctan(\dot{x}_{ij}/\Delta x_{ij}), \quad (15)$$

where the numerator is the normalized velocity of the time series of the pendulum i at sample j (calculated from the third step, described above) divided by Δx_{ij}, defined as the displacement of the time series at sample j minus the average displacement for the trial (calculated from the second step, described above). The relative phase ϕ was calculated as $\phi_{ball} - \phi_{bat}$. The mean of ϕ for each trial was taken as the estimate of the stable stationary state and was calculated as the average over the cycles making up the time series. Finally, by averaging ϕ from all participants, an overall mean ϕ was obtained for each $\Delta\omega$ under each Δaxis. Similarly, $SD\phi$ was obtained from the fluctuations of ϕ for each trial. Then, $SD\phi$ was averaged for each $\Delta\omega$ for each participant and, by averaging mean values from each participant, we obtained the overall mean $SD\phi$ for each $\Delta\omega$ under each Δaxis.

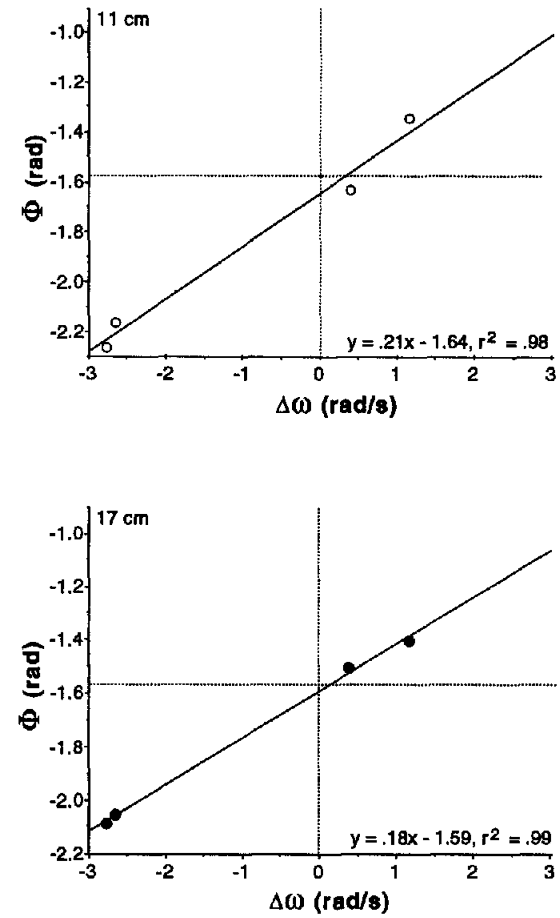

Figure 3. Mean relative phase as a function of $\Delta\omega$ and Δaxis (11 vs. 17 cm) in Experiment 1.

Results and Discussion

Mean ϕ and mean $SD\phi$ as a function of $\Delta\omega$ and Δaxis are plotted in Figure 3 and Figure 4. A repeated measures analysis of variance (ANOVA) was conducted on ϕ, with $\Delta\omega$ and Δaxis as independent variables. There was a significant main effect of $\Delta\omega$, $F(3, 42) = 66.61$, $p < .0001$, no significant effect of Δaxis, $F(1, 14) < 1$, and no significant interaction between $\Delta\omega$ and Δaxis, $F(3, 42) = 1.09$, $p > .05$. A second ANOVA on $SD\phi$, with $\Delta\omega$ and Δaxis as independent variables, revealed main effects of both $\Delta\omega$, $F(3, 42) = 12.71$, $p < .0001$, and Δaxis, $F(1, 14) = 4.78$, $p < .05$, but no interaction, $F(3, 42) < 1$.

A comparison of Figures 3 and 4 with Figure 2 revealed that Predictions 1–4 were closely approximated in both conditions of Δaxis. First, when $\Delta\omega < 0$, $\phi < -\pi/2$, and when $\Delta\omega > 0$, $\phi > -\pi/2$. Second, larger values of $|\Delta\omega|$ induced greater deviations of ϕ from $-\pi/2$. Third, the linear regression of ϕ on $\Delta\omega$ yielded intercepts at $\Delta\omega = 0$ of $\phi = -91.04°$ and $\phi = -94.12°$ for Δaxis = 17 cm and Δaxis = 11 cm, respectively, values that were statistically indistinguishable ($p > .05$) from the expected $-90°$ ($-\pi/2$ rad). Fourth, larger values of $|\Delta\omega|$ were associated with greater magnitudes of $SD\phi$ ($\propto \tau_{rel}$). A possible exception was $\Delta\omega = 1.164$. A t test (Tukey's) showed that there was no statistical difference ($p > .05$) between the $SD\phi$ at $\Delta\omega = 0.386$ and $\Delta\omega = 1.164$ for either Δaxis. Further comparison of Figure 4 with Figure 2 (right side) suggested that even an apparently odd feature of $SD\phi$, specifically, the sharp increase in $SD\phi$ between $\Delta\omega = -2.64$ and $\Delta\omega = -2.78$, was expected from Equation 8. (This is indicated for the smallest value of k in Figure 2.)

In addition to the continuous measure of ϕ, we computed a discrete measure of ϕ made once per cycle. We did this to provide a check on possible contaminations of the variability estimate. Computations of $SD\phi$ based on the continuous relative phase could have been affected by the possibility that the bat–ball contact might have changed ϕ_{ball} more rapidly than ϕ_{bat} to the extent that ϕ underwent large variations around the impact points. For oscillations at higher frequencies, a larger proportion of the cycle would have been consumed by the variability induced by bat–ball contact, with a consequent confounding of the $SD\phi$ measure by frequency per se. An additional confound might arise from the imperfect sinusoidal motion of the bat in the phase plane creating a source of deterministic variability (Fuchs & Kelso, 1994). Consequently, $SD\phi$ was computed for each trial of each condition using the discrete estimate of ϕ. The patterning of "discrete" $SD\phi$ as a function of the conditions of the experiment replicated the patterning of $SD\phi$ on the basis of the continuous measure of ϕ shown in Figure 4 and the results of the corresponding ANOVA. The two sets of numbers differed only in magnitude (with discrete being smaller than continuous). Therefore, we can be confident that Figure 4 depicts the relative stabilities of the coordination patterns assembled under the conditions of this experiment.

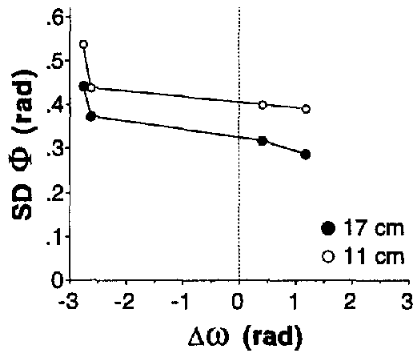

Figure 4. Standard deviation of relative phase as a function of $\Delta\omega$ and Δaxis in Experiment 1.

Despite the high degree of agreement between the results and Equation 8, one feature of the data requires further examination: the significant tendency for $SD\phi$ to be greater by a constant amount under Δaxis = 11 cm than Δaxis = 17 cm. The absence of an effect of Δaxis on ϕ suggests that the influence of Δaxis was not exerted via a modulation of k in Equation 8. Indeed, setting $k = 5$ (rad s^{-1}) in Equation 8 produced ϕ magnitudes of -2.160, -2.127, -1.493, and -1.336 rad, corresponding closely to the observed values of ϕ averaged over Δaxis of -2.173, -2.105, -1.570, and -1.371 rad, respectively. The implication is that the stationary states mapped onto the attractors of the order parameter dynamics of Equation 8 under $k \approx 5$. Consequently, an account of the aspect of the $SD\phi$ data noted earlier must be sought elsewhere. At this juncture, the only candidate is Q, the strength of the stochastic force. As is evident from Equation 12, τ_{rel} is a deterministic feature of the order parameter dynamics. τ_{rel} follows strictly from Equation 8 for specific values of $\Delta\omega$ and k. By contrast, $SD\phi$ is the result of both deterministic and stochastic processes; it follows from the product of τ_{rel} and Q, as is evident from Equation 13. Q seems to have been generally less under Δaxis = 17 cm than Δaxis = 11 cm. Calculation of Q using Equation 13 with τ_{rel} calculated from Equations 8 and 9 yielded means of 0.93 and 1.43 for the 17-cm and 11-cm separations, respectively.

The stochastic term of Equation 8 has two sources: The subsystems of the participant producing the batting motions and the parameters of the ball pendulum. Research on human interlimb rhythmic coordination provides little reason to assume that Q of an individual participant varies with conditions (e.g., R. C. Schmidt & Turvey, 1994; Sternad et al., 1996; Treffner & Turvey, 1995), although motor behavior in other tasks might suggest otherwise (e.g., R. A. Schmidt & Sherwood, 1982). The more likely source of variation in the current experiment is in the parameters governing oscillations of the ball pendulum. If the manner in which the ball was struck created a time-dependent variation in the cord by which the ball was suspended from its rotational axis (e.g., a time-dependent variation in the axis-to-ball distance), then the stiffness of the ball pendulum (the restoring torque due to gravity) would be variable during oscillation. The manipulation of Δaxis could have affected the conditions of bat–ball contact, with the resultant variations in the ball pendulum's parameters greater at the smaller Δaxis. In this regard, the coupled frequency at which the bat and ball phase locked was less on the average for Δaxis = 17 cm than for Δaxis = 11 cm; the contrast was 8.67 rad s^{-1} (1.38 Hz) versus 9.93 rad s^{-1} (1.58 Hz).

The order parameter hypothesis under consideration contrasts with an alternative hypothesis that could be generated from the more familiar treatment of a force-driven, damped harmonic oscillator. The alternative hypothesis is that the participant behaves simply to maximize the transfer of power from the bat to the ball. This hypothesis can be referred to as the *resonance hypothesis*. Specifically, the participant should adjust ω (the coupling frequency) to the eigenfrequency of the ball so that the phase lock guaranteeing maximum power absorption by the ball, namely, $\phi = -\pi/2$, always is achieved by simply satisfying $\omega/\omega_{ball} = 1$. That is, although the eigenfrequency of the driven system remains fixed, the driver tries to adjust its running frequency to the preferred frequency of the driven system. The plot of ϕ against ω/ω_{ball} presented in Figure 5 contradicts the resonance hypothesis. Inspection of Figure 5 reveals that for none of the conditions did $\omega/\omega_{ball} = 1$ and for none of the conditions did phase locking occur at $\phi = -\pi/2$. Rather, as both Figure 3 and the statistical analyses confirm, the bat–ball coordination dynamics conformed to the order parameter hypothesis: According to Equation 8, a stable coordination exists at $\phi = -\pi/2$ only when $\Delta\omega = 0$.

The failure of the resonance hypothesis and the insignificance of Δaxis in determining the equilibria of the bat–ball coordination complement each other in underscoring the importance of the coordination dynamics represented by Equation 8. Participants in the experiment oriented to the stabilities dictated by the coordination dynamics of Equation 8 rather than the resonance associated with the task's physical dynamics, and they did so identically over variations in the physical dynamics (pattern of batting forces, ball's trajectory) necessarily induced by differences in the bat-to-ball axes.

In summary, Experiment 1 provided support for the order

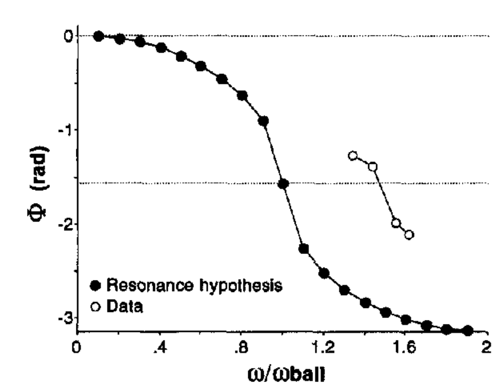

Figure 5. Data from Experiment 1 on stable relative phase compared with predictions of the resonance hypothesis.

parameter hypothesis of the bat–ball task depicted in Figure 1: It seems reasonable to assume that $\phi = (\phi_{ball} - \phi_{bat})$ is the relevant collective variable or order parameter and that Equation 8 expresses the intrinsic dynamics of the bat–ball coordination. We can now examine the generalization of these dynamics to situations in which batting the ball must satisfy certain goals that demand departures from the stable coordinations defined intrinsically.

Experiment 2

In the child's game of repetitively hitting a ball against a wall after a bounce, an important contribution to the coordination dynamics is provided by the fact that the wall and floor provide resistances greater than the ball's own momentum. The ball rebounds from a resistance with a force that depends on the magnitude of the resistance (firmness of wall and floor), the speed and mass of the ball, the ball's coefficient of restitution, the friction of the impact, and the energy lost at impact (Broer & Zernicke, 1979). When the child's paddle meets the ball, the total resulting momentum is the sum of the ball's momentum (Mass × Velocity) and the paddle's momentum, with the resulting motion in the direction of the paddle's (greater) momentum. That is, the ball is given reverse momentum proportional to the sum of the original momentum minus the momentum still possessed by the paddle. In Experiment 2, this essential impact feature of the task of batting a ball against a wall was reproduced in the pendular bat–ball task of Figure 1 by the introduction of a hard, planar surface oriented perpendicular to the ground plane and at a specific distance from the ball's axis of rotation. Introducing a rigid "goal surface" means that there are now two forcing terms in the bat–ball dynamics: one from the bat and one from the goal surface. Additionally, in Experiment 2 the distance of the stationary goal surface was manipulated to vary systematically the minimal force per contact required to satisfy the cyclic task of propelling the ball to the goal surface.

In the dynamical theory of behavioral patterns (e.g., Schöner & Kelso, 1988a, 1988c), any physical feature of the environment that affects the behavioral pattern, but that is itself unaffected by the behavioral pattern, is treated parametrically. Stated in terms of Equation 3, the proposal is to incorporate the environment as an additional c. Two assumptions define the strategy by which environmental factors are to be incorporated (Schöner & Kelso, 1988a, 1988c), and they can be stated in terms of the variation of the simple bat–ball task studied in Experiment 2. First, the hard goal surface introduces an additional potential into the dynamics of the order parameter $\phi = (\phi_{ball} - \phi_{bat})$. Second, the coordination dynamics expressed by Equation 8, manifest in the absence of a resistive goal surface, endures in the presence of the resistive goal surface and the rebound forces it introduces. The latter assumption means that the intrinsic dynamics will still be apparent in the particular behavioral pattern produced in batting the ball against the resistive goal surface.

Patently, the key idea is that the resistive goal surface introduces an additional forcing term represented by a second potential that, when added to the original potential, changes the geometry of the potential governing the order parameter dynamics. How should this second potential be defined? The first step, following the insights of Schöner and Kelso (1988a, 1988c), is recognizing that the added environmental feature must be expressed in terms of $\phi = (\phi_{ball} - \phi_{bat})$. The original potential expresses the coupling of the bat and ball and is asymmetric with its minimum at $\phi = -\pi/2$ when $\Delta\omega = (\omega_{ball} - \omega_{bat}) = 0$ (see Equation 6). By contrast, the new potential expresses the coupling between the ball and the resistive goal surface. It readily can be shown that this new potential is symmetric with its minimum at $\phi = (\phi_{ball} - \phi_{bat}) = 0$.

The rebound force provided by the fixed resistive surface will be proportional to the impact force of the ball with the surface, which is in turn proportional to the force delivered to the ball by the bat. Assuming that $\Delta\omega = (\omega_{ball} - \omega_{bat}) = 0$, and a constant, sinusoidal driving force provided by the bat, power absorption by the ball is proportional to the Lorentz function

$$f(X) = 1/(1 + X^2), \tag{16}$$

where $X = \cot\phi$ (Kittel, Knight, Ruderman, Helmholz, & Moyer, 1973). (From the cotangent function's relation to the sine function, it follows that the right-hand side of Equation 16 is formally equivalent to $\sin^2\phi$). We have noted already, albeit indirectly, that the maximum of Equation 16 occurs when $\phi = -\pi/2$ ($\cot\phi = 0$). Its minimum occurs when $\phi = 0$ ($\cot\phi$ is indefinitely large and negative). When ball and bat are in phase, the ball simply is displaced back and forth by the bat acting against the restoring force of gravity. Consequently, at $\phi = (\phi_{ball} - \phi_{bat}) = 0$, the power transferred from the bat to the ball will be minimal and therefore the rebound force delivered by the resistive surface will be minimal. Therefore, the potential brought in by adding the environmental feature of a resistive surface can be expressed as

$$V(\phi)_{\text{resistive goal}} = -k_2 \cos\phi, \tag{17}$$

where k_2 is the coupling coefficient. Abiding by the proposals for incorporating the environment into the order parameter dynamics (Schöner & Kelso, 1988a, 1988c), the full expression of the potential function for the intended task of batting the ball against a resistive surface would be

$$V(\phi) = k_1 \sin\phi - k_2 \cos\phi, \tag{18}$$

where $k_1 \sin\phi$ is the intrinsic potential. In summary, where the force required to displace the ball from its rest position to the resistive surface is governed by a potential with a minimum at $\phi = -\pi/2$, the force provided by the resistive surface is governed by a potential with a minimum at $\phi = 0$. A plot of Equation 18 is given in Figure 6. Following the steps discussed in the introduction to this article, the appropriate motion equation for repetitively batting the ball against the resistive surface can be generated by (a) taking the derivative of the negative of Equation 18 and (b) adding the frequency competition and stochastic force. Thus,

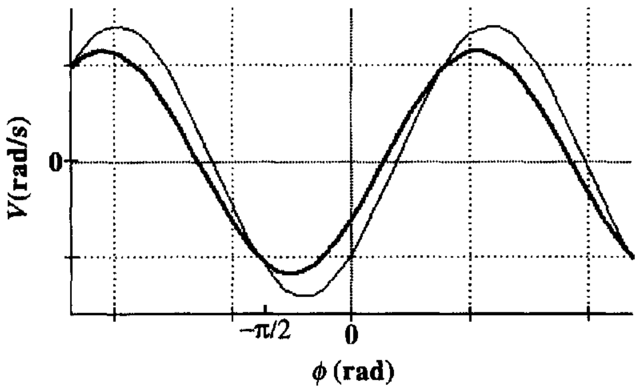

Figure 6. A plot of the potential defined by Equation 18 for $k_1 = 5$, $k_2 = 5$ (light curve) and $k_1 = 5$, $k_2 = 3$ (dark curve).

$$\dot{\phi} = \Delta\omega - k_1 \cos \phi - k_2 \sin \phi + \sqrt{Q}\zeta_t. \quad (19)$$

Experiment 2 is an evaluation of Equation 19 using two goal distances and two $\Delta\omega$ magnitudes (−2.78 and 1.163) from Experiment 1. A rigorous experimental test of Equation 19 requires explicit predictions that must be based on reasonable estimates of the parameters k_1 and k_2. Pursuing a minimalist strategy (Schöner & Kelso, 1988a, 1988b, 1988c), k_1 as the parameter of the intrinsic dynamics is assumed to remain fixed over the introduction of specific environmental influences on the bat–ball coordination. From Experiment 1, k_1 should be approximately 5 rad s^{-1}. Another minimal assumption is that k_2 need not be identical for any two different placements (distances) of the resistive goal surface. From considerations of just the ball-resistive surface system, other things being equal, the closer the ball is to its maximum angular excursion at the point of contact with the resistive surface, the smaller will be the force of rebound. As a pendulum closes in on its maximum displacement, its angular velocity approaches zero. Consequently, it may be supposed that a nearer resistive surface (relative to the ball's axis of rotation) will contribute more to the motion of the ball, and thereby to the dynamics of bat–ball coordination, than a further resistive surface. Figure 7 shows the ϕ and τ_{rel} predictions of Equation 19 derived numerically for a near surface, with $k_1 = 5$, $k_2 = 5$, and a far surface, with $k_1 = 5$, $k_2 = 3$ (see Figure 6). The main predictions can be stated simply as follows:

1. For $\Delta\omega < 0$ and $\Delta\omega > 0$, $\phi > -\pi/2$.
2. The smaller the value of $|\Delta\omega|$, the greater the deviation of ϕ from $-\pi/2$ in the direction of zero.
3. The greater the distance of the surface (or the smaller the value of k_2), the greater the deviation of ϕ from $-\pi/2$.
4. The larger the value of $|\Delta\omega|$, the greater the magnitude of $SD\phi$ ($\propto \tau_{rel}$).
5. The greater the distance of the surface (or the smaller the value of k_2), the greater the magnitude of $SD\phi$ ($\propto \tau_{rel}$).

Except for Prediction 4, the predictions from Equation 19 are nonintuitive. Consider, for example, Prediction 1. It says that regardless of the gradient of frequency competition (a bat of higher uncoupled frequency than the ball or vice versa), the equilibria of the bat–ball coordination dynamics drifts toward in phase. Consider also Prediction 2. It says that as the frequency competition between the bat and ball is reduced, the equilibrium point drifts away from rather than toward the value of $-\pi/2$, defining the stable fixed point of bat–ball oscillations in the absence of competition. Experiment 2 therefore provides a strong test of the modeling strategy behind Equations 8 and 19.

Method

Participants. Eight undergraduates at the University of Connecticut participated in partial fulfillment of an introductory psychology course requirement.

Materials and procedure. The materials and procedure were the same as in Experiment 1, with the addition of a rectangular wooden board (30 × 34 × 0.5 cm) positioned perpendicular to the ground plane at either 9 or 16 cm from the axes of rotation of the ball. The two goal line distances were selected on the basis of the average distance reached by the ball in the nonconstrained batting of Experiment 1 (under the condition of a 11-cm distance between the axes of bat and ball). The 16-cm location was farther than this average distance, and the 9-cm location was closer than this average distance. Participants were instructed simply to bat the ball against the board continuously and as consistently as possible for the 30-s period of each trial.

Design. A 2 × 2 factorial design of two within-subject variables ($\Delta\omega$ and target distance) was used for Experiment 2. The two extreme values of $\Delta\omega$ were chosen from Experiment 1: $\Delta\omega$ = 1.163, $\Delta\omega$ = −2.78. Each participant repeated each of the four conditions five times, for a total of 20 trials. Trials were randomized within each block, with a whole session lasting about 50 min.

Results and Discussion

Mean ϕ and mean $SD\phi$ as a function of $\Delta\omega$ and distance of the resistive goal surface are plotted in Figure 8. An ANOVA conducted on ϕ with $\Delta\omega$ and distance showed a significant main effect of $\Delta\omega$, $F(1, 7) = 21.33$, $p < .05$, a significant interaction between $\Delta\omega$ and distance, $F(1, 7) = 6.07$, $p < .05$, but no main effect of distance, $F(1, 7) = 4.57$, $p > .05$. A second ANOVA on $SD\phi$ with $\Delta\omega$ and distance as independent variables showed a main effect of $\Delta\omega$, F(1,

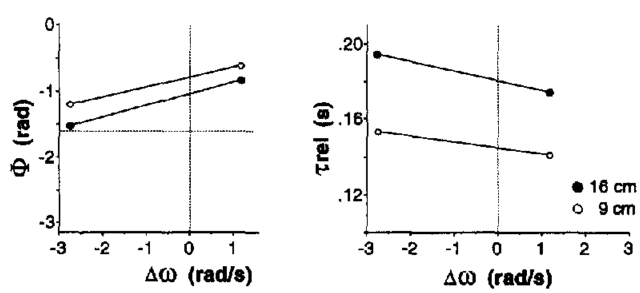

Figure 7. Predicted stable relative phase (left) and predicted standard deviations in relative phase (proportional to τ_{rel}, right) from Equation 18 as a function of $\Delta\omega$ and distance of the resistive surface in Experiment 2.

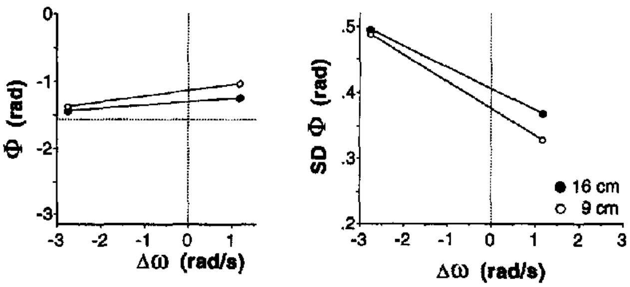

Figure 8. Stable relative phase (left) and standard deviation of relative phase (right) as a function of $\Delta\omega$ and distance of the resistive surface in Experiment 2.

7) = 17.44, $p < .005$, but no effect of distance, $F(1, 7) < 1$, and no interaction, $F(1, 7) > 1$.

It is apparent from the comparison of Figures 7 and 8 that the major predictions of Equation 19—Predictions 1, 2, and 4—were confirmed: $\phi \in (0, -\pi/2)$ was satisfied for both $\Delta\omega$ values and both distances; ϕ under $\Delta\omega = 1.16$ deviated more from $-\pi/2$ in the direction of zero than ϕ under $\Delta\omega = -2.78$; and $SD\phi$ under $\Delta\omega = 1.16$ was less than $SD\phi$ under $\Delta\omega = -2.78$. Although the results involving the distance of the resistive surface were in the predicted directions, they failed to reach statistical significance. A possible implication (assuming that experimental noise was not swamping the effect) is that the difference between k_2 values for the 9- and 16-cm distances was less than the $k_2 = 5$ (9-cm) and $k_2 = 3$ (16-cm) parameter choices used to generate Figure 7.

In summary, the results of Experiment 2 provide support for a major assumption of the dynamical theory of behavioral patterns (Schöner & Kelso, 1988a, 1988b, 1988c). The assumption is that an environmental contribution to bat–ball coordination in our simple bat–ball task can be incorporated at the level of the order parameter dynamics in the form of an additional potential. In the current case, the additional potential was symmetrical about and minimal at $\phi = 0$. This potential, added to the asymmetrical potential (minimal at $\phi = -\pi/2$) of the intrinsic dynamics, deflected the bat–ball coordination in the direction of in-phase coordination.

Experiment 3

Recalling again the child's game of repetitively hitting a ball against a wall after a bounce, an important restraint on the skill can be introduced by adding a line to the wall, parallel to the ground, such that the ball's contact with the wall must always be either above the line, below the line, or on the line. Performing this task, in which a specific behavioral requirement must be satisfied, calls for a departure from the intrinsic dynamics shaped by the physical properties of the ball, paddle, wall, and floor and the physics of collisions. In Experiment 3, the strictly informational aspect of this particular environmental amendment to the task of batting a ball against a wall is mimicked in the bat–ball task of Figure 1. A nonresistive target line is introduced at a specific distance from the ball's axis of rotation that must be reached by the ball. Successful performance of the task in Experiment 3 means that ϕ must achieve a particular value such that the ball, when batted, attains the goal line perfectly (no undershoot or overshoot).

An important conceptual move in the dynamical theory of behavioral patterns (e.g., Schöner & Kelso, 1988a, 1988c) is to treat environmental information about required ϕ as being functionally similar to a force supplied by the environment, such as the force provided by the ball's contact with the resistive surface of Experiment 2. This conceptual move was motivated by an experiment reported by Tuller and Kelso (1989; see also Zanone & Kelso, 1992) using a procedure similar to that introduced by Yamanishi, Kawato, and Suzuki (1980). This procedure provides a means of probing the attainability and stability of each $\phi \in (0, 2\pi)$ in bimanual 1:1 frequency locking for a given $\Delta\omega$. Briefly, a bimanual movement pattern is paced by two metronomes of identical frequency whose relative phase can be varied. On a particular trial, if the left metronome leads the right metronome by 10°, then the participant's task on that trial is to produce the ϕ that matches the metronomically established ϕ, referred to as ϕ_ψ, with ψ designating "intended." The fundamental result is that $\phi_\psi = 0$ and $\phi_\psi = \pi$ are the least variable patterns performed and that they attract neighboring conditions—the plot of $|\phi - \phi_\psi|$ against ϕ_ψ passes through zero and π with negative slopes (Tuller & Kelso, 1989; Yamanishi et al., 1980).

The central idea in addressing the observed dynamics using this procedure is consonant with that applied to the dynamics of Experiment 2: that the potential function $V(\phi)$ identified in Equation 6 represents the coordinated state of the system when functioning under intrinsic criteria and that this feature of the potential should persist when extrinsic criteria are imposed (Schöner & Kelso, 1988c). Consequently, the strategy is to include ϕ_ψ in the order parameter dynamics by adding a term to $V(\phi)$ that attracts the intrinsic relative phase to the required relative phase ϕ_ψ. Detailed theoretical analysis (Schöner & Kelso, 1988a, 1988b) of the Tuller and Kelso (1989) task and data leads to

$$V(\phi) = -a\cos(\phi) - b\cos(2\phi) - c\cos(\phi - \phi_\psi)/2. \quad (20)$$

As argued earlier, because a potential is the integral of a force, the terms associated with a and b and the term associated with c can be conceptualized, respectively, as intrinsic and extrinsic "forces" affecting the collective variable. These forces can either cooperate or compete, as indexed by the ratio $c/(4b + a)$ (Schöner & Kelso, 1988a).

The bimanual task to which Equation 20 applies is different in important ways from the bat–ball task of Experiment 3. In the Tuller and Kelso (1989) experiment, the phasing of the metronomes provided environmental information for the participant about the relative phase that the participant was to produce. Thus, the required dynamics ϕ_ψ is well defined. In this experiment, the goal line is information about the required extremum of the ball's displacement; it is not information about the required ϕ. If the ball reaches the goal line each cycle, without either under- or overshoot, then the required ϕ would (patently) have been achieved. Therefore, the required ϕ must be a stable solution of an order parameter equation that includes (a) the intrinsic dynamics of the bat and ball and (b) the boundary condition defined by the goal line specifying the spatial extremum of the ball's trajectory. It can be assumed that Equation 6 accommodates the potential of relevance to the intrinsic dynamics of the bat and ball. At issue is how to accommodate the potential for the boundary condition defined by the goal line specifying the spatial extremum of the ball's trajectory. Experiment 3 was designed and conducted in a manner similar to Experiment 2 to facilitate the comparison of a dynamics whose essential structure is unknown (the task in Experiment 3) and one whose essential structure seems to be reasonably well understood (the task in Experiment 2).

Method

Participants. Sixteen undergraduates at the University of Connecticut participated in this experiment: 8 participants in each of two groups defined by the distance of the goal line. The students participated in partial fulfillment of a requirement for the introductory psychology course.

Materials and procedure. A lightweight "surface" consisting of strips of paper was used as a nonresistant goal line. It was hung vertically in a plane perpendicular to the ball's plane of motion depicted in Figure 1 on a movable bar that was affixed to the upper edge of the wall of the box adjacent to the participant. That is, the goal line was parallel to the ball's rotation axis. As in Experiment 2, the goal line was located at either 16 cm or 9 cm from the axis of rotation of the ball. The two goal line distances were selected on the basis of the average distance reached by the ball in the nonconstrained batting of Experiment 1. The 16-cm location was farther than this average distance, and the 9-cm location was closer than this average distance. To ensure precision in the control of batting in Experiment 2, participants were instructed to brush the goal line lightly with the ball and to guard against both under- and overshoot. With relatively little practice, the 16 participants were able to perform the task with the requisite accuracy.

Design. One between-subjects variable (target distance) and two within-subject variables ($\Delta\omega$ and mode) constituted a 2 × 2 × 2 mixed factorial design. The two mode conditions were defined by whether the visual target was present (goal mode) or not (i.e., comfortable batting mode). Two extreme values of $\Delta\omega$ were chosen from Experiment 1: $\Delta\omega = 1.163$ and $\Delta\omega = -2.78$. Each participant repeated each of the four conditions five times, resulting in a total of 20 trials. Trials were randomized within each block, with a session lasting about 60 min.

Results and Discussion

Mean ϕ and mean $SD\phi$ as a function of $\Delta\omega$ and goal line distance (9 cm, 16 cm, and no goal line) are plotted in Figure 9. A repeated measures ANOVA was conducted on ϕ, with $\Delta\omega$ and goal line distance as independent variables. The ANOVA revealed significant main effects of $\Delta\omega$, $F(1, 14) = 68.36$, $p < .0001$, and distance, $F(1, 14) = 22.28$, $p < .001$, but no interaction between $\Delta\omega$ and distance, $F(1, 14) = 4.16$, $p > .05$. Two ANOVAs also were conducted on ϕ with $\Delta\omega$ and mode as independent variables, one ANOVA for each goal line distance. For the 16-cm target, there were significant main effects of $\Delta\omega$, $F(1, 7) = 105.27$, $p < .0001$, and mode, $F(1, 7) = 6.56$, $p < .05$, but no interaction, $F(1, 7) = 1.43$, $p > .05$. For the 9-cm target, there were significant main effects of $\Delta\omega$, $F(1, 7) = 35.89$, $p < .001$, and mode, $F(1, 7) = 15.60$, $p < .01$, together with a significant interaction, $F(1, 7) = 8.65$, $p < .05$.

An ANOVA on $SD\phi$ with $\Delta\omega$ and distance as independent variables revealed main effects of $\Delta\omega$, $F(1, 14) = 37.91$, $p < .0001$, and distance, $F(1, 14) = 9.07$, $p < .01$, and a significant interaction, $F(1, 14) = 5.93$, $p < .05$. Additional ANOVAs were conducted on $SD\phi$ with $\Delta\omega$ and mode as independent variables, one ANOVA for each of the two distances. For the 16-cm group, there was a main effect of $\Delta\omega$, $F(1, 7) = 23.04$, $p < .01$, but neither mode, $F(1, 7) < 1$, nor the interaction, $F(1, 7) < 1$, was significant. For the 9-cm group, there were significant main effects of both $\Delta\omega$, $F(1, 7) = 34.60$, $p < .001$, and mode, $F(1, 7) = 30.07$, $p < .001$, and a significant interaction, $F(1, 7) = 7.41$, $p < .05$.

Finally, ANOVAs were conducted on the intrinsic dynamics of the 9-cm group and the intrinsic dynamics of the 16-cm group. That is, the non-goal-line bat–ball coordination (the same coordination as studied in Experiment 1) was compared across the two groups of participants. The ANOVAs revealed that the two distance groups did not differ in respect to either ϕ or $SD\phi$, $F(1, 14) < 1$ for both measures. Both measures, however, were significantly affected by $\Delta\omega$: ϕ, $F(1, 14) = 122.33$, $p < .0001$; $SD\phi$, $F(1, 14) = 23.85$, $p < .001$. Comparison of the non-goal-line data of Figure 9 with Figure 3 suggests that the intrinsic dynamics observed in Experiment 1 were replicated in Experiment 3.

As stated earlier, the primary purpose of conducting Experiment 3 in a manner similar to that of Experiment 2 was to facilitate the comparison between a dynamics whose essential structure is unknown (the task in Experiment 3) with a dynamics whose essential structure seems to be reasonably well understood (the task in Experiment 2). Furthermore, whereas the task in Experiment 2 had a definite additional force provided by a resistive "wall," the corresponding force in Experiment 3 was fictive. As identified earlier, a major hypothesis from the dynamical theory of behavioral patterns is that information about required distance should function within the order parameter dynamics much like the resistive goal surface of Experiment 2 (i.e., as a force influencing the behavior of ϕ). A comparison of Figure 9 with Figure 8 suggests the following about the order parameter dynamics of the present task: The hypothesized potential function associated with the nonresistive goal line (environmental information) was symmetrical, similar to the potential function associated with the impact force of Experiment 2. However, unlike the symmetrical extrinsic potential in Experiment 2 (see Equation 17), the symmetrical potential function for the environmental information of Experiment 3 was of the opposite sign for the two goal line distances. Specifically, we can hypothesize that the full potential function for the goal line task, when $\Delta\omega = 0$, is of the same form as Equation 18:

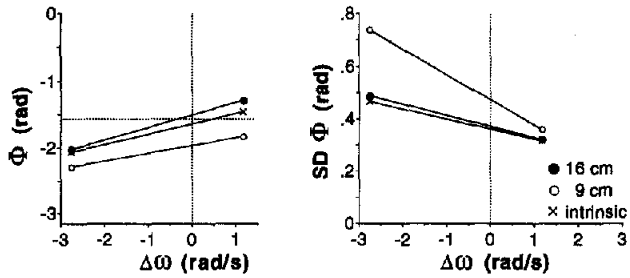

Figure 9. Stable relative phase (left) and standard deviation of relative phase (right) as a function of $\Delta\omega$ and distance of goal line in Experiment 3.

$$V(\phi) = k \sin(\phi) - g \cos(\phi), \tag{21}$$

with the difference being that the extrinsic coefficient g can assume positive and negative values. When $g > 0$, the order parameter dynamics are drawn away from $-\pi/2$ toward zero (in phase); when $g < 0$, the order parameter dynamics are drawn away from $-\pi/2$ toward $-\pi$ (antiphase). By inference, $g > 0$ defines the extrinsic potential associated with the 16-cm goal line and $g < 0$ defines the extrinsic potential associated with the 9-cm goal line. What is clear from Figure 9 is that the stable states for the 16-cm goal line were closer to zero than those of the corresponding nongoal line (or intrinsic dynamics) and, conversely, the stable states of the 9-cm condition were closer to $-\pi$ than those of the corresponding nongoal line (or intrinsic dynamics).

From Equation 21, the motion equation follows as the negative derivative with frequency competition and noise added:

$$\dot{\phi} = \Delta\omega - k \cos \phi - g \sin \phi + \sqrt{Q}\zeta_t. \tag{22}$$

Setting $k = 5$ (the standard parameter value of the intrinsic dynamics derived from Experiment 1), and setting $g = 2$ for the 16-cm condition and $g = -1.5$ for the 6-cm condition, produced the dependence of ϕ on $\Delta\omega$ as a function of goal line distance shown in Figure 10 (left side) and the dependence of $SD\phi$ on $\Delta\omega$ as a function of goal line distance shown in Figure 10 (right side). The motivation for the values of g was twofold. First, we assumed that the influence of a soft constraint such as information would be less than a hard mechanical constraint (the resistive goal surface); therefore, we chose g values to be smaller than the k_2 values used in implementing Equation 15. Second, we assumed that achieving the farther goal line would involve more force than the nearer goal line.

The similarities between the corresponding conditions of Figure 10 and Figure 9 lend support to the proposal that environmental information in our simple bat–ball skill can be formulated as a potential function. The coefficient g on the extrinsic potential is a continuous function of the goal line distance. A continuous increase in the goal line, from the zero distance of the static rest position of the ball pendulum to the farthest horizontal distance D (the ball pendulum length), moves the order parameter from $-\pi$ through $-\pi/2$ to zero. Clearly, from Equations 21 and 22, $g = 0$ when the goal line distance is equal to the horizontal distance that the ball achieves when batted without a specific behavioral goal (intrinsic dynamics). Therefore, the intrinsic dynamics set the origins of the coordinate system within which g relates to goal distance. Distances less than this intrinsically defined zero will mean $g < 0$ and a tendency for the equilibrium state to enter the range $(-\pi, -\pi/2)$; distances greater than this intrinsically defined zero will mean $g > 0$ and a tendency for the equilibrium state to enter the range $(-\pi/2, 0)$.

General Discussion

The major goal of our research was to examine the dynamical theory of behavioral patterns advanced by Schöner and Kelso (1988a, 1988b, 1988c) within the domain of bat–ball coordinations. Results of the three experiments show that this theory addressed both the stability and variability of the behavioral patterns observed within the simple bat–ball task of Figure 1. Specifically, in agreement with the theory (a) the observed bat–ball coordination patterns were characterized successfully by a coordination- or task-specific collective variable; (b) the dynamics of this collective variable—its equation of motion—predicted successfully the changes in the bat–ball coordination patterns; (c) the intrinsic dynamics of the bat oscillator and ball oscillator, and of the system they formed when coupled, proved to be integral to the bat–ball coordination dynamics in different environmental settings; (d) a hard surface functioning as a wall, and the forces it introduced, were defined successfully as parameters in the same space as the collective variable characterizing the intrinsic bat–ball coordination pattern; and (e) a nonresistant goal line functioning as information, and the fictive forces it introduced, also could be accommodated as a parameter in the same space as the collective variable characterizing the intrinsic bat–ball coordination pattern. The importance of the last item is that it lends support to the argument that information connects closely with dynamics and that information's properties, when properly construed for biological action–perception systems, are dynamical in a nontrivial sense of the word (Beek, Turvey, & Schmidt, 1992; Kelso, 1994; Kugler, Kelso, & Turvey, 1980, 1982; Kugler & Turvey, 1987; Schöner & Kelso, 1988c).

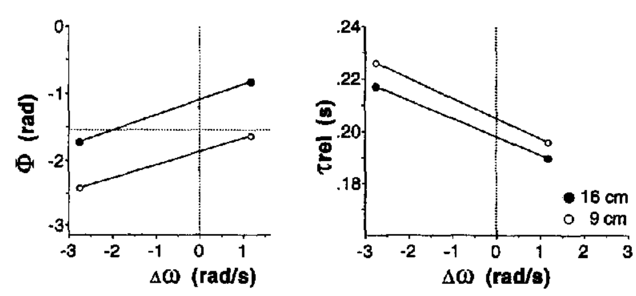

Figure 10. Predictions of stable relative phase (left) and τ_{rel} (right) from Equation 21 for the conditions of Experiment 3.

As underscored in the introduction, the task investigated in this research constitutes a minimal example of bat–ball skills. Within the classification schemes of Poulton (1957) and Gentile (1972), such skills tend to be "open," meaning that they are performed in an uncertain, changing environment. In tennis, for example, both the movements of the opposing player and the trajectories of the ball vary dramatically. Unlike tennis, the variations in our task were severely limited. The participant could always expect the ball's trajectories to reside in a well-circumscribed spatial region and to exhibit well-defined temporal characteristics. Consequently, our bat–ball task might be considered more of a closed skill than an open skill. Note, however, that there are bat–ball skills outside the laboratory that closely approximate the features of our task. As noted earlier, the simple bat–ball skill under study in Experiments 1–3 was inspired by the child's game of repetitively batting a ball against a wall after a bounce. Additionally, there are set practices in the games of tennis, baseball, and cricket, in which a ball is projected by a machine at a fixed interprojection interval and with fairly well-constrained kinematics. Although these latter cases are at some remove from the full-fledged games that they partially simulate, it is nonetheless true that each provides a challenge for the general theory of human perception–action capabilities. Consequently, in relation to our second goal, our research identified the simple bat–ball task of Figure 1 as a model experimental system whose coordination dynamics are both open to systematic investigation and a potential source of insight into the abstract coordination principles governing the ability to execute controlled strikes of a ball.

Overall, we think that our research, the research of Beek and his colleagues (Beek, 1989a, 1989b; Beek & Turvey, 1992; Beek & van Santvoord, 1992), and the research of Schaal, Sternad, and Atkeson (1996) is encouraging with respect to the development of a dynamical theory of ball skills. Consistent with the ideas of Schöner and Kelso (1988a, 1988b, 1988c), the themes identified in items (a) to (e) above expressed in their most general form could be the cornerstones of such a theory.

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